Pricing, Monopolists, and Discrimination

ISyE 6230

Single-Product Monopolist

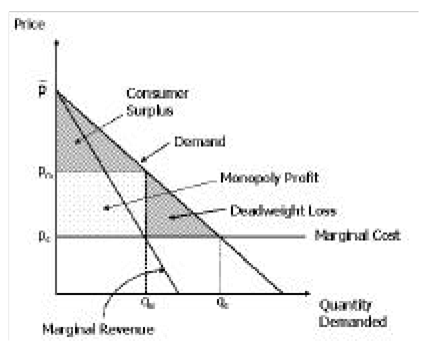
- Demand is $q=D(p)$;
  Inverse demand function $p = P(q)$
  - Assume $D'(p) < 0$ and Revenue concave
- $C(q)$ is cost of producing q units
  - $C'(q) > 0$
- Monopolist
  - $\max_p [pD(p) - C(D(p))]$

Single-Product Monopolist

- First order conditions:
  - p^m – $C'(D(p^m)) = -D(p^m)/D'(p^m)$
  - Or $(p^m - C')/p^m = 1/\varepsilon$ (1)
    - Where $\varepsilon = -D'p^m/D$ is demand elasticity at the monopoly price p^m
  - Lerner Index = relative "markup"
- (Equivalently, if q^m ´ $D(p^m)$,
  - $MR(q^m)$ ´ $P(q^m)+P'(q^m)q^m = C'(q^m)$)
- Implications?

Definitions

- Consumer surplus:
  - What customers would pay in excess of what they already spend
- Total surplus = consumer + supplier
- Social welfare = consumer surplus + firm's profit
- Dead-weight loss:
  - Reduction in benefit due to inefficient allocation of resources



Elasticity

- Given a function B = f(A...)
- Elasticity of B with respect to A ($e_{B,A}$)
  = % change in B / % change in A
  = (**M**B/B)/(**M**A/A) = $(\partial B/\partial A) * (A/B)$

Elasticity

- Price Elasticity of Demand ($e_{Q,P}$)
  - $e_{Q,P} = (\partial Q/\partial P) * (P/Q)$
  - 3 regions:
    - $e_{Q,P} < -1$ elastic
    - $e_{Q,P} = -1$ unit elastic
    - $e_{Q,P} > -1$ inelastic
- If we raise price by 1%, then what happens to demand?

Elasticity

- Income Elasticity of Demand
  - $e_{Q,I} = (\partial Q/\partial I)*(I/Q)$

- If $e_{Q,I}$=2.0 for auto, then a 10% increase in income will lead to what?

Elasticity

- Cross-Price Elasticity
  - How does quantity demand change with respect to price change in another good?
  - $e_{Qi,Pj} = (\partial Q_i/\partial P_j)*(P_j/Q_i)$
  - $e_{Qi,Pj} > 0$ → i and j are substitutes
  - $e_{Qi,Pj} < 0$ → i and j are complements
  - Examples?

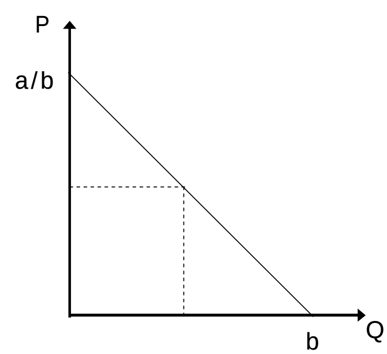
Representative Income & Price Elasticities

Item	Income Elasticity	Price Elasticity
Food	0.28	-0.21
Auto	3	-1.2
Electricity	0.61	-1.14
Medical Services	0.22	-0.2
Charitable Giving	0.71	-1.29
Gasoline	1.06	-0.54
Housing: rent	1	-0.18
Housing: ownership	1.2	-1.2

- Necessities versus luxury items?
- Price elasticies?

Linear Demand Curve

- Linear demand → $Q = a + bP$ $(b < 0)$
- $e_{Q,P} = (\partial Q/\partial P) * (P/Q)$
  $=$
- Implications?



