Signaling

- An agent may be willing to reveal private information if he obtains greater utility
- Will not want to reveal information if the signal is too costly

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Signaling Games

- Nature draws a type t_i for the Sender from a set of feasible types $T=\{t_1,\ldots,t_I\}$ according to a probability distribution $p(t_i)$, where $p(t_i)>0$ for every i and $p(t_1)+\ldots+p(t_1)=1$.
- The Sender observes t_i and then chooses a message m_j from a set of feasible messages $M=\{m_1,\ldots,m_J\}$
- The Receiver observes m_j (but not t_i) and then chooses an action a_k from a set of feasible actions $A=\{a_1,\ldots,a_K\}$.
- Payoffs are given by $U_s(t_i,m_j,a_k)$ and $U_R(t_i,m_j,a_k)$

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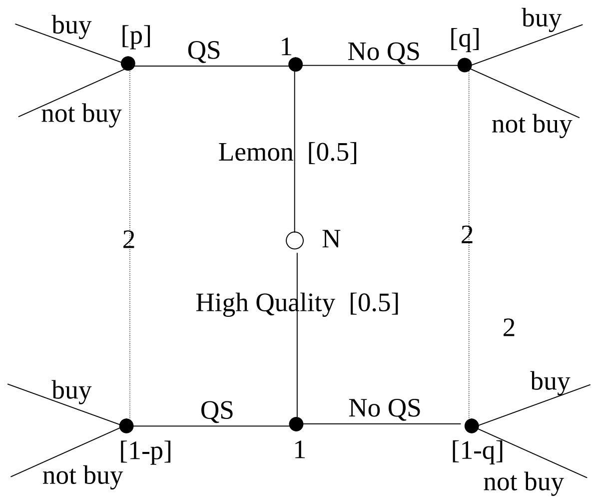
Signaling Example



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Signaling Game Pure Strategies

- Sender:
  - Strategy 1: Play m_1 if nature draws either type
  - Strategy 2: Play m_1 if nature draws t_1 and play m_2 if nature draws t_2
  - Strategy 3: Play m_2 if nature draws t_1 and play m_1 if nature draws t_2
  - Strategy 4: Play m_2 if nature draws either type
- Receiver
  - Strategy 1: Play a_1 no matter what message the Sender chooses
  - Strategy 2: Play a_1 if the Sender chooses m_1 and play a_2 if the sender chooses m_2
  - Strategy 3: Play a_2 if the Sender chooses m_1 and play a_1 if the sender chooses m_2
  - Strategy 4: Play a_2 no matter what message the Sender chooses

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Signaling Requirements

- Requirement 1: After observing any message m_j from M, the Receiver must have a belief about which types could have sent m_j. Denote this belief by the probability distribution $\mu(t_i|m_j)$, where $\mu(t_i|m_j) \geq 0$ for each t_i in T, and $\sum_{t_i \in T} \mu(t_i|m_j)=1$.

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Signaling Requirements

- Requirement 2R: For each m_j in M, the Receiver's action $a^*(m_j)$ must maximize the Receiver's expected utility, given the belief $\mu(t_i|m_j)$ about which types could have sent m_j. That is, $a^*(m_j)$ solves
  $$\max_{a_k \in A} \sum_{t_i \in T} \mu(t_i|m_j)U_R(t_i,m_j,a_k).$$
- Requirement 2S: For each t_i in T, the Sender's message $m^*(t_i)$ must maximize the Sender's utility, given the Receiver's strategy $a^*(m_j)$. That is, $m^*(t_i)$ solves
  $$\max_{m_j \in M} U_S(t_i,m_j,a^*(m_j)).$$

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Signaling Requirements

- Given the sender's strategy, let T_j denote the set of types that send the message m_j
- Requirement 3: For each m_j in M, if there exists t_i in T such that $m^*(t_i)=m_j$, then the Receiver's belief at the information set corresponding to m_j must follow from Bayes' rule and the Sender's strategy:

  $\mu(t_i|m_j)=p(t_i)/\sum_{t_i \in T_j} p(t_i)$.
- Definition: A pure-strategy perfect Bayesian equilibrium in a signaling game is a pair of strategies $m^*(t_i)$ and $a^*(m_j)$ and a belief $\mu(t_i|m_j)$ satisfying Signaling Requirements (1), (2R), (2S), and (3).

