The Effects of Customer Rebates and Retailer Incentives on a Manufacturer's Profits and Sales*

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Abstract

In some industries such as automotive, production costs are largely fixed and therefore maximizing revenue is the main objective. Automotive manufacturers use promotions directed to the end customers and/or retailers (dealers) in their distribution channels to increase sales and market share. We study a game theoretical model to examine the impact of "retailer incentive" and "customer rebate" promotions on the manufacturer's pricing and the retailer's ordering/sales decisions. The main tradeoff is that customer rebates are given to every customer, while the use of retailer incentives are controlled by the retailer. We consider several models with different demand characteristics and information asymmetry between the manufacturer and a price discriminating retailer, and determine which promotion would benefit the manufacturer under which market conditions. When demand is deterministic, we find that retailer incentives may increase the manufacturer's profits (and sales) while customer rebates do not unless they lead to market expansion. When the uncertainty in demand ("market potential") is high, a customer rebate can be more profitable than the retailer incentive for the manufacturer. We provide additional insights through numerical examples.

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1 Introduction

Much of supply chain management focuses on matching supply and demand through various means such as production flexibility, lead-time management, and channel coordination. In recent years, there has also been a particular focus on manipulating demand through the use of dynamic pricing or promotions to achieve higher profits, e.g., Elmaghraby and Keskinocak [15] and Kim et al. [23]. This is especially true in industries such as automotive and airlines, where significant capacity changes are difficult and costly to do. Our goal is to determine under what market conditions (such as demand uncertainty in market potential and price sensitivity) offering customer rebates versus retailer incentives or combined promotions are more profitable or lead to higher sales for the manufacturer. This problem lies at the interface of marketing and operations management.

The main motivation for our research comes from the practices in the automotive (auto) industry and our discussions with a major auto manufacturer. The production and labor costs of American auto manufacturers are nearly fixed in part due to union costs, and hence, they focus on increasing revenues, e.g., through the use of promotions. Jakobson [22] states that "Detroit's costs are roughly the same whether a plant is churning out as many cars as it can or standing idle part of the time - so the Big Three produce more cars than their market share justifies, creating gluts that force them to offer large cash incentives to move the excess." Busse et al. [9] mention the auto manufacturers' rigid pricing strategy and state that "Although retail demand for an automobile fluctuates due to changing economic conditions, seasonality, and the stage of the model's life cycle, manufacturers rarely vary published retail and invoice prices of a particular model over the course of the model year." The authors also mention that the auto manufacturers use the "incentive promotions" as an important market strategy tool to respond to fluctuating demand conditions.

Promotions have been frequently used by manufacturers in different industries as a means to increase sales, revenues and market share against competitors by increasing consumers' awareness about their brand, to reduce the inventories of the slow-moving items, or to price discriminate. In the auto industry, different promotional programs such as customer rebate, low percent financing, and employee pricing programs, are offered. In addition to these direct-to-customer promotions, auto manufacturers sometimes offer incentives to the retailers that are generally not publicized to the end customers. Retailer incentives may encourage the dealers to advertise or negotiate with their customers to generate more sales.

As stated by Priddle and Zoia [29], promotions in the auto industry date back to 1912 with rebates offered by Henry Ford on Model Ts. Other developments include the introduction of Chrysler's rebate program (1975) and General Motors' 0% financing program (2001). Auto manufacturers offered $56 billion in incentives to sell 16.6 million cars and trucks in 2003 (Smith [32]) and the average rebate per model offered by the American auto manufacturers was more than triple that of the Japanese auto manufacturers in 2004 (Henderson [21]). In fact, American auto manufacturers are known to use frequent and deep customer rebates or cashback, whereas the non-Americans, especially, Japanese, are more inclined to offer incentives to their dealers. Figure 1, based on data sets provided by R. L. Polk & Co. and a major market research firm, shows that the rebate percentages used by the American auto manufacturers are higher and show an increasing trend compared to the Japanese auto manufacturers who offer steadily lower rebates. The promotional choice may depend on the demand characteristics faced by the manufacturers.

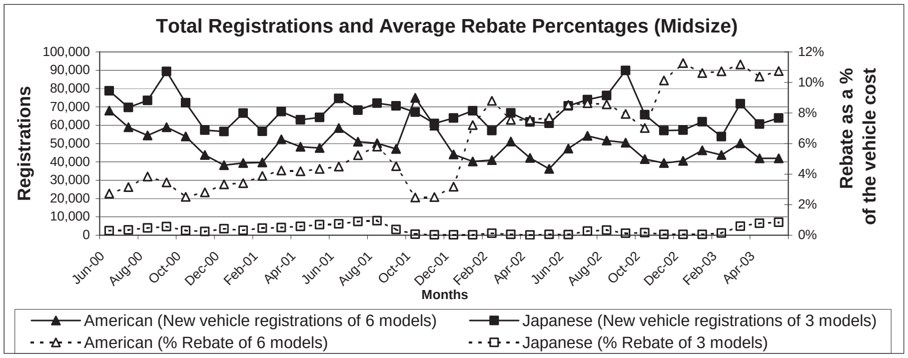


Figure 1: Total registrations and average rebate percentages of midsize models

It is essential to know which promotion provides higher sales and profits under what kind of market conditions. In this paper, we examine these questions by focusing on two different promotions, namely, the *retailer incentive* and the *customer rebate*. In the former, the manufacturer offers a (lump-sum) incentive to the retailer which can be used in a flexible way by the dealer; in the latter, the manufacturer offers a per-unit customer rebate directly to the end customers. Dealer incentives are offered in different forms by manufacturers, e.g., as lump-sum or per-unit incentive for each vehicle sold. Referring to lump-sum incentives Edmunds.com president Jeremy Anwyl states "here is $100,000 for the dealer to use as he sees fit" (Anwyl [2]). By contrast, in per-unit incentive scheme, the dealer usually receives a higher payment for each additional vehicle sold beyond a quota set by the manufacturer. The basic tradeoff is that customer rebates are given to every customer, while the retailer controls the use of the retailer incentive and decides how much to give to each customer. While customer rebates are widely publicized, dealer incentives are not. The retailers can also offer their own promotions mostly in the form of customer rebates, but these are generally offered locally and at smaller amounts; thus they are outside the scope of our study.

This paper is organized as follows. We provide a literature review in Section 2, followed by our models, their analysis and comparisons in Section 3. We state our conclusions in Section 4.

2 Literature Review

Several articles on promotions focus on their role as a price discriminating tool, considering single or multiple firms selling directly to the end customers with no intermediary in between. (e.g., Gerstner and Holthausen [18] and Narasimhan [27].) Gerstner and Hess [16] introduce channel issues by analyzing different types of promotions (trade deals and rebates) in a single manufacturer and single retailer setting, where the market has two customer segments with high and low reservation prices. They find that even when all consumers exercise their rebates and price discrimination does not occur, the manufacturers may still find it profitable to offer rebates.

The retailer pass through rate (the percentage of a trade promotion passed through to consumers) and forward buying (or stockpiling) are two issues stemming from the use of promotions directed towards retailers. Lal et al. [26] analyze the motivations for manufacturers to offer promotions when the retailers forward buy and do not pass the price discounts to the customers. Kim and Staelin [24] analyze manufacturer allowances (similar to our lump-sum incentives) and retailer pass-through rates in a supply chain with two manufacturers selling through two non-exclusive retailers. In their model, the lump-sum payments are used by the retailers as price discounts for each unit of sales. They find that the manufacturers give retailers side payments (allowances) even though they know the retailers will pocket some portion of it. This finding is mainly a result of the competition among the manufacturers.

In recent years, there have been some extensions within the framework of supply chains analyzing sales promotions as a tool for channel coordination including Gerstner and Hess [17] and Krishnan et al. [25]. Chen et al. [12] and Aydin and Porteus [3] analyze the effects of rebates on the manufacturer's and retailer's profits in a 2-stage supply chain with stochastic demand. Chen et al. [12] show that rebates always benefit the manufacturer unless all of the buyers redeem their rebates; otherwise they do not necessarily increase the manufacturer's profits. Aydin and Porteus [3] compare per-unit retailer rebate and per-unit customer rebate. The authors conclude that neither the manufacturer nor the retailer always prefers one particular rebate to the other. Sohoni et al. [33] analyze the effects of dealer incentives on sales variability focusing on the auto industry and show that manufacturers may increase profits and decrease sales variability by offering an appropriate stair-step dealer incentive when their dealer is exclusive. Biller et al. [5] are also motivated by the auto industry but their focus is on the manufacturers' capacity and flexibility investment decisions where the authors analyze a delayed pricing strategy to cope with uncertainty. Manufacturers' flexible capacity decisions were also investigated empirically by Goyal et al. [20] in the auto industry.

Motivated by practices in the auto industry, our work differs from the cited articles in that the retailer can price discriminate rather than choose a fixed retail price for all customers, which we believe captures the nature of sales by the auto dealers. We analyze retailer incentives that are in the form of lump-sum amounts rather than a wholesale price deduction, motivated by the practices of auto manufacturers who generally keep wholesale prices constant for the model year and offer periodic incentives to the dealers.

