Supplemental Appendix

Supplemental Appendix is for referees and editors only.

Analysis of the model with market expansion

We model this situation with an increased market potential $a^D = (1+\alpha)a$, $\alpha \geq 0$, when the manufacturer offers a rebate R. In this case, the retailer's best response to a given R, w, and α is $Q_{R'} = \left(\frac{(1+\alpha)a+R-w-m}{b}\right)^+$. The manufacturer's problem is to find the optimal values for R and α to maximize profit as follows:

$$\max_{R\geq 0, \alpha\geq 0, w\geq c+R} \quad (w-c-R)\left(\frac{(1+\alpha)a+R-w-m}{b}\right)^+ - e\alpha^2$$

When the manufacturer offers a customer rebate and the rebate leads to market expansion, the SPNE is shown in Table 9.

Table 9: Equilibrium decisions when customer rebates expand market potential

	$a^2 \geq 4be$ or $be \leq a^2 \leq 2be$	$a^2 \leq be$ or $2be \leq a^2 < 4be$
$w_{R'} - R$	$a - m$	$\frac{a^2c+2be(m-c-a)}{a^2-4be}$
α	$\frac{(a-m-c)a}{2be}$	$\frac{(a-m-c)a}{4be-a^2}$
$Q_{R'}$	$\frac{(a-m-c)a^2}{2b^2e}$	$\frac{(a-m-c)2e}{4be-a^2}$
$\Pi^M_{R'}$	$\frac{(a-m-c)^2a^2}{4b^2e}$	$\frac{(a-m-c)^2e}{4be-a^2}$
$\Pi^D_{R'}$	$\frac{a^2(a-m-c)(a^2(a-m-c)+4bem)}{8b^3e^2}$	$\frac{2e(a-m-c)(be(a+3m-c)-a^2m)}{(4be-a^2)^2}$
$\Pi^{SC}_{R'}$	$\frac{a^2(a-m-c)(a^2(a-m-c)+2be(a+m-c))}{8b^3e^2}$	$\frac{e(a-m-c)(-a^2(a+m-c)+2be(a+m-3c))}{(4be-a^2)^2}$

In Table 10, we compare the manufacturer's profits and sales with the retailer incentive and the customer rebate in two feasible regions and we see that the manufacturer can be better off with alternate promotions depending on the system parameters.

Proof of Observation 1 Observation 1(i) is trivial since $Q_o = Q_R$ and $\Pi^M_o = \Pi^M_R$. Observation 1(ii) follows from Table 11.

Analysis of fixed retail price

The analysis with perfect price discrimination has the demand function of the form $P(Q) = a - bQ$. We model the alternative case where the the retailer sets a fixed price r instead of charging a different price for each unit that is equal to the customer's willingness to pay. For a fair

Table 10: Comparison of retailer incentive and customer rebate for the deterministic demand model when rebates lead to market expansion

Condition		$\Pi_I^M - \Pi_{R'}^M$	$Q_I - Q_{R'}$
$a \leq 2m + c$	$eb \leq a^2 \leq 2be$	$\frac{(2be-a^2)(a-m-c)^2}{4b^2e} \geq 0$	$\frac{(2be-a^2)(a-m-c)}{2b^2e} \geq 0$
	$2be \leq a^2 \leq 4be$	$\frac{(2be-a^2)(a-m-c)^2}{2b(4be-a^2)} \leq 0$	$\frac{(2be-a^2)(m+c-a)}{b(a^2-4be)} \leq 0$

Table 11: Comparison of retailer incentive and customer rebate for the deterministic demand model