Alternative Demand Curve

- Suppose that $Q = aP^b$, $b < 0$
  - $e_{Q,P} = (\partial Q/\partial P)*(P/Q) = (baP^{b-1})(P/(aP^b))$
  - We also have log Q = log a + b log P
- Generalized version:
  - $X = aP_x^b P_y^c I^d$
  - $\log X = \log a + b \log P_x + c \log P_y + d \log I$
  - $e_{X,Px}=b$, $e_{x,Py}=c$, $e_{x,I}=d$

Estimating Demand Curves

- Run experimentation
  - Change prices and see how demand changes
  - Issues:
    - Do you have representative customers?
    - Can you deal with confounding factors?
- Use historical data
  - Past sales
  - Past economic data
  - Issue:
    - Can you account for confounding issues (e.g., growth, income changes)?

Multi-Product Monopolist

- Produces goods i = 1,…,n
- Charges prices $p=(p_1,\dots,p_n)$
- Sells quantities $q=(q_1,\dots,q_n)$ where $q_i = D_i(p)$ is the demand for good i
- Cost of producing is $C(q_1,\dots,q_n)$

Multi-Product Monopolist

- Maximizes
  - $\sum_{i=1}^{n} p_i D_i(p) - C(D_1(p),\dots,D_n(p)) \rightarrow$

$$\left(D_i + p_i \frac{\partial D_i}{\partial p_i}\right) + \sum_{j\neq i} p_j \frac{\partial D_j}{\partial p_i} = \sum_j \frac{\partial C}{\partial q_j}\frac{\partial D_j}{\partial p_i} \quad \text{for all i} \qquad (2)$$

- If $C(q_1,\dots,q_n) = \sum_{i=1}^{n} C_i(q_i)$, and let R_i ´ $p_i D_i$, then algebra gives us:

$$\frac{p_i - C_i'}{p_i} = \frac{1}{e_{ii}} - \sum_{j\neq i} \frac{(p_j - C_j')D_j e_{ij}}{R_i e_{ii}}$$

- where ε_{ii} ´ $-(\partial D_i/\partial p_i)(p_i/D_i)$ & ε_{ij} ´ $-(\partial D_j/\partial p_i)(p_i/D_j)$

Multi-Product Monopolist

- Multi-product monopolist compared to before?
- Goods that are substitutes ($\varepsilon_{ij} < 0$)
  - (An increase in p_i raises demand for good j)
  - RHS side is now bigger than before, so Lerner exceeds inverse of its own elasticity
- Goods that are complements ($\varepsilon_{ij} > 0$)
  - (A decrease in p_i raises the demand for good j)
  - RHS side is now smaller than before, so Lerner index is less than inverse of own elasticity of demand
  - Could even sell some goods below marginal cost (→ Lerner index < 0) so as to raise demand for other goods!

Price discrimination

- Same commodity sold at different prices to different consumers
  - Entrance ticket to Disneyland
  - (But not always…freight charges may not be!)
- These price differences cannot be explained by the difference in the marginal cost of making the goods available for the various consumers
- An economic good is defined not only according to its physical properties but also to the space, time, and state of the world at which it is available for consumption (Debreu 1959)

Necessary for Price-Discrimination

- Firm must have some market power
- Firm controls the sale of its products
  - Secondary markets make this more difficult
- Consumers should have heterogeneous utilities from the good
  - I.e., different price elasticities of demand

First-degree (Perfect) Price Discrimination

- Seller charges each individual consumer his reservation price
- Seller must know how much the consumer is willing to pay
- Examples?

Second-degree discrimination

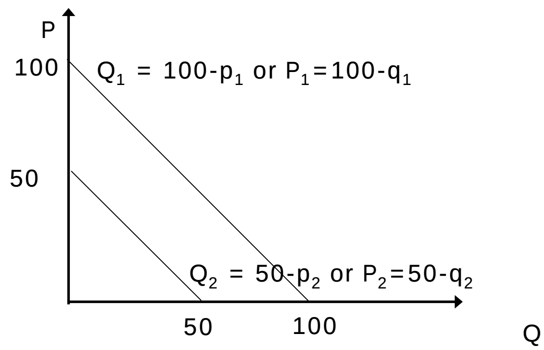
- Firm makes n separate prices and offers them (all) to each group (who purchase at a price smaller or equal to their reservation prices
  - 1st, 2nd, or 3rd railway tickets
- Other ways to achieve this are through quantity discounts and two-part tariffs
- Each consumer "self-selects" by their own choice

Third-degree discrimination

- Seller separates consumers into groups and offers the monopoly price to each class
- Uses a direct signal about demand
- Examples?