Bruce et al. [7, 8] analyze trade promotions (wholesale discounts after a sales quantity target) and cash rebates in the durable goods market, such as automobile, by explicitly incorporating a durability measure for the manufacturer's products to focus on the intertemporal effects of the promotions. In the former research, the authors find in a competitive setting that the manufacturer of the more durable product benefits more from trade promotions. In the latter research, they analyze cash rebates in a single manufacturer and single retailer setting similar to ours. They find that as the durability of the manufacturer's products decreases, the manufacturer finds it more profitable to offer deeper cash rebates. Our research differs from [7, 8] in several aspects. We focus on the effect of demand uncertainty on the manufacturer's choice of promotions, while they focus on the durability of the manufacturers' products. In their analysis, the wholesale price of the manufacturer is contracted simultaneously with the promotions in a deterministic setting, while we allow the wholesale price to be determined under uncertainty. Further, our work focuses on comparing the performances of the customer rebates and retailer incentives in the same model setting where the retailer can price discriminate.

There is a large body of empirical research investigating how promotions work, focusing mostly on non-durables. See Blattberg et al. [6] for a review. Some empirical analysis in durables, such as automobiles includes Busse et al. [9] and Pauwels et al. [28].

In our research, we focus mainly on the manufacturer's wholesale price and promotion decisions and their impact on profits and sales in a decentralized decision framework where the retailer chooses the sales price (for each customer) and the sales quantity. We also contribute to the economics literature by analyzing first-degree (perfect) price discrimination, where different prices may be given to every customer rather than just to segments of customers, which has received little attention by researchers. To the best of our knowledge, Spulber [34] and White and Walker [35] are only two that analyze perfect price discrimination in detail. Spulber [34] analyzes a model where a group of firms that do perfect price discrimination select their output levels simultaneously. The author shows the existence of a unique non-cooperative equilibrium in output levels. In this analysis the firms sell directly to the end customers and no sales promotions are considered. White and Walker [35] analyze the case where there is a variable cost associated with perfect price discrimination. The authors propose a model where perfect price discrimination is selectively practiced only on one portion of the linear demand function. Their setting is simplistic, and they analyze only one firm's sales quantity decision, where the firm sells directly to the end customers, while we analyze sales quantity and promotional decisions in two-stage supply chains.

3 Models

We consider a two-stage supply chain with a single manufacturer and a single retailer, operating in a market where demand is characterized by a linear inverse demand function, i.e., $P(Q) = a - bQ$, where a is the market potential, b is the price sensitivity and $P(Q)$ is the price when Q units of the product are sold. (Note that $\frac{a}{b}$ is the maximum quantity that may be sold when the price is zero. Since we are interested in changing a while keeping b fixed, we denote a as the market potential for the rest of the paper.) The retailer (or dealer) buys from the manufacturer at a unit wholesale price denoted by w, and has a reservation price, $w + m$, below which he would not be willing to sell. We assume that the retailer can do perfect (first degree) price discrimination. While perfect price discrimination by the retailer can be costly, we do not expect this cost to differ significantly from one customer to another even when the customers differ in terms of their willingness to pay. Therefore, we assume that the cost of price discrimination is constant for each unit of sale. This allows us to absorb the price discrimination cost as part of the retailer's reservation price.

Our setting is applicable to channels where the retailer is a franchisee selling one manufacturer's products exclusively, e.g., the auto industry. Linear demand models have been used by major auto manufacturers in practice, e.g., Biller et al. [4]. In the auto industry, manufacturers usually commit to a constant wholesale price. (Cachon and Lariviere [10] mention that manufacturers hold wholesale prices constant even when capacity is scarce.) Our model mimics the practice, where a retailer's reservation price may depend on the car invoice price (Scott Morton et al. [30], and Zettelmeyer et al. [37]). In our setting, the reservation price of the retailer may be compensated by a combination of the sales price (which should be at least $w + m$ if there is no incentive) and the lump-sum incentive offered by the manufacturer.

The auto dealers have the opportunity to negotiate individually with each customer and collect information about the customer's willingness to pay. Zettelmeyer et al. [36] mention price discrimination as one reason for the retailers to negotiate the prices and state that "Given the high price of a new car, it would not be surprising if the cost of gaining information about a customer's willingness to pay is, in comparison, small enough to make the dealer's effort to assess a customer's valuation and negotiate individual prices more profitable than posting a fixed price." Goldberg [19] empirically finds that dealers can achieve price discrimination through the car model, market-specific properties, and the type of purchase transaction such as first-time purchase and trade-in. Scott Morton et al. [31] add that individual characteristics of car buyers, such as income, education, and search costs are significant factors that affect the dealers' pricing of the cars. Price discrimination through negotiations may also be observed during purchases and sales of houses or other less expensive products through bidding over the internet.

We assume that the manufacturer has ample capacity, and there is no cost to customers associated with rebate redemptions. Since we are interested in comparing the performances of different promotions, we also assume that the administrative cost of offering promotions is zero. We analyze a single selling season; this can be thought of as snapshot in time of a repeated process. Finally, we assume that the manufacturer and the retailer are risk neutral, and both seek to maximize their own (expected) profits.

Table 1 summarizes our notation. Note that the manufacturer cannot make positive profits unless she sells above cost, therefore she chooses $w \geq c$. By the retailer's reservation price, $P(0) \geq w + m$ must hold, which together with $w \geq c$, implies that $a \geq c + m$.

Table 1: Notation

a^j : Market potential in demand state $j = l, h$ ($l =$ low, $h =$ high)
b^j : Price sensitivity of customers in demand state $j = l, h$
Q_i^j : Retailer's order/sales quantity in demand state $j = l, h$ when promotion type $i \in (o, I, R, R', C)$ is used ($o =$no promotion, $I =$retailer incentive, $R =$customer rebate, $R' =$customer rebate leading to market expansion, $C =$combined)
$P(Q)$: Retail price when Q units are sold
$\Pi_i^{M_j}$: Profit of the manufacturer under promotion i in demand state j (Π_i^M when demand is deterministic)
$\Pi_i^{D_j}$: Profit of the retailer under promotion i in demand state j when manufacturer makes her w decision under uncertainty (Π_i^D when demand is deterministic)
Π_i^{SC} : Profit of the supply chain under promotion i
c : Production cost of the manufacturer
w_i : Wholesale price under promotion type i
$w_i + m$: Reservation price of the retailer under promotion i
K^j : Lump-sum incentive given to the retailer in demand state j
R^j : Per unit customer rebate in demand state j

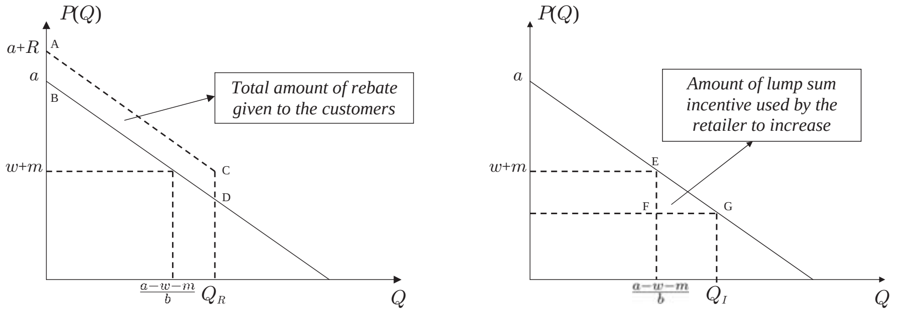
We analyze three demand settings. In Section 3.1, we present a deterministic model where there is no uncertainty in the system parameters. (Deterministic demand models have been commonly used in the literature to provide insights and for analytical tractability; see for example, Choi [13] and Corbett and Karmarkar [14].) In Sections 3.2 and 3.3, we introduce uncertainty ("high" with probability β and "low" with probability $1-\beta$) to the market potential (a) and price sensitivity (b) parameters of the demand function, respectively. This type of uncertainty could correspond to scenario planning as is used by many manufacturers in strategic level decisions.

We formulate all models as Stackelberg games, where the manufacturer is the leader and the retailer is the follower. We use backward induction to find the subgame-perfect Nash equilibrium (SPNE). We find the optimal decisions for the manufacturer and the retailer for each model and compare the promotions based on the manufacturer's profit and total sales. We also analyze the profits of the retailer and the supply chain, but our focus is on the manufacturer's choice of promotions. Our goal is to determine situations under which one promotion type is better than another for the manufacturer.

3.1 Deterministic Demand Model

In this benchmark setting, demand is deterministic and is common knowledge to both the manufacturer and the retailer. The sequence of decisions starts with the manufacturer determining the wholesale price. Given the manufacturer's decision, the retailer then decides the order/sales quantity.

When the manufacturer offers customer rebate or retailer incentive, Figure 2 describes how we would expect these promotions to affect the retailer's decisions. In the case of no promotion, the retailer has no incentive to sell at a price less than $w+m$, where the corresponding order/sales quantity is $Q_o = \left(\frac{a-w-m}{b}\right)^+$. The manufacturer's purpose in offering promotions is to induce the retailer to order more than Q_o. If she offers customers a per-unit rebate of R applied to each buyer's price, this has the effect of shifting the demand function up, i.e., $P(Q)-R = a-bQ$ or $P(Q) = (a+R)-bQ$ so that the y-axis is intersected at $a+R$ (Figure 2(a)). The total amount of rebate given to the end customers is the area of $ABCD$. On the other hand, if the manufacturer offers a lump-sum incentive to the retailer, this will have no effect on the shape of the demand function since end customers are not made aware of this incentive, but will make the retailer move downward on the demand function from E down to G as long as he receives the reservation price from the sales of each additional unit (Figure 2(b)). In this case, the triangular area EFG represents the total amount of the lump-sum incentive the retailer is going to use to increase sales, while his



(a) Customer rebates

(b) Retailer incentives

Figure 2: The impact of retailer incentives and customer rebates on the retailer's decisions under deterministic demand

reservation price is compensated by this incentive.