	$a \leq 2m + c$	$a \geq 2m + c$
$Q_I - Q_R$	$\frac{(a-m-c)}{2b} \geq 0$	$\frac{m}{2b} \geq 0$
$\Pi_I^M - \Pi_R^M$	$\frac{(a-m-c)^2}{4b} \geq 0$	$\frac{m(2a-2c-3m)}{4b} \geq 0$
$\Pi_I^D - \Pi_R^D$	$\frac{(a-m-c)(5m+c-a)}{8b} \geq 0$	$\frac{m(7m+2c-2a)}{8b} \geq 0$ for $a \leq \frac{7}{2}m + c$
$\Pi_I^{SC} - \Pi_R^{SC}$	$\frac{(a-m-c)(a+3m-c)}{8b} \geq 0$	$\frac{m(2a+m-2c)}{8b} \geq 0$

comparison of these models, we derive the following correspondence between the demand functions: $r = P(Q) = a - bQ \Rightarrow Q = \frac{a-r}{b}$. Then, the retailer's decision becomes to find the optimal retail price instead. We make the same assumptions as in the base model such as the retailer's reservation price requirement, and analyze the no-promotion, retailer incentive, and customer rebate cases with deterministic demand.

<u>No Promotion</u>

The Retailer's Problem:

$$\begin{aligned} \max_{r} \quad & (r - w)\left(\frac{a - r}{b}\right) \\ \text{s.t.} \quad & r \geq w + m \end{aligned}$$

$r^* = \max\left\{w + m, \frac{a+w}{2}\right\}$

Case 1) $w \geq a - 2m \Rightarrow r^* = w + m$.

The Manufacturer's Problem:

$$\begin{aligned} \max_{w} \quad & (w - c)\left(\frac{a - w - m}{b}\right) \\ \text{s.t.} \quad & a - m \geq w \geq a - 2m \\ & w \geq c \end{aligned}$$

$w^* = max\left\{a-2m, min\left\{\frac{a-m+c}{2}, a-m\right\}\right\}$

Case 1.a) $a \leq 3m+c$

$w^* = \frac{a-m+c}{2}$; $r^* = \frac{a+m+c}{2}$; $\Pi^M = \frac{(a-m-c)^2}{4b}$

Case 1.b) $a \geq 3m+c$

$w^* = a-2m$; $r^* = a-m$; $\Pi^M = \frac{m(a-2m-c)}{b}$

Case 2) $w \leq a-2m \Rightarrow r^* = \frac{a+w}{2}$.

The Manufacturer's Problem:

$$\begin{array}{ll} \max\limits_{w} & (w-c)\left(\dfrac{a-w}{2b}\right) \\ \text{s.t.} & c \leq w \leq a-2m \end{array}$$

$w^* = min\left\{\frac{a+c}{2}, a-2m\right\}$

Case 2.a) $a \geq 4m+c$

$w^* = \frac{a+c}{2}$; $r^* = \frac{3a+c}{4}$; $\Pi^M = \frac{(a-c)^2}{8b}$

Case 2.b) $a-2m \leq a \leq 4m+c$

$w^* = a-2m$; $r^* = a-m$; $\Pi^M = \frac{m(a-2m-c)}{b}$

The SPNE for the no-promotion case is summarized in Table 12. The SPNE for the no-promotion case in perfect price discrimination setting is found in Section 3.1 as follows:

$w_o = \frac{a+c-m}{2}$; $Q_o = \frac{a-m-c}{2b}$; $\Pi_o^M = \frac{1}{b}\left(\frac{a-m-c}{2}\right)^2$; $\Pi_o^D = \frac{(a-m-c)(a+3m-c)}{8b}$;
$\Pi_o^{SC} = \frac{(a-m-c)(3a+m-3c)}{8b}$.

We can show that both the retailer and the manufacturer are strictly better off in terms of profits and sales when the retailer has the ability to perfectly price discriminate versus when he sets a fixed retail price, except when $c+m \leq a \leq c+3m$, where the sales and the manufacturer's profit are identical in both models.

<u>Retailer Incentive</u>

The Retailer's Problem:

$$\begin{array}{ll} \max\limits_{r} & (r-w)\left(\dfrac{a-r}{b}\right) + K \\ \text{s.t.} & (w+m-r)\left(\dfrac{a-r}{b}\right) \leq K \end{array}$$

Case 1) $w \leq a-2m \Rightarrow r^* = \frac{a+w}{2}$

The Manufacturer's Problem:

Table 12: The SPNE with no promotion where the retailer sets a fixed retail price