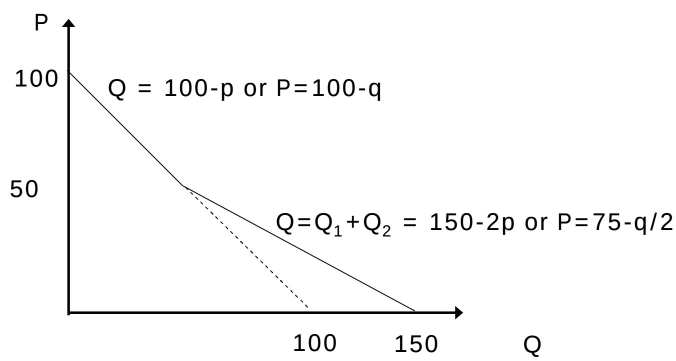
Third-degree Example

- Firm can distinguish customer segments
  - $P_1(q_1) = 100 - q_1$
  - $P_2(q_2) = 50 - q_2$
- $C(q) = cq$ where $c=\$0$



Third-degree Example

- If monopolist cannot price discriminate on type:



Third-degree Example

- Uniform price in the market:
  - If sell with uniform price to both customer types
    - $\max_p pD(p) = p(150-2p) = 150p-2p^2$
    - → $p^m = 75/2$, $q^m = 75$, $\Pi^m = 2812.5$
  - If sell with uniform price to only p=100-q customers
    - → $p^m = 50$, $q^m = 50$, $\Pi^m = 2500$
- Suppose can price discriminate
  - $p^1 = 50$, $q^1 = 50$, $\Pi^1 = 2500$
  - $p^2 = 25$, $q^2 = 25$, $\Pi^2 = 625$
  - → $\Pi^{total} = 3725$
- Can monopolist ever be worse under 3rd degree PD?

Third-degree Multi-market

- Monopolist charges linear tariff for each market,
  - $\{p_1, \ldots, p_i, \ldots, p_m\}$
  - $\{q_1 = D_1(p_1), \ldots q_i, \ldots, q_m\}$
  - $Q = \sum_{i=1}^{m} D_i(p_i)$
- Monopolist
  - $\max_p [\sum_{i=1}^{m} p_i D_i(p_i) - C(\sum_{i=1}^{m} D_i(p_i))]$
  - → $(p_i - C'(q))/p_i = 1/\varepsilon_i$
  - Implications?

Second-degree Discrimination

- Monopolist cannot tell customers apart but can offer different bundles
- Needs to make sure that consumer does not choose a bundle directed towards another customer

- Examples?
- How would he do this?

Second-degree Discrimination

- $T(q) = A + pq$ is two-part tariff
- Customer i pays T_i and consumes q_i
- Customer utility is $\theta_i V(q) - T_i$ if they pay T_i and consume q units; 0 otherwise
  - $V(0) = 0$, $V'(q) > 0$, $V''(q) < 0$
- λ is probability customer is of type 1

Second-degree Discrimination

- Monopolist:
  - $\max_{q1,T1,q2,T2} \lambda(T_1 - q_1 C) + (1-\lambda)(T_2 - q_2 C)$
- s.t.
  - $\theta_1 V(q_1) - T_1 \geq 0$ (IR$_1$)
  - $\theta_2 (V(q_2) - T_2 \geq 0$ (IR$_2$)
  - $\theta_1 V(q_1) - t_1 \geq \theta_1 V(q_2) - t_2$ (IC$_1$)
  - $\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1$ (IC$_2$)
- We know that IR_1 and IC_2 are active and the others are not
- Solve the problem as before with similar insights

Tie-in Sales as Price Discrimination

- Tie-in sales:
  - “basic” good consumed in a fixed quantity
  - a complementary good that may be consumed in variable quantities
  - Examples?
- Insight:
  - Price of the complementary (tied) good is higher under a tie-in sale
  - Price of the basic good is lower
- Tie-in sale reduces welfare as long as the manufacturer serves both types of customers, but prohibiting tie-in sales make it more likely the manufacturer serves only the best consumer

Commodity Bundling

- Pure Commodities
  - Firm sells each product separately
- Pure Bundling
  - Firm offers only bundles
- Mixed Bundling
  - Firm sells products separately as well as bundled together

- Examples?

Commodity Bundling

- Pure bundling can result if there are economies of scale
- Otherwise, mixed bundling seems optimal but is not always

Resources

- The Theory of Industrial Organization (Jean Tirole)
- The Economics of Price Discrimination (Louis Phlips)