Case 0: (No-promotion) The retailer solves the problem in (P1) to decide how much to order/sell to maximize his profit given the wholesale price decision of the manufacturer, and finds that $Q_o = \left(\frac{a-w-m}{b}\right)^+$ where $(x)^+ = \max\{0, x\}$. Note that, for the retailer to satisfy his reservation price, $P(Q) = a - bQ \geq w + m$, i.e., $Q \leq \frac{a-w-m}{b}$.

$$\text{(P1)} \qquad \Pi_o^D = \max_{Q\geq 0} \int_0^Q (a-bQ)dQ - wQ = (a-w)Q - \frac{bQ^2}{2}$$
$$\text{s.t.} \quad Q \leq \frac{a-w-m}{b}$$

The manufacturer's problem is to choose the optimal wholesale price that maximizes her profit, i.e., $\max\limits_{w\geq c}(w-c)Q_o$.

Case 1: (Retailer incentive) The manufacturer offers an incentive (K) to the retailer. In this case, the retailer chooses his order/sales quantity (Q_I) by solving the problem in (P2), and the manufacturer determines the wholesale price and incentive value simultaneously by solving $\max\limits_{K\geq 0, w\geq c}(w-c)Q_I - K$.

$$\text{(P2)} \qquad \Pi_I^D = \max_{Q\geq 0} \quad (a-w)Q - \frac{bQ^2}{2} + K$$
$$\text{s.t.} \quad \int_{\frac{a-w-m}{b}}^{Q} ((w+m) - (a-bQ))dQ \leq K$$

Note that, given the manufacturer's decisions, (w, K), the retailer has the option of not transferring the incentive to the end customers and choosing $Q_I = \frac{a-w-m}{b}$, where he pockets all of K, and the price of the last unit sold is equal to his reservation price. However, note also that the unconstrained maximizer of the retailer's profit function $\left(Q=\frac{a-w}{b}\right)$ is greater than $\frac{a-w-m}{b}$, i.e., there is potential for the retailer to further increase his profits if he agrees to sell some additional units below his reservation price. On the other hand, to sell additional units beyond $Q=\frac{a-w-m}{b}$ and still make his reservation price $w+m$, the retailer must be compensated by the lump-sum incentive, which is ensured by the constraint. In other words, being the first decision maker in this game, the manufacturer has the advantage of choosing (w,K) to make the retailer voluntarily transfer some part of the incentive to the end customers and increase sales. We also note that while any $K\geq 0$ increases the retailer's profit, it results in higher sales only until $\left(Q=\frac{a-w}{b}\right)$, after which the retailer starts to pocket the incentive. In equilibrium, the manufacturer finds the optimal incentive value such that incentive is completely passed to the end customers and no pocketing occurs. This model captures a dealer behavior observed in practice, where the auto dealers transfer some part of the manufacturers' incentives to the end users. This is investigated by Busse et al. [9] among others. The turn-and-earn system for inventory allocation, and the fact that dealers who sell a high volume may receive a better selection of vehicles, e.g., vehicles with higher profit margin in the future, also motivate this behavior.

Case 2: (Customer rebate) The manufacturer gives the end customers a rebate, R, for each unit purchased. Therefore, the purchasing power of the customers increases by R and the inverse demand function is written as: $P(Q)=a+R-bQ$. The retailer's and the manufacturer's problems are similar to the no-promotion case, where the retailer determines order/sales quantity, and the manufacturer determines the rebate R and the wholesale price w that maximize their own profits.

Theorem 1 *When demand is deterministic, the SPNE corresponding to the cases where the manufacturer offers no promotion, a lump-sum amount of incentive to the retailer, and a per-unit rebate to the end customers are summarized as follows:*

(i) No promotion: $w_o=\frac{a+c-m}{2}$; $Q_o=\frac{a-m-c}{2b}$; $\Pi_o^M=\frac{1}{b}\left(\frac{a-m-c}{2}\right)^2$; $\Pi_o^D=\frac{(a-m-c)(a+3m-c)}{8b}$; $\Pi_o^{SC}=\frac{(a-m-c)(3a+m-3c)}{8b}$.

(ii) Retailer incentive: see Table 2.

(iii) Customer rebate: $w_R-R=\frac{a+c-m}{2}$; $Q_R=\frac{a-m-c}{2b}$; $\Pi_R^M=\frac{1}{b}\left(\frac{a-m-c}{2}\right)^2$; $\Pi_R^D=\frac{(a-m-c)(a+3m-c)}{8b}$; $\Pi_R^{SC}=\frac{(a-m-c)(3a+m-3c)}{8b}$.

Table 2: The SPNE for the deterministic demand model with retailer incentive

	$a \leq 2m + c$	$a \geq 2m + c$
w_I	$a - m$	$\frac{a+c}{2}$
Q_I	$\frac{a-m-c}{b}$	$\frac{a-c}{2b}$
K	$\frac{(a-m-c)^2}{2b}$	$\frac{m^2}{2b}$
Π_I^M	$\frac{(a-m-c)^2}{2b}$	$\frac{(a-c)^2-2m^2}{4b}$
Π_I^D	$\frac{(a-m-c)m}{b}$	$\frac{(a-c)^2+4m^2}{8b}$
Π_I^{SC}	$\frac{(a-c)^2-m^2}{2b}$	$\frac{3(a-c)^2}{8b}$

We present the proof of this theorem in the Appendix.

In Theorem 1(iii), we find $w_R - R = \frac{a+c-m}{2}$ in equilibrium. Since for any given wholesale price we can find another wholesale price and rebate pair (w, R) resulting in the same profits, this is equivalent to the optimal wholesale price decision for the no-promotion case. Moreover, the manufacturer's sales and profit do not improve with the customer rebate promotion. Bruce et al. [8] find a similar relation between w^* and R^*, and show that the manufacturer does not always find it profitable to give customer rebates, especially when the administrative cost of rebate promotion is high. In our analysis, as a special case of their model, the cost of rebates to the manufacturer is zero. The ineffectiveness of the customer rebates in our setting is mainly a result of the retailer's ability of perfect price discrimination which allows him to capture all the market surplus when the manufacturer offers rebates directly to the end customers.

In our basic model, the effect of the rebates is limited by an additive function of the cash-back amount, i.e., the market potential increases to $a + R$. However, in practice, rebates may cause an additional increase in the market potential (market expansion) since they are often advertised by the manufacturers and/or the retailers, which increase customer awareness. For example, in the auto industry both the dealers and the auto manufacturers use the newspapers and broadcasting for advertising. Since it is costly to advertise these promotions, it is essential that the manufacturer finds the correct balance between the cost of promoting and the revenue from the additional sales. We model this situation with a market expansion factor (α) affecting market potential, and an advertising cost that is a convex increasing function of this expansion factor ($e\alpha^2$). We observe that when rebates lead to market expansion, neither of the promotions is always preferred to another by the manufacturer.

Using the results in Theorem 1, we compare the retailer incentive and the customer rebate promotions with respect to two measures that might be of interest to the manufacturer, namely, total profit and quantity sold. The latter is related with the market share, which is a particularly important measure in the auto industry. Although we focus on the manufacturer's promotional decisions, we also compare the retailer's and the supply chain's profits under different promotions to better understand their impact on the entire chain.

Observation 1 *When rebates do not lead to market expansion (i) offering customer rebates is not effective in increasing the quantity sold and does not change the manufacturer's or retailer's profits (ii) the retailer incentive is always better than the customer rebate and no promotion in terms of total sales, the manufacturer's profit, and the supply chain's profit, but not necessarily the retailer's profit.*

Observation 1(i) holds because when rebates do not lead to market expansion, for any (w, R) combination, the manufacturer can choose a wholesale price $w - R \geq 0$ and achieve the same results as in no promotion. Observation 1(ii) is driven by the tradeoff in using the promotion for every buyer (customer rebate) versus only for those who need it (retailer incentive), as well as the retailer's reservation price. Recall that, the retailer is not willing to sell below his reservation price $(w + m)$. By using the incentive to cover the difference between the price that the customer pays and his reservation price (only for those customers who cannot afford to pay his reservation price), the retailer is able to generate more sales than the no-promotion case. Although the manufacturer obtains higher profits with higher sales, the retailer's profit does not always increase. For example, if the market potential is already high ($a \geq 3.5m + c$), when the manufacturer offers an incentive, she raises her wholesale price at the same time, which in turn results in higher sales but lower profits for the retailer. (Bruce et al. [8] make a similar observation regarding customer rebate, i.e., the manufacturer offering a cash rebate increases her wholesale price more than the rebate amount. Bruce et al. [7] analyze trade promotions with two manufacturers and two retailers, and find that each retailer makes less profit when both manufacturers offer trade promotions.) A modelling alternative is based on principal-agent framework, where the manufacturer chooses the wholesale price and the retailer incentive such that the retailer's profit with the incentive is at least as high as his profit with no promotion. However, this model may have limited applicability in the auto industry where the retailers do not have the flexibility to accept or reject each contract in the case of promotions. Moreover, a dealer generally has a long-standing relationship with the manufacturer, and a dealership does not easily switch from one manufacturer to another.