	$c+m \leq a \leq c+3m$	$c+3m \leq a \leq c+4m$	$a \geq c+4m$
w_o	$\frac{a+c-m}{2}$	$a-2m$	$\frac{a+c}{2}$
r_o	$\frac{a+c+m}{2}$	$a-m$	$\frac{3a+c}{4}$
Q_o	$\frac{a-m-c}{2b}$	$\frac{m}{b}$	$\frac{a-c}{4b}$
Π_o^M	$\frac{(a-m-c)^2}{4b}$	$\frac{m(a-2m-c)}{b}$	$\frac{(a-c)^2}{8b}$
Π_o^D	$\frac{m(a-m-c)}{2b}$	$\frac{m^2}{b}$	$\frac{(a-c)^2}{16b}$
Π_o^{SC}	$\frac{(a-m-c)(a+m-c)}{4b}$	$\frac{m(a-m-c)}{b}$	$\frac{3(a-c)^2}{16b}$

$$\begin{aligned}\max_{w,K} \quad & (w-c)\left(\frac{a-w}{2b}\right) - K \\ \text{s.t.} \quad & c \leq w \leq a-2m\end{aligned}$$

$w^* = min\{\frac{a+c}{2}, a-2m\}$; $K^* = 0$

Case 1.a) $a \geq 4m+c$

$w^* = \frac{a+c}{2}$; $r^* = \frac{3a+c}{4}$; $K^* = 0$; $\Pi^M = \frac{(a-c)^2}{8b}$; $\Pi^D = \frac{(a-c)^2}{16b}$

Case 1.b) $a-2m \leq a \leq 4m+c$

$w^* = a-2m$; $r^* = a-m$; $K^* = 0$; $\Pi^M = \frac{m(a-2m-c)}{b}$; $\Pi^D = \frac{m^2}{b}$

Case 2) $w \geq a-2m \Rightarrow r^* = \left(min\left\{\frac{a+w}{2}, \frac{a+w+m-\sqrt{(a-w-m)^2+4Kb}}{2}\right\}\right)^+$

Case 2.a) $\frac{a+w}{2} \geq \frac{a+w+m-\sqrt{(a-w-m)^2+4Kb}}{2} \geq 0$

$$\begin{aligned}\max_{K} \quad & (w-c)\left(\frac{a-w-m+\sqrt{4Kb+(a-w-m)^2}}{2b}\right) - K \\ \text{s.t.} \quad & \frac{a(w+m)}{b} \geq K \geq \frac{m^2-(a-w-m)^2}{4b}\end{aligned}$$

$$\begin{aligned}K^* &= \max\left\{\frac{m^2-(a-w-m)^2}{4b}, \min\left\{\frac{(a-m-c)(2w+m-a-c)}{4b}, \frac{a(w+m)}{b}\right\}\right\} \\ &= \max\left\{\frac{m^2-(a-w-m)^2}{4b}, \frac{(a-m-c)(2w+m-a-c)}{4b}\right\}\end{aligned}$$

Case 2.a.1) $\frac{m^2-(a-w-m)^2}{4b} \geq \frac{(a-m-c)(2w+m-a-c)}{4b} \Rightarrow (w-c)^2 \leq m^2$

$K^* = \frac{m^2-(a-w-m)^2}{4b}$

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w}{2b}\right) - \frac{m^2-(a-w-m)^2}{4b} \\ \text{s.t.} \quad & a-2m \leq w \\ & (w-c)^2 \leq m^2 \\ & w \geq c \end{aligned}$$

$w^* = max\{a-2m, m+c\}$

Case 2.a.1.a $a \geq 3m+c$

$w^* = a-2m$; $K^* = 0$; $\Pi^M = \frac{a(a-2m-c)}{b}$

Case 2.a.1.b $2m+c \leq a \leq 3m+c$

$w^* = m+c$; $K^* = \frac{(3m-a+c)(a-m-c)}{4b}$; $\Pi^M = \frac{(a-m-c)^2}{4b}$

Case 2.a.2) $\frac{m^2-(a-w-m)^2}{4b} \leq \frac{(a-m-c)(2w+m-a-c)}{4b} \Rightarrow (w-c)^2 \geq m^2$