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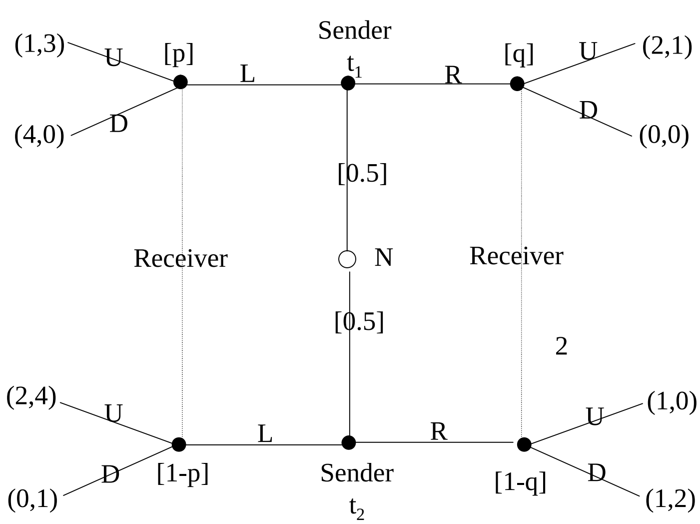
Example



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Possible Pure Strategy PBE

1) Pooling on L
2) Pooling on R
3) Separation with t_1 playing L and t_2 player R
4) Separation with t_1 playing R and t_2 player L

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Pooling on L

- Suppose a Sender's strategy is (L,L)
  - Receiver's belief (p,1-p) is on the equilibrium path, and her belief at this information set is determined by Bayes' rule and the Sender's strategy: p = .5
  - Receiver's best response on the equilibrium path is to play U
  - Sender earns 1 (if t_1) or 2 (if t_2)
- How would Receiver react to R?
  - If Receiver plays U to R, then Sender type t_1 gets 2, which is better than he got by playing L.
  - If Receiver plays D to R, then Sender is better off by playing L instead
  - So if an pooling PBE (L,L) exists, Receiver must play (U,D) to L and R.

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Pooling on L

- When will a Receiver Play D to R?
  - She plays D if Π(D) ¸ Π(U), or q(0) + (1-q)(2) ¸ q(1) + (1-q)(0) → q · 2/3
- So [(L,L),(U,D),p=0.5,q] is a pooling PBE for any q · 2/3

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Pooling on R

- Suppose a Sender's strategy is (R,R)
  - Then q = 0.5
  - Receiver's best response to R is D
  - Sender earns 0 (t_1) or 1 (t_2)
- But Sender type t_1 could earn 1 by playing L (knowing Receiver's best response is U for any value of p)
- So there is no equilibrium in which the Sender plays (R,R)

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Separation with t_1 playing L

- If Sender plays (L, R) then both of the Receiver's information sets are on the equilibrium path
  - So p = 1 and q = 0
  - Receiver's best response to these beliefs are U and D respectively
  - Both Sender types earn 1
- Given this Receiver response, is the Sender strategy optimal?
  - If type t_2 deviates by playing L instead, then the Receiver responds with U, earning t_2 a payoff of 2
- This separation equilibrium does not exist

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Separation with t_1 playing R

- If Sender plays (R, L)
  - p = 0 and q = 1
  - Receiver's best response is (U, U)
  - Both Sender types earn 2
- If t_1 were to deviate by playing L
  - Receiver responds with U
  - t_1's payoff is 1
  - So no incentive to deviate
- If t_2 were to deviate by playing R
  - Receiver responds with U
  - t_2's payoff is 1
  - So no incentive to deviate
- So [(R,L),(U,U),p=0,q=1] is a separating PBE

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