The retailer's reservation price can be related to his operational costs. However, if we treat m only as a cost incurred for every unit sold, in the retailer incentive case, the entire K is pocketed by the retailer, and therefore, the manufacturer does not offer any incentive. As a result, the customer rebate and retailer incentive cases become identical to the no-promotion case. Assuming the retailer's reservation price is similar to what we observe for customer behavior (willing to buy if price is below reservation price), as also mentioned in several other studies in the automobile industry (Scott Morton et al. [30], and Zettelmeyer et al. [37]), we assume that the reservation price of the retailer does not only correspond to an operational cost per-unit sold, but rather it is the lowest acceptable price for each unit that the retailer is willing to sell.

We analyze the manufacturer's promotional decisions in three other cases: 1) the retailer sets a fixed retail price rather than price discriminating, 2) the retailer incentive is a per-unit payment rather than a lump-sum amount, 3) the retailer has a specific pass-through rate when offered an incentive. In the first case, we find that the manufacturer is always better off with the retailer incentive than the customer rebate in terms of profit, but not necessarily in terms of sales, which is different than our result for the price discrimination case. This also shows the importance of modelling price discrimination since increased sales with incentives are observed in the auto industry. In the second case, we find that the manufacturer is always better off with a per-unit retailer incentive than a customer rebate (just as we found for lump-sum incentive.) Moreover, when $a \geq 3m + c$, the manufacturer's profit is higher with a per-unit incentive than a lump-sum incentive. Therefore, we expect that our insights with lump-sum incentive would hold even more strongly for other incentive schemes that depend on quantity. Finally, in the third case, we find that our qualitative results in the comparison of promotions do not change, i.e., the manufacturer is better off with a retailer incentive than a customer rebate as long as the pass-through rate is not zero; otherwise, the manufacturer would not offer any incentive.

By Observation 1, we conclude that when the manufacturer and the retailer have the same (and accurate) information about the market conditions and when rebates do not lead to market expansion, the manufacturer always prefers the retailer incentive over customer rebate. If this is the case, then why do the manufacturers offer customer rebates? In practice, rebates are frequently used especially by the American auto manufacturers with the goal of increasing market share. One reason for the use of rebates might be the increased awareness and market potential. In addition, in practice, market demand is most likely stochastic rather than deterministic. In such situations, the timing of the decisions, as well as the information possessed at the time of the decisions become critical for the success of promotions. In the next section, we show that demand uncertainty may explain the American auto manufacturers' choice of customer rebates over retailer incentives.

3.2 Uncertain Market Potential Model

In this section, we introduce uncertainty to the demand through the market potential by considering two demand states, high (h) and low (l), with respective probabilities β and $1-\beta$, $\beta \in [0,1]$. Correspondingly, we represent the inverse demand functions as $P(Q^j) = a^j - bQ^j$, where $j = l, h$. Although this is a simplification of demand uncertainty in reality, it helps us to capture the effects of uncertainty on the decisions of the retailer and the manufacturer. In the auto industry, the dealers are closer to the end market and have more information about the customers, therefore we assume that the retailer knows the demand state. In addition, dealers do not have to report sales directly back to manufacturers, therefore manufacturers may not have timely and accurate demand information. In recent years, it has become possible for the manufacturers to buy demand information although there may be a delay and transactions may be averaged. The manufacturers usually have a forecast based on past sales, or they may estimate whether demand is high or low based on signals from the market, web interest or economic indicators, which may include uncertainties.

In the presence of uncertainty, the manufacturer makes her decisions in two stages. In the first stage, she determines the wholesale price; in the second stage, after the demand state is revealed, she chooses to offer either a retailer incentive, customer rebate or no promotion. As in the deterministic model, the retailer determines the optimal order/sales quantity given the manufacturer's decisions.

Similar to Section 3.1, we analyze three cases, i.e., the retailer incentive, customer rebate and no promotion, and then compare the results. The solution procedure for finding the SPNE is again

Table 3: The SPNE for the uncertain market potential model with no promotion

	$a^h - a^l \leq a^l - m - c$	$a^h - a^l \geq a^l - m - c$	
	$\beta \in [0,1]$	$\beta \leq \overline{\beta}$	$\beta \geq \overline{\beta}$
w_o	$\frac{\overline{a}+c-m}{2}$		$\frac{a^h-m+c}{2}$
Q_o^l	$\frac{(1+\beta)a^l-\beta a^h-m-c}{2b}$		0
Q_o^h	$\frac{(2-\beta)a^h-(1-\beta)a^l-m-c}{2b}$		$\frac{a^h-m-c}{2b}$
$\Pi_o^{D_l}$	$\frac{(\beta a^h-(1+\beta)al+m+c)(\beta a^h-(1+\beta)a^l-3m+c)}{8b}$		0
$\Pi_o^{D_h}$	$\frac{((\beta-2)a^h+(1-\beta)al+m+c)((\beta-2)a^h+(1-\beta)al-3m+c)}{8b}$		$\frac{(a^h-m-c)(a^h-c+3m)}{8b}$
Π_o^M	$\frac{(\overline{a}-m-c)^2}{4b}$		$\frac{\beta(a^h-m-c)^2}{4b}$

backward induction, starting with the retailer's problem, which we solve for both high and low demand states. Next, we solve the manufacturer's problem of determining the optimal promotion values for both high and low states. The final step in the induction is to maximize the manufacturer's expected profit, i.e., $\Pi_i^M = \beta\Pi_i^{M_h} + (1-\beta)\Pi_i^{M_l}$ by choosing the optimal wholesale price, where $\Pi_i^{M_j}$; $j = l, h$ is the manufacturer's profit in demand state j with promotion type i. We denote the expected market potential with $\overline{a} = \beta a^h + (1-\beta)a^l$.

Theorem 2 *When the market potential is uncertain, the SPNE corresponding to the cases where the manufacturer offers no promotion, a lump-sum amount of incentive to the retailer, and a per-unit rebate to the end customers are summarized as follows:*

(i) No promotion: see Table 3, where $\overline{\beta} = \frac{(a^l-m-c)^2}{(a^h-a^l)^2}$.

(ii) Retailer incentive: see Table 5 in the Appendix.

(iii) Customer rebate: $w_R - R^j = \frac{a^j+c-m}{2}$*;* $Q_R^j = \frac{a^j-m-c}{2b}$*,* $j = l, h$*;* $\Pi_R^M = \frac{\beta(a^h-m-c)^2+(1-\beta)(a^l-m-c)^2}{4b}$*;* $\Pi_R^{D_j} = \frac{(a^j-m-c)(a^j+3m-c)}{8b}$*,* $j = l, h$*.*

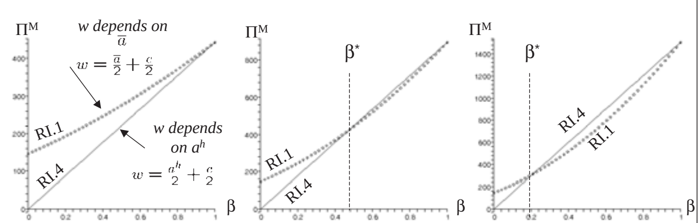
In the cases of no-promotion and retailer incentive, we obtain a *unique* equilibrium for a given set of system parameters. In both cases, the feasible solutions that are candidates for being the unique equilibrium are of two types: the wholesale price is either driven by the expected market potential, $\overline{a}$, (expectation driven wholesale price or solution) or only by the high market potential, a^h, (high-demand driven wholesale price or solution). In the former, the manufacturer is able to make positive sales whether the realized demand is low or high, whereas in the latter, she cannot generate any sales when the realized demand is low. The manufacturer faces a tradeoff between the loss of profit she incurs in the following two events. In the first event, she keeps the wholesale price "average" (by considering both states) and sees a high demand, which happens with a probability of β. In the second event, she keeps the wholesale price "high" (by only considering the high state) and sees a low demand, which happens with a probability of $1 - \beta$. Being a risk neutral decision maker, the manufacturer chooses the event that brings highest expected profit. Intuitively, we expect that the loss incurred in the former event would increase in β. For all expectation driven solutions, we can show that the expected loss of profit in the former event increases in a^h and decreases in a^l. This suggests that the high-demand driven wholesale price brings higher expected profit to the manufacturer when β and $a^h - a^l$ are "high".

Observation 2 *In the case of no-promotion, when the difference between high and low market potentials is relatively "small" (i.e., $a^h - a^l \leq a^l - m - c$), or β is "low" (i.e., $\beta \leq \overline{\beta}$), the equilibrium for the manufacturer is driven by the expectation over the high and low market potentials. Otherwise, the wholesale price depends on the high market potential, but not on the low market potential. Finally, as the gap between a^h and a^l increases, $\overline{\beta}$ decreases.*

In Figure 3, we see that the manufacturer's equilibrium behavior in the retailer incentive case is similar to the no-promotion case. In this example, two feasible solutions corresponding to the system parameters are RI.4 (high-demand driven solution) and RI.1 (expectation driven solution) in Table 5. When $a^h - a^l$ is "low", RI.1 provides a higher profit for the manufacturer. As we increase a^h in Figures 3(b) and 3(c), when $\beta \geq \beta^* = \frac{(a^l-c)^2-2m^2}{(a^h-a^l)^2}$, RI.4 results in a higher profit for the manufacturer. Note also that β^* decreases in $(a^h - a^l)$.