$K^* = \frac{(a-m-c)(2w+m-a-c)}{4b}$

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w-m+\sqrt{4K^*b+(a-w-m)^2}}{2b}\right) - K^* \\ \text{s.t.} \quad & a-2m \leq w \\ & (w-c)^2 \geq m^2 \\ & w \geq c \end{aligned}$$

Objective function becomes independent of w when $K^* = \frac{(a-m-c)(2w+m-a-c)}{4b}$. Any (w, K) pair that satisfies the following relations is optimal with profit $\Pi^M = \frac{(a-m-c)^2}{4b}$:

$K^* = \frac{(a-m-c)(2w+m-a-c)}{4b}$; $w \geq a-2m$; $(w-c)^2 \geq m^2$; $w \geq \frac{a+c-m}{2}$.

Case 2.b) $0 \leq \frac{a+w}{2} \leq \frac{a+w+m-\sqrt{(a-w-m)^2+4Kb}}{2}$

$$\begin{aligned} \max_{K} \quad & (w-c)\left(\frac{a-w}{2b}\right) - K \\ \text{s.t.} \quad & K \leq \frac{m^2-(a-w-m)^2}{4b} \end{aligned}$$

Trivially, $K^* = 0$.

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w}{2b}\right) \\ & a-2m \leq w \leq a \\ & c \leq w \end{aligned}$$

$w^* = max\{\frac{a+c}{2}, a-2m\}$

Case 2.b.1) $a \leq 4m + c$

$w^* = \frac{a+c}{2}$; $r^* = \frac{3a+c}{4}$; $K^* = 0$; $\Pi^M = \frac{(a-c)^2}{8b}$

Case 2.b.2) $a \geq 4m + c$

$w^* = a - 2m$; $r^* = a - m$; $K^* = 0$; $\Pi^M = \frac{m(a-2m-c)}{b}$

Summary of the SPNE:

- When $a \leq c + 3m$, SPNE is the solution obtained in Case 2.b.1 ($w_I = \frac{a+c}{2}$; $K = 0$; $r_I = \frac{3a+c}{4}$; $Q_I = \frac{a-c}{4b}$; $\Pi_I^M = \frac{(a-c)^2}{8b}$; $\Pi_I^D = \frac{(a-c)^2}{16b}$).
- When $c + 3m \leq a \leq c + 4m$, solution obtained in Case 2.a.2 or Case 2.b.1 can be the unique equilibrium depending on the parameters, where $\Pi_I^M = max\left\{\frac{(a-c)^2}{8b}, \frac{(a-m-c)^2}{4b}\right\}$
- When $a \geq c + 4m$, the solution obtained in Case 2.a.2 is the unique equilibrium, where $\Pi_I^M = \frac{(a-m-c)^2}{4b}$

For each region we can show that $\Pi_I^M \geq \Pi_o^M$ when the retailer uses fixed price. This is identical to our result in the case of perfect price discrimination, where offering a retailer incentive increases the manufacturer's profits and sales. However, when the retailer sets a fixed retail price, offering a retailer incentive does not always increase sales. (This result is trivial to prove in cases that are not listed below.)

- When $a \leq c + 2m$, $\Pi_I^M = \frac{(a-c)^2}{8b} \geq \frac{(a-m-c)^2}{4b} = \Pi_o^M$ follows since $a - c \leq 2m$ and $a - c \leq 2a - 2c - m$. $Q_o = \frac{a-m-c}{2b} \geq Q_I = \frac{a-c}{4b}$.
- When $c + 2m \leq a \leq c + 3m$, $\frac{(a-c)^2}{8b} - \frac{(a-m-c)^2}{4b}$ is decreasing in a, and takes its smallest value when $a = c + 3m$. We show that $\frac{(a-c)^2}{8b} - \frac{(a-m-c)^2}{4b} \geq 0$ when $a = c + 3m$.
- When $a \geq c + 4m$, $\frac{(a-m-c)^2}{4b} - \frac{(a-c)^2}{8b}$ is increasing in a, and takes its smallest value when $a = c + 4m$. We show that $\frac{(a-m-c)^2}{4b} - \frac{(a-c)^2}{8b} \geq 0$ when $a = c + 4m$.