Finally, we analyze the equilibrium in the customer rebate case stated in Theorem 2(iii). Note that, although the manufacturer determines the wholesale price before knowing the demand state, she has the opportunity to adjust it with a rebate according to the demand state. This is obviously an advantage for the manufacturer and as we will see in Observation 3, depending on the system parameters, the manufacturer may be better off offering a customer rebate instead of retailer incentive when there is demand uncertainty.

Next, we compare the promotions under uncertain market potential. From Theorem 2(i)-(iii),



(a) $a^h = 75$, $a^l = 50$; (b) $a^h = 100$, $a^l = 50$, (c) $a^h = 125$, $a^l = 50$.

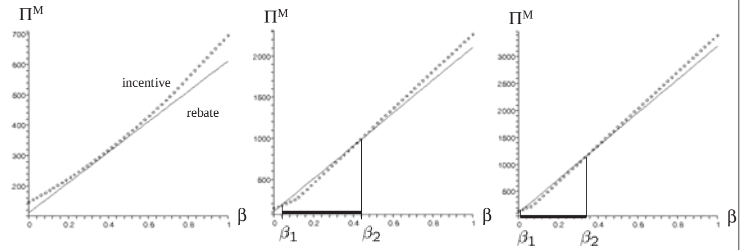
Figure 3: Retailer incentive equilibrium wholesale price with increasing a^h $(m = 5, b = 2, c = 15)$

we can show that offering no promotion is the least profitable option and generates the lowest sales for the manufacturer. The comparison of the retailer incentive and customer rebate promotions shows that unlike the deterministic demand case in Section 3.1, when there is demand uncertainty neither the retailer incentive nor the customer rebate has an absolute dominance over the other. Depending on system parameters, especially related to uncertainty, the manufacturer may find customer rebate or retailer incentive more profitable.

Observation 3 *When the market potential is uncertain and the uncertainty is "high", i.e., when $(a^h - a^l)$ is "large" and β is in the "middle" of the range, offering a customer rebate may be more profitable than a retailer incentive for the manufacturer. (See Figure 4 for an example.)*

In Figure 4, we continue with the example illustrated in Figure 3. We observe that as $(a^h - a^l)$ increases, the β range (β_1, β_2) where the customer rebate is more profitable than the retailer incentive shifts to the left approaching the origin. We analytically derive this comparative static result for this example in Proposition 1. Note that β^* denotes the threshold value where the equilibrium switches from RI.1 to RI.4. When $\beta_1 \leq \beta \leq \beta^*$, customer rebate gives a higher profit than RI.1, and when $\beta^* \leq \beta \leq \beta_2$, customer rebate gives a higher profit than RI.4. Therefore (β_1, β_2) denotes the range where the manufacturer is better off with a customer rebate. We also observe that (β_1, β_2) becomes smaller as $(a^h - a^l)$ increases (Figure 4(c)). This suggests that uncertainty can be one reason for auto manufacturers to offer rebates.

Proposition 1 *When $\beta_1 \leq \beta^* \leq \beta_2$, as a^h increases and a^l decreases (i.e., $(a^h - a^l)$ increases), β^*, β_1, and β_2 decrease. ($\beta^* = \frac{(a^l-c)^2-2m^2}{(a^h-a^l)^2}$, $\beta_1 = \frac{a^h-a^l-2m-\sqrt{(a^h-a^l-2m)^2+4m(3m+2c-2a^l)}}{2(a^h-a^l)}$, and*



(a) $a^h = 90$, $a^l = 50$, (b) $a^h = 150$, $a^l = 50$, (c) $a^h = 180$, $a^l = 50$.

Figure 4: Comparison of retailer incentive and customer rebate with increasing a^h ($m = 5, b = 2, c = 15$)

$\beta_2 = \frac{(a^l-m-c)^2}{(a^l-c)^2+2m(a^h-a^l-m)}$)

3.3 Uncertain Price Sensitivity Model

In this section, we consider uncertainty in price sensitivity, i.e., $P(Q^j) = a - b^j Q^j$; $j = l, h$, where $P(Q^j)$ is the price when Q^j units are sold with price sensitivity b^j in state $j = l, h$. We repeat the analysis in Section 3.2, and find the SPNE for the cases of no promotion, retailer incentive, and customer rebate. (See Table 6 in the Appendix for a summary of equilibrium solutions.)

We observe that when there is uncertainty in price sensitivity, offering customer rebate is identical to offering no promotion from the manufacturer's point of view. Moreover, for any market condition, the manufacturer's profit and the quantity sold is higher under retailer incentive compared to customer rebate or no promotion in both demand states. This observation arises because the wholesale price decision is independent of price sensitivity and therefore the manufacturer's decisions are identical to the deterministic demand case.

3.4 Combined Promotions: Retailer Incentive and Customer Rebate

In practice, manufacturers sometimes choose to offer a combination of retailer incentive and customer rebate. In this section, we aim to obtain insights on how effective it is to offer both promotions at the same time. We summarize the equilibrium solutions in Table 7 in the Appendix.

When demand is deterministic, the manufacturer's profit and total sales when she offers retailer incentive and customer rebate simultaneously are equal to her profit and total sales when she offers retailer incentive alone. In Section 3.1 we have seen that the manufacturer's profit remains the same when she offers a customer rebate instead of no promotion, while she increases her profits by offering a retailer incentive. It is indeed expected that combined promotions, being a hybrid of the customer rebate and the retailer incentive promotions, will not do any better than the retailer incentive itself for the deterministic demand case. With the same reasoning, we also expect that combining two promotions will not improve the profits when there is uncertainty in price sensitivity, and confirm by Table 7(b).

However, for the uncertain market potential model, we have observed situations where customer rebate was performing better than the retailer incentive as well as situations where the opposite holds. Clearly, by combining these promotions, the manufacturer is expected to do at least as well as the case when she uses each promotion individually. The question is whether the manufacturer is able to do strictly better when she offers both promotions simultaneously. Our analysis of the equilibrium decisions shows that this is true, i.e., the manufacturer's total profit is higher if she offers two promotions (retailer incentive and customer rebate) at the same time rather than offering any one of them individually. Moreover, the combined promotions generate more sales than the retailer incentive, except for some instances with $a^h \geq a^l \geq 2m + c$ or $a^l \leq 2m + c \leq a^h$. Under all market conditions, combined promotions generate more sales than the customer rebate alone.

In the example shown in Figure 4, the manufacturer is able to increase her profits by as much as 18% by combining two promotions. The improvement in the manufacturer's profits decreases as the gap between the market potentials $(a^h - a^l)$ increases. We can explain this behavior as follows. As $(a^h - a^l)$ increases, the manufacturer is better off with a high-demand driven equilibrium, i.e., behaving as if she is in a deterministic setting with a market potential of a^h. We have seen that when demand is deterministic, the retailer incentive is always better than the customer rebate. Therefore, we expect that the profit increase achieved by combining the two promotions in an effectively deterministic setting is not as high as the profit increase in a setting with uncertainty where both promotions may improve the manufacturer's profits.

4 Conclusions

In this paper, we have analyzed two different types of promotions, customer rebates and retailer (dealer) incentives, which are commonly used by manufacturers in the auto industry. We considered several models with different demand characteristics to determine under which market conditions one promotion is more effective than the other in terms of increasing the manufacturer's profits and sales. Our setting is a two stage supply chain with an uncapacitated manufacturer and a retailer (or dealer) who can do perfect price discrimination in a market with price sensitive demand.

We showed that when demand is deterministic and rebates do not lead to market expansion, customer rebates are not effective in increasing the manufacturer's sales and profits, and the manufacturer is always better off with a retailer incentive. If the rebates lead to an increase in the market potential possibly through advertising but at an additional cost, the manufacturer may prefer the customer rebate over the retailer incentive when the increase in the market potential is sufficient enough to cover the cost of promoting.

We showed that when the market potential is uncertain, neither the retailer incentive nor the customer rebate has an absolute dominance over the other. We observed that when the uncertainty is high, the customer rebate performs better than the retailer incentive. On the other hand, when the uncertainty is on the price sensitivity parameter of the demand function, we obtained identical results to the deterministic setting where the profits and sales generated with a retailer incentive are higher than those of a customer rebate. We also showed that offering combined promotions improves the manufacturer's profits and sales only if there is uncertainty in the market potential.