Analysis of the deterministic demand model with per-unit retailer incentive

Let k denote the per-unit payment given by the manufacturer to the retailer for each unit of sale. We start the backward induction steps by solving the retailer's problem:

$$\begin{aligned} \Pi_I^D = \quad & \max_{Q\geq 0} && \int_0^Q (a-bQ)dQ-(w-k)Q \\ & \text{s.t.} && \int_{\frac{a-w-m}{b}}^Q ((w+m)-(a-bQ))dQ \leq kQ \\ &&& Q \geq \tfrac{a-w-m}{b} \end{aligned}$$

$Q^* = \left(\min\left\{\frac{a-w+k}{b}, \frac{a-w-m+k+\sqrt{k^2+2k(a-w-m)}}{b}\right\}\right)^+.$

In the next step, we solve the manufacturer's problem in a two step procedure similar to our analysis for the case of lump-sum incentive.

Case 1) $0 \leq \frac{a-w+k}{b} \leq \frac{a-w-m+k+\sqrt{k^2+2k(a-w-m)}}{b} \Rightarrow m^2 \leq k^2+2k(a-w-m)$

$$\begin{aligned} & \max_{k} && (w-c-k)\left(\frac{a-w+k}{b}\right) \\ & \text{s.t.} && k \geq (w-a)^+ \\ &&& k^2+2k(a-w-m)-m^2 \geq 0 \end{aligned}$$

$k^* = \max\left\{w-\frac{a+c}{2}, -a+w+m+\sqrt{(a-w-m)^2+m^2}\right\}$

Case 1.a) $a \geq 2m+c$; $\frac{(a-2m-c)^2}{4} \geq (a-w-m)^2+m^2 \Rightarrow k^* = w-\frac{a+c}{2}$

$$\begin{aligned} & \max_{w} && \frac{(a-c)^2}{4b} \\ & \text{s.t.} && c+k^* \leq w \leq a-m \\ &&& \frac{(a-2m-c)^2}{4} \geq (a-w-m)^2+m^2 \\ &&& (k^*)^2+2k^*(a-w-m)-m^2 \geq 0 \end{aligned}$$

Note that when $a \geq 4m+c$, any w that satisfies $a-m-\frac{1}{2}\sqrt{(a-c)(a-c-4m)} \leq w \leq a-m$ is a feasible solution with $\Pi_I^M = \frac{(a-c)^2}{4b}$.

Case 1.b) $a \geq 2m+c$; $\frac{(a-2m-c)^2}{4} \leq (a-w-m)^2+m^2 \Rightarrow k^* = -a+w+m+\sqrt{(a-w-m)^2+m^2}$

Case 1.c) $a \leq 2m+c \Rightarrow k^* = -a+w+m+\sqrt{(a-w-m)^2+m^2}$

In Case 1.b and Case 1.c, $c+k^* \leq w \Leftrightarrow \sqrt{(a-w-m)^2+m^2} \leq a-m-c$. Note also that $c \leq w \leq a-m \Rightarrow \sqrt{(a-m-c)^2+m^2} \leq a-m-c$, which is a contradiction. So there are no feasible solutions obtained in these regions.

Case 2) $\frac{a-w+k}{b} \geq \frac{a-w-m+k+\sqrt{k^2+2k(a-w-m)}}{b} \geq 0 \Rightarrow m^2 \geq k^2 + 2k(a-w-m)$

$$\begin{aligned} \max_{k} \quad & (w-c-k)\left(\frac{a-w-m+k+\sqrt{k^2+2k(a-w-m)}}{b}\right) \\ \text{s.t.} \quad & k \leq w-c \\ & k^2+2k(a-w-m)-m^2 \leq 0 \end{aligned}$$

$k^* = \min\left\{\frac{(w-c)^2}{2(a-m-c)}, -a+w+m+\sqrt{(a-w-m)^2+m^2}\right\}$
Case 2.a) $\frac{(w-c)^2}{2(a-m-c)} \leq -a+w+m+\sqrt{(a-w-m)^2+m^2}$

$$\begin{aligned} \max_{w} \quad & (w-c-k^*)\left(\frac{a-w-m+k^*+\sqrt{(k^*)^2+2k^*(a-w-m)}}{b}\right) \\ \text{s.t.} \quad & c+k^* \leq w \leq a-m \\ & (k^*)^2+2k^*(a-w-m)-m^2 \leq 0 \\ & \frac{(w-c)^2}{2(a-m-c)} \leq -a+w+m+\sqrt{(a-w-m)^2+m^2} \end{aligned}$$

where $k^* = \frac{(w-c)^2}{2(a-m-c)}$.