We have seen that uncertainty can be an important factor in determining whether to offer a customer rebate or retailer incentive. We investigate whether our analytical findings are in line with what we observe in practice, in particular, whether different promotions offered by the American auto manufacturers (customer rebates based promotions) and the Japanese auto manufacturers (retailer incentives based promotions) can be explained by the variability of the demand that they observe. Since the actual demand data is not available, we use sales as an approximation of demand and we take the variability of sales to represent uncertainty in demand for this analysis unless described otherwise. We focus on the following two questions: 1) Is there any statistical evidence for the rebates by the American auto manufacturers being higher than those by the Japanese auto manufacturers? 2) Is the sales variability of the American vehicles higher than the Japanese

Table 4: Summary statistics

	Total number of Registrations				Rebate Percentages			
	Mean		CoV*		Mean %		CoV	
Segment	US	Japan	US	Japan	US	Japan	US	Japan
Midsize	48,574.58	66,942.86	0.18	0.13	6.24	0.33	0.48	0.85
Compact	35,977.94	42,078	0.20	0.16	9.48	0.50	0.33	0.86
Utility	52,459.75	18,729.42	0.29	0.17	2.87	1.02	0.57	0.64

$^*\text{CoV} = \frac{\text{standard deviation}}{\text{mean}}$

vehicles?

Table 4 summarizes the basic statistics related with the data sets for the compact, midsize, and utility vehicle segments. Although these values may help to test the hypotheses, one should be careful using them because the registrations and the rebates are time series data with possible autocorrelations and therefore may lack independence. In order to test the variability differences, ideally we need to separate out the effects of uncertainty from seasonality. For these reasons, instead of the actual registrations, we use the normalized data where each data point is normalized with the total registrations in the corresponding month, and we then divide the normalized data points into non-overlapping batches and take the batch means as the data points. (Batch means method is frequently used for the estimation of mean and variances in simulation output analysis; e.g., see Alexopoulos et al. [1].)

Our results suggest that the rebates by the American auto manufacturers are statistically significantly higher than the rebates by the Japanese auto manufacturers at the 95% confidence level. We have weaker results for the variability differences; at 90% confidence level, there is evidence that the sales variability of the American auto manufacturers is higher than that of the Japanese auto manufacturers for the midsize and utility vehicle segments, but not for the compact segment. As we mentioned before, we relate the variability in sales with the uncertainty in demand. Therefore, the higher variability in demand by the American auto manufacturers could explain their choice of rebates as an important promotion mechanism as our analytical results suggested that the rebates improve profits and sales most when there is high uncertainty. However, it would be useful to further investigate the effect of rebates on the variability of demand. It might be the case that the American auto manufacturers started using the rebates for some other reason, but then as they offered high rebates in some periods and no rebates in others, they added more variability to the demand that they see. Identifying if rebates cause variability or if rebates are offered because of variability is very difficult and we leave this for future research.

Our work analytically compares different promotions used in the auto industry, which is very important for the manufacturers. However, there are many further questions of interest in this area. For example, it would be useful to add the sales effort decisions of the dealers into the analysis, since effort may play important role in increasing sales. It is also important to consider the impact of competition since it is a component of the auto industry. In an extension of this research, we analyze the problem in an environment with multiple manufacturers and retailers where the effects of competition can be captured (see Caliskan Demirag et al. [11])). Issues of supply chain coordination in this setting could also be useful as well as the aspects of customer behavior such as strategic buying.

Appendix

Proof of Theorem 1.

No Promotion : The results follow from the backward induction steps below.

Step 1. The retailer's order quantity decision: Given w, find Q that maximizes the retailer's profit by solving problem (P1) in Section 3.1. Π_o^R is concave in Q ($\frac{\partial^2 \Pi_o^R(Q)}{\partial Q^2} = -b < 0$). From first order conditions (FOC) we get $Q = \frac{a-w}{b}$. Considering the upper bound on Q, we find the retailer's best response to w as $Q_o = max\left\{0, min\left\{\frac{a-w}{b}, \frac{a-w-m}{b}\right\}\right\} = \left(\frac{a-w-m}{b}\right)^+$.

Step 2. The manufacturer's wholesale price decision: Given the best response of the retailer to w, find w that maximizes the manufacturer's profit by solving the following problem.

$$\Pi_o^M = \max_{w \geq c} \quad (w-c)\left(\frac{a-w-m}{b}\right)^+$$

When $\frac{a-w-m}{b} \geq 0$, the objective function is concave in w ($\frac{\partial^2 \Pi_o^M}{\partial w^2} = \frac{-2}{b} < 0$). It follows from FOC and the lower bound on w that $w^* = max\left\{c, min\left\{\frac{a+c-m}{2}, a-m\right\}\right\} = \frac{a+c-m}{2}$. Therefore, $w_o = \frac{a+c-m}{2}$; $Q_o = \frac{a-m-c}{2b}$; $\Pi_o^M = \frac{1}{b}\left(\frac{a-m-c}{2}\right)^2$; $\Pi_o^D = \frac{(a-m-c)(a+3m-c)}{8b}$; $\Pi_o^{SC} = \frac{(a-m-c)(3a+m-3c)}{8b}$.

Retailer Incentive: The results follow from the backward induction steps below.

Step 1. The retailer's order quantity decision: Given w and K, find Q that maximizes the retailer's profit by solving the problem (P2) in Section 3.1. Π_I^R is concave in Q ($\frac{\partial^2 \Pi^R}{\partial Q^2} = -b$).

Table 5: Dominating feasible solutions for the uncertain market potential demand model with retailer incentive

Feasible Region (F.R.)			Solution
$a^l \leq 2m + c$	$a^h \leq 2m + c$		RI.2, RI.6
	$2m + c \leq a^h \leq 2a^l - c$	$\beta \leq \frac{2m+c-a^l}{a^h-a^l}$	RI.2, RI.3
		$\beta \geq \frac{2m+c-a^l}{a^h-a^l}$	RI.1, RI.3
	$a^h \geq 2a^l - c$	$\beta \leq \frac{2m+c-a^l}{a^h-a^l}$	RI.2, RI.3
		$\frac{2m+c-a^l}{a^h-a^l} \leq \beta \leq \frac{a^l-c}{a^h-a^l}$	RI.1, RI.3
		$\beta \geq \frac{a^l-c}{a^h-a^l}$	RI.3
$a^l \geq 2m + c$	$2m + c \leq a^h \leq 2a^l - 2m - c$		RI.5, RI.1
	$2a^l - 2m - c \leq a^h \leq 2a^l - c$		RI.4, RI.1
	$a^h \geq 2a^l - c$	$\beta \leq \frac{a^l-c}{a^h-a^l}$	RI.4, RI.1
		$\beta \geq \frac{a^l-c}{a^h-a^l}$	RI.4

	RI.1	RI.2	RI.3	RI.4	RI.5	RI.6
w^*	$\frac{\overline{a}+c}{2}$	$\overline{a} - m$	$\frac{a^h+c}{2}$		$a^l - m$	$a^h - m$
Q^{l*}	$\frac{-\beta a^h+(1+\beta)a^l-c}{2b}$	$\frac{a^l-m-c}{b}$	0		0	0
Q^{h*}	$\frac{(2-\beta)a^h-(1-\beta)a^l-c}{2b}$	$\frac{a^h-m-c}{b}$	$\frac{a^h-c}{2b}$		$\frac{a^h-a^l+m}{b}$	$\frac{a^h-m-c}{b}$
K^{l*}	$\frac{m^2}{2b}$	$\frac{(\overline{a}-m-c)^2}{2b}$	0		0	0
K^{h*}	$\frac{m^2}{2b}$	$\frac{(\overline{a}-m-c)^2}{2b}$	$\frac{m^2}{2b}$		$\frac{m^2}{2b}$	$\frac{(a^h-m-c)^2}{2b}$
$\Pi^{D_l^*}$	$\frac{(\beta a^h-(1+\beta)a^l+c)^2+4m^2}{8b}$	$\frac{\beta^2(a^h-a^l)^2}{2b}$ $+\frac{2m(a^l-m-c)}{2b}$	0		0	0
$\Pi^{D_h^*}$	$\frac{(\overline{a}-2a^h+c)^2+4m^2}{8b}$	$\frac{(a^h-a^l)^2(\beta-1)^2}{2b}$ $+\frac{2m(a^h-m-c))}{2b}$	$\frac{(a^h-c)^2+4m^2}{8b}$		$\frac{(a^h-a^l+m)^2+m^2}{2b}$	$\frac{m(a^h-m-c)}{b}$
Π^M	$\frac{(a^l-c)^2-2m^2+\beta^2(a^h-a^l)^2}{4b}$ $+\frac{2\beta(c(a^l-a^h)+a^h a^l-(a^l)^2)}{4b}$	$\frac{(\overline{a}-m-c)^2}{2b}$	$\frac{\beta((a^h-c)^2-2m^2)}{4b}$		$\frac{\beta(a^l-m-c)(a^h-a^l+m)}{b}$ $-\frac{\beta m^2}{2b}$	$\frac{\beta(a^h-m-c)^2}{2b}$
F.R:	$\overline{a} \geq 2m + c$ $\beta \leq \frac{a^l-c}{a^h-a^l}$	$\overline{a} \leq 2m + c$	$a^h \geq 2m + c$ $a^l \leq 2m + c$	$a^l \geq 2m + c$ $a^h \geq 2a^l - m - c$	$a^l \geq 2m + c$ $a^h \leq 2a^l - m - c$	$a^h \leq 2m + c$

Table 6: The SPNE for the uncertain price sensitivity model

(a) No promotion

w_o	$\frac{a+c-m}{2}$
Q_o^j	$\frac{a-m-c}{2b^j};\ j=l,h$
$\Pi_o^{D_j};\ j=l,h$	$\frac{(a-m-c)(a+3m-c)}{8b^j}$
Π_o^M	$\frac{(a-m-c)^2}{4}\left(\frac{\beta}{b^h}+\frac{(1-\beta)}{b^l}\right)$