The objective function can be simplified to: $(w-c-k^*)\left(\frac{(a-w-m)+k^*+\frac{(w-c)}{2(a-m-c)}|w-2a+2m+c|}{b}\right)$. Note that $w-2a+2m+c \geq 0$ conflicts with $w \leq a-m$. Therefore, $w-2a+2m+c \leq 0$.

$$\begin{aligned} \max_{w} \quad & \frac{(w-c)(-w+2a-2m-c)}{2b} \\ \text{s.t.} \quad & w \leq a-m \\ & \frac{(w-c)^4}{(2a-2c-2m)^2} + \frac{2(w-c)^2(a-w-m)}{(2a-2m-2c)} - m^2 \leq 0 \\ & \frac{(w-c)^2}{2(a-m-c)} \geq -a+w+m+\sqrt{(a-w-m)^2+m^2} \end{aligned}$$

$$w^* = \begin{cases} a-m & \text{if } a \leq 3m+c \\ a-m-\sqrt{(a-m-c)(a+m-c)} & \text{if } a \geq 3m+c \end{cases} \tag{2}$$

Case 2.a.1) $a \leq 3m+c \Rightarrow w^* = a-m$; $k^* = \frac{a-m-c}{2}$; $Q^* = \frac{a-m-c}{b}$; $\Pi_I^M = \frac{(a-m-c)^2}{2b}$

Case 2.a.2) $a \geq 3m+c \Rightarrow w^* = a-m-\sqrt{(a-m-c)(a+m-c)}$; $k^* = a-c-\sqrt{(a-m-c)(a+m-c)}$; $\Pi_I^M = \frac{(a-m-c)^2-(a-m-c)(a+m-c)}{2b}$

Case 2.b) $\frac{(w-c)^2}{2(a-m-c)} \geq -a+w+m+\sqrt{(a-w-m)^2+m^2}$

$$
\begin{aligned}
\max_{k} \quad & (w-c-k^*)\left(\frac{a-w-m+k^*+\sqrt{(k^*)^2+2k^*(a-w-m)}}{b}\right) \\
\text{s.t.} \quad & c+k^* \le w \le a-m \\
& \frac{(w-c)^2}{2(a-m-c)} \le -a+w+m+\sqrt{(a-w-m)^2+m^2}
\end{aligned}
$$

where $k^* = -a+w+m+\sqrt{(a-w-m)^2+m^2}$.

Note that $c+k^* \le w \Leftrightarrow \sqrt{(a-w-m)^2+m^2} \le a-m-c$. Note also that $c \le w \le a-m \Rightarrow \sqrt{(a-m-c)^2+m^2} \le a-m-c$, which is a contradiction. So there is no feasible solution obtained in this region.

The SPNE solutions are summarized as follows:

- When $a \le 3m+c$, SPNE is the solution obtained in Case 2.a.1, where $w^* = a-m$; $k^* = \frac{a-m-c}{2}$; $Q^* = \frac{a-m-c}{b}$; $\Pi_I^M = \frac{(a-m-c)^2}{2b}$.
- When $a \ge 3m+c$, SPNE is the solution obtained in Case 1.a, where $a-m-\frac{1}{2}\sqrt{(a-c)(a-c-4m)} \le w^* \le a-m$; $\Pi_I^M = \frac{(a-c)^2}{4b}$.

We can show that in each region, $\Pi_I^M \ge \Pi_o^M$. Therefore, the manufacturer is always better off with a per-unit retailer incentive. (Same result holds for a lump-sum incentive.) Moreover, when $a \ge 3m+c$, the manufacturer's profit is higher with a per-unit incentive than a lump-sum incentive.