(b) Customer rebate

$w_R - R^j$	$\frac{a+c-m}{2};\ j=l,h$
Q_R^j	$\frac{a-m-c}{2b^j};\ j=l,h$
$\Pi_R^{D_j};\ j=l,h$	$\frac{(a-m-c)(a+3m-c)}{8b^j}$
Π_R^M	$\frac{(a-m-c)^2}{4}\left(\frac{\beta}{b^h}+\frac{(1-\beta)}{b^l}\right)$

(c) Retailer incentive

	$a \leq 2m+c$	$a \geq 2m+c$
w_I	$a-m$	$\frac{a+c}{2}$
$Q_I^j;\ j=l,h$	$\frac{a-m-c}{b^h}$	$\frac{a-c}{2b^h}$
$K^j;\ j=l,h$	$\frac{(a-m-c)^2}{2b^j}$	$\frac{m^2}{2b^j}$
$\Pi_I^{D_j};\ j=l,h$	$\frac{(a-m-c)m}{b^j}$	$\frac{(a-c)^2+4m^2}{8b^j}$
Π_I^M	$\frac{(a-m-c)^2}{2}\left(\frac{\beta}{b^h}+\frac{(1-\beta)}{b^l}\right)$	$\frac{(a-c)^2-2m^2}{4}\left(\frac{\beta}{b^h}+\frac{(1-\beta)}{b^l}\right)$

From FOC we get $Q = \frac{a-w}{b}$, and by considering the boundary condition on Q enforced with the constraint, we find the retailer's best response as $Q_I = \left(min\left\{\frac{a-w}{b}, \frac{a-w-m+\sqrt{2Kb}}{b}\right\}\right)^+$.

Step 2. The manufacturer's wholesale price and retailer incentive decision: Given the retailer's best response, find w and K that maximizes the manufacturer's profit:

$$\Pi_I^M = \max_{K\geq 0, w\geq c} \quad (w-c)\left(min\left\{\frac{a-w}{b}, \frac{a-w-m+\sqrt{2Kb}}{b}\right\}\right)^+ - K$$

In order to solve the manufacturer's problem, we proceed in two steps; first, we characterize the optimal value for the retailer incentive, K^*, for a given w, and next, we find the optimal wholesale price, by embedding K^* in the manufacturer's objective function and maximizing it over w. In the first step, we obtain the expression in (1) for K^*.

$$K^* = \begin{cases} min\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\} & \text{if } K \leq \frac{m^2}{2b} \text{ and } \sqrt{2Kb} \geq w-(a-m) \text{ where } w \leq a \\ 0 & \text{if } K \leq \frac{m^2}{2b} \text{ and } \sqrt{2Kb} \leq w-(a-m) \text{ where } w \geq a-m \\ \frac{m^2}{2b} & \text{if } K \geq \frac{m^2}{2b} \text{ where } w \leq a. \end{cases} \tag{1}$$

Note that K^* is identified under different cases which lead to different Q decisions. For example,

Table 7: The SPNE for the combined promotions

(a) Deterministic demand model

	$a \leq 2m + c$	$a \geq 2m + c$
$w_C - R$	$a - m$	$\frac{a+c}{2}$
Q_C	$\frac{a-m-c}{b}$	$\frac{a-c}{2b}$
K	$\frac{(a-m-c)^2}{2b}$	$\frac{m^2}{2b}$
Π_C^D	$\frac{(a-m-c)m}{b}$	$\frac{(a-c)^2+4m^2}{8b}$
Π_C^M	$\frac{(a-m-c)^2}{2b}$	$\frac{(a-c)^2-2m^2}{4b}$

(b) Uncertain price sensitivity model

	$a \leq 2m + c$	$a \geq 2m + c$
$w_C - R^j;\ j = l, h$	$a - m$	$\frac{a+c}{2}$
$Q_C^j;\ j = l, h$	$\frac{a-m-c}{b^j}$	$\frac{a-c}{2b^j}$
$K^j;\ j = l, h$	$\frac{(a-m-c)^2}{2b^j}$	$\frac{m^2}{2b^j}$
$\Pi_C^{D_j};\ j = l, h$	$\frac{(a-m-c)m}{b^j}$	$\frac{(a-c)^2+4m^2}{8b^j}$
Π_C^M	$\frac{(a-m-c)^2}{2}\left(\frac{\beta}{b^h} + \frac{(1-\beta)}{b^l}\right)$	$\frac{(a-c)^2-2m^2}{4}\left(\frac{\beta}{b^h} + \frac{(1-\beta)}{b^l}\right)$

(c) Uncertain market potential model

	$a^h \geq a^l \geq 2m + c$	$a^l \leq 2m + c \leq a^h$	$a^l \leq a^h \leq 2m + c$
$w_C - R^l$	$\frac{a^l+c}{2}$	$a^l - m$	$a^l - m$
$w_C - R^h$	$\frac{a^h+c}{2}$	$\frac{a^h+c}{2}$	$a^h - m$
Q_C^l	$\frac{a^l-c}{2b}$	$\frac{a^l-m-c}{b}$	$\frac{a^l-m-c}{b}$
Q_C^h	$\frac{a^h-c}{2b}$	$\frac{a^h-c}{2b}$	$\frac{a^h-m-c}{b}$
K^l	$\frac{m^2}{2b}$	$\frac{(a^l-m-c)^2}{2b}$	$\frac{(a^l-m-c)^2}{2b}$
K^h	$\frac{m^2}{2b}$	$\frac{m^2}{2b}$	$\frac{(a^h-m-c)^2}{2b}$
$\Pi_C^{D_l}$	$\frac{(a^l-c)^2+4m^2}{8b}$	$\frac{m(a^l-m-c)}{b}$	$\frac{(a^l-m-c)m}{b}$
$\Pi_C^{D_h}$	$\frac{(a^h-c)^2+4m^2}{8b}$	$\frac{(a^h-c)^2+4m^2}{8b}$	$\frac{(a^h-m-c)m}{b}$
Π_C^M	$\beta\left(\frac{(a^h-c)^2}{4b} - \frac{m^2}{2b}\right)$ $+(1-\beta)\left(\frac{(a^l-c)^2}{4b} - \frac{m^2}{2b}\right)$	$\beta\left(\frac{(a^h-c)^2}{4b} - \frac{m^2}{2b}\right)$ $+(1-\beta)\frac{(a^l-m-c)^2}{2b}$	$\beta\frac{(a^h-m-c)^2}{2b}$ $+(1-\beta)\frac{(a^l-m-c)^2}{2b}$

in the first case, $Q = \frac{a-w-m+\sqrt{2Kb}}{b}$ and for a fixed w we solve the following problem:

$$\begin{aligned} \max_{K \geq 0} \quad & (w-c)\left(\frac{a-w-m+\sqrt{2Kb}}{b}\right) - K \\ \text{s.t.} \quad & K \leq \tfrac{m^2}{2b} \\ & \sqrt{2Kb} \geq w-(a-m) \end{aligned}$$

The objective function is concave in K, $(\frac{\partial^2 \Pi^M}{\partial K^2} = \frac{-(w-c)\sqrt{2}}{4\sqrt{b}\sqrt{K^3}} \leq 0)$. By FOC we get $K = \frac{(w-c)^2}{2b}$. Considering the bounds on K, it follows that $K^* = \max\left\{\frac{(w-(a-m))^2}{2b}, \min\left\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\right\}\right\}$. Note that a feasibility condition for $K \leq \frac{m^2}{2b}$ and $\sqrt{2Kb} \geq w-(a-m)$ to hold simultaneously is $w \leq a$. This condition and our assumption $a \geq c+m$ imply that $\frac{(w-(a-m))^2}{2b} \leq \frac{(w-c)^2}{2b}$ and $\frac{(w-(a-m))^2}{2b} \leq \frac{m^2}{2b}$. Hence, $K^* = \min\left\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\right\}$. The analysis of the other cases is similar.

Characterizing K^* for a given w, in the second step, we find the optimal wholesale price, w_I, by embedding K^* in the manufacturer's objective function and maximizing it over w in the subregions defined by the branching in Figure 5. (We eliminate the case with $K = 0$ and $Q = 0$ (resulting in zero profit), since $w = c$ and $K = 0$ is always feasible with the same profit.) We show the computations for a representative feasible solution (FS.2, standing for "feasible solution 2") and present all feasible solutions in Table 8. The SPNE in Table 2 (Section 3.1) is the solution that is feasible in a region and has the highest profit. (In the last row of Table 8, the SPNE that dominates the feasible solution in the respective column is shown under the title D.S, standing for "dominating solution".)

Derivation of FS.2 When $K \leq \frac{m^2}{2b}$ and $\sqrt{2Kb} \geq w-(a-m)$, $K^* = min\left\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\right\}$. In the region where $w \geq m+c$, $K^* = \frac{m^2}{2b}$ and the retailer's order quantity is $\frac{a-w}{b}$. In order to find w^*, we solve the following problem:

$$\begin{aligned} \max_{w \geq c} \quad & (w-c)(\frac{a-w}{b}) - \frac{m^2}{2b} \\ \text{s.t.} \quad & m+c \leq w \leq a. \end{aligned}$$

By FOC, $w = \frac{a+c}{2}$. Therefore, $w^* = max\left\{m+c, min\left\{a, \frac{a+c}{2}\right\}\right\} = max\left\{m+c, \frac{a+c}{2}\right\}$. When $a \geq 2m+c$, $w^* = max\left\{m+c, \frac{a+c}{2}\right\} = \frac{a+c}{2}$, $K^* = \frac{m^2}{2b}$, $Q^* = \frac{a-c}{2b}$ and $\Pi^M = \frac{(a-c)^2-2m^2}{4b}$.

<u>Customer Rebate</u>: The retailer's best response is $Q_R^* = \left(\frac{a+R-w-m}{b}\right)^+$, and the manufacturer's problem is as follows:

$$\max_{w \geq c+R, R \geq 0} \quad (w-c-R)\left(\frac{a+R-w-m}{b}\right)^+$$

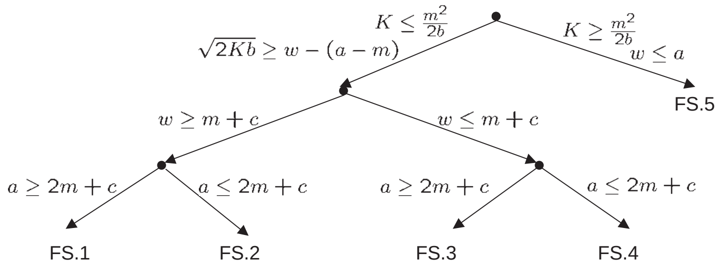


Figure 5: Decomposition of the feasible region for determining w in the deterministic demand model with retailer incentive

Table 8: All feasible solutions for deterministic demand model with retailer incentive

	FS.1	FS.5	FS.2	FS.3	FS.4
F.R:	$a \geq 2m + c$	$a \geq m + c$	$a \leq 2m + c$	$a \geq 2m + c$	$a \leq 2m + c$
w^*	$\frac{a+c}{2}$		$m + c$		$a - m$
Q^*	$\frac{a-c}{2b}$		$\frac{a-m-c}{b}$		$\frac{a-m-c}{b}$
K^*	$\frac{m^2}{2b}$		$\frac{m^2}{2b}$		$\frac{(a-m-c)^2}{2b}$
Π^M	$\frac{(a-c)^2-2m^2}{4b}$		$\frac{m(2a-2c-3m)}{2b}$		$\frac{(a-m-c)^2}{2b}$
D.S:	-		FS.4	FS.1	-

We solve the manufacturer's problem in a two-step procedure, where in the first step we characterize the optimal rebate value given a wholesale price, and in the second step we substitute the optimal rebate value back into the manufacturer's profit function to solve for the optimal wholesale price. We find $R = w - \frac{a+c-m}{2}$. After back substitution, the objective function becomes independent of w, and therefore any w, R satisfying $w_R - R = \frac{a+c-m}{2}$ is optimal.

References

[1] C. Alexopoulos, D. Goldsman, N.T. Argon, and G. Tokol. Overlapping variance estimators for simulations. *Proceedings of the 2004 Winter Simulation Conference*, pages 737–745, 2004.

[2] J. Anwyl. A transcription of comments made at the Society of Automo-

tive Analysts 16th annual Automotive Outlook Conference in Detroit, 2004. http://www.cybersaa.org/precis/04anwyl.pdf.

[3] G. Aydin and E.L. Porteus. The effects of retailer versus consumer rebates in a supply chain. Working Paper, University of Michigan, 2005.

[4] S. Biller, L.M.A. Chan, D. Simchi-Levi, and J. Swann. Dynamic pricing and the direct-to-customer model in the automotive industry. *Electronic Commerce Journal special issue on Dynamic Pricing*, 5(2):309–334, 2005.

[5] S. Biller, A. Muriel, and Y. Zhang. Impact of price postponement on capacity and flexibility investment decisions. *Production and Operations Management*, 15(2):198–214, 2006.

[6] R.C. Blattberg, R. Briesch, and E.J. Fox. How promotions work. *Marketing Science*, 14(3-2):G122–G132, 1995.

[7] N. Bruce, P. Desai, and R. Staelin. The better they are, the more they give: Trade promotions of consumer durables. *Journal of Marketing Research*, 42(1):54–66, 2005.

[8] N. Bruce, P. Desai, and R. Staelin. Enabling the willing: Consumer rebates for durable goods. *Marketing Science*, 25(4):350–366, 2006.

[9] M. Busse, J. Silva-Risso, and F. Zettelmeyer. $1000 cash back: The pass-through of auto manufacturer promotions. *American Economic Review*, 96(4):1253–1270, 2006.

[10] G.P. Cachon and M.A. Lariviere. Capacity allocation using past sales: When to turn-and-earn. *Management Science*, 45(5):685–703, 1999.

[11] O. Caliskan Demirag, P. Keskinocak, and J. Swann. Customer rebates and retailer incentives in the presence of competition. Working paper, Georgia Institute of Technology, 2007.

[12] X. Chen, L. Chung-Lun, and D. Simchi-Levi. The impact of manufacturer rebates on supply chain profits. Working Paper, MIT, 2005.

[13] S.C. Choi. Price competition in a channel structure with a common retailer. *Marketing Science*, 10(4):271–296, 1991.

[14] C. J. Corbett and U.S. Karmarkar. Competition and structure in serial supply chains with deterministic demand. *Management Science*, 47(7):966–978, 2001.

[15] W. Elmaghraby and P. Keskinocak. Dynamic pricing in the presence of inventory considerations: Research overview, current practices and future directions. *Management Science*, 49(10):1287–1309, 2003.

[16] E. Gerstner and J.D. Hess. A theory of channel price promotions. *The American Economic Review*, 81(4):872–886, 1991.

[17] E. Gerstner and J.D. Hess. Pull promotions and channel coordination. *Marketing Science*, 14(1):43–60, 1995.

[18] E. Gerstner and D.M. Holthausen. Profitable pricing when market segments overlap. *Marketing Science*, 5(1):55–69, 1986.

[19] P.K. Goldberg. Dealer price discrimination in new car purchases: Evidence from the consumer expenditure survey. *The Journal of Political Economy*, 104(3):622–654, 1996.

[20] M. Goyal, T. Randall, and S. Netessine. Deployment of manufacturing flexibility: an empirical analysis of the automotive industry. Working paper, University of Maryland and The University of Pennsylvania, 2005.

[21] T. Henderson. What drives Detroit: The economic and technological challenges facing the auto industry. www.facsnet.orgtoolsbiz_econdetroit_auto.php, 2005.

[22] L. Jakobson. Bribing the customer. *Incentive*, 179(8):12, 2005. www.incentivemag.com/incentive/search/article_display.jsp?vnu_content_id=1001000969.

[23] B.D. Kim, M. Shi, and K. Srinivasan. Managing capacity through reward programs. *Management Science*, 50(4):503–520, 2004.

[24] S.Y. Kim and R. Staelin. Manufacturer allowances and retailer pass-through rates in a competitive environment. *Marketing Science*, 18(1):59–76, 1999.

[25] H. Krishnan, R. Kapuscinski, and D.A. Butz. Coordinating contracts for decentralized supply chains with retailer promotional effort. *Management Science*, 50(1):48–63, 2004.

[26] R. Lal, J.D.C. Little, and J.M. Villas-Boas. A theory of forward buying, merchandising, and trade deals. *Marketing Science*, 15(1):21–37, 1996.

[27] C. Narasimhan. A price discrimination theory of coupons. *Marketing Science*, 3:128–147, 1984.

[28] K. Pauwels, J. Silva-Risso, S. Srinivasan, and D. Hanssens. New products, sales promotions, and firm value: The case of the automobile industry. *Journal of Marketing*, 68(4):142–156, 2004.

[29] A. Priddle and E.D. Zoia. Incentives. *Ward's Auto World*, 39(8):36–42, 2003.

[30] F. Scott Morton, F. Zettelmeyer, and J. Silva-Risso. A test of bargaining theory in the auto retailing industry. Working Paper, Yale University.

[31] F. Scott Morton, F. Zettelmeyer, and J. Silva-Risso. Consumer information and discrimination: Does the internet affect the pricing of new cars to women and minorities? *Quantitative Marketing and Economics*, 1:65–92, 2003.

[32] H. Smith. Ford boss wishes incentives to buy automobiles would go away. *Knight Ridder Tribune Business News*, January 2004.

[33] M. Sohoni, S. Chopra, U. Mohan, and M.N. Sendil. Impact of stair-step incentives and dealer structures on a manufacturer's sales variance. Working Paper, Indian School of Business, 2005.

[34] D. Spulber. Noncooperative equilibrium with price discriminating firms. *Economic Letters*, 4:221–227, 1979.

[35] D.L. White and M.C. Walker. First degree price discrimination and profit maximization. *Southern Economic Journal*, 81(4):313–318, 1973.

[36] F. Zettelmeyer, F. Scott Morton, and J. Silva-Risso. Inventory fluctuations and price discrimination: The determinants of price variation in car retailing. Working Paper, Yale University.

[37] F. Zettelmeyer, F. Scott Morton, and J. Silva-Risso. How the internet lowers prices: Evidence from matched survey and automobile transaction data. *Journal of Marketing Research*, 43(2):168–181, 2006.