Analysis of the retailer incentive case in the deterministic demand model with pass-through rate $0 < \rho \le 1$

The Retailer's Problem:

$$
\begin{aligned}
\Pi^R = \quad & \max_{Q \ge 0} \quad \int_0^Q (a-bQ)dQ - wQ \\
& \text{s.t.} \quad \int_{\frac{a-w-m}{b}}^{Q} ((w+m)-(a-bQ))dQ \le \rho K \\
& \qquad\quad Q \ge \frac{a-w-m}{b}
\end{aligned}
$$

$Q^* = \left(min\left\{\frac{a-w}{b}, \frac{a-w-m+\sqrt{2\rho K b}}{b}\right\}\right)^+$.

Case 1) $K \leq \frac{m^2}{2b\rho}$; $\sqrt{2\rho K b} \geq w - (a-m)$; $\Rightarrow Q^* = \frac{a-w-m+\sqrt{2\rho K b}}{b}$. First we solve for K:

$$\begin{aligned} \max_{K} \quad & (w-c)\left(\frac{a-w-m+\sqrt{2\rho K b}}{b}\right) - K \\ \text{s.t.} \quad & 0 \leq K \leq \frac{m^2}{2b\rho} \\ & \sqrt{2\rho K b} \geq w - (a-m) \end{aligned}$$

$K^* = min\{\frac{\rho(w-c)^2}{2b}, \frac{m^2}{2b\rho}\}$. Next, we solve for w. We need to consider the following cases: $w \leq c + \frac{m}{\rho}$ and $w \geq c + \frac{m}{\rho}$, where $K^* = \frac{\rho(w-c)^2}{2b}$ and $K^* = \frac{m^2}{2b\rho}$, respectively.

Case 1.a) $w \geq c + \frac{m}{\rho}$

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w}{b}\right) - \frac{m^2}{2b\rho} \\ \text{s.t.} \quad & \frac{m}{\rho} + c \leq w \leq a \end{aligned}$$

$w^* = max\{\frac{m}{\rho} + c, min\{a, \frac{a+c}{2}\}\}$

Case 1.a.1) $\rho a \geq 2m + \rho c \Rightarrow w^* = \frac{a+c}{2}$; $Q^* = \frac{a-c}{2b}$; $K^* = \frac{m^2}{2b\rho}$; $\Pi^M = \frac{(a-c)^2}{4b} - \frac{m^2}{2b\rho}$.

Case 1.a.2) $\rho a \leq 2m + \rho c \Rightarrow w^* = \frac{m}{\rho} + c$; $Q^* = \frac{a - \frac{m}{\rho} - c}{b}$; $K^* = \frac{m^2}{2b\rho}$; $\Pi^M = \frac{(m+\rho c)(\rho a - m - \rho c)}{\rho^2 b} - \frac{m^2}{2b\rho}$.

Case 1.b) $w \leq c + \frac{m}{\rho}$

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-(1-\rho)w-m-\rho c)}{b}\right) - \frac{\rho(w-c)^2}{2b} \\ \text{s.t.} \quad & c \leq w \leq c + \frac{m}{\rho} \end{aligned}$$

$w^* = max\{c, min\{c + \frac{m}{\rho}, \frac{a-m-(\rho-1)c}{2-\rho}\}\}$

Case 1.b.1) $\rho a \geq 2m + \rho c \Rightarrow w^* = \frac{m}{\rho} + c$; $Q^* = \frac{a - \frac{m}{\rho} - c}{b}$; $K^* = \frac{m^2}{2b\rho}$; $\Pi^M = \frac{(m+\rho c)(\rho a - m - \rho c)}{\rho^2 b} - \frac{m^2}{2b\rho}$.

Case 1.b.2) $\rho a \leq 2m + \rho c \Rightarrow w^* = \frac{a-m-(\rho-1)c}{2-\rho}$; $Q^* = \frac{a-m-c}{(2-\rho)b}$; $K^* = \frac{\rho(a-m-c)^2}{(\rho-2)^2 2b}$; $\Pi^M = \frac{(a-m-c)^2}{2b(2-\rho)}$.

Case 2) $K \leq \frac{m^2}{2b\rho}$; $w \leq a \Rightarrow Q^* = \frac{a-w}{b}$. We obtain the following solution:

$w^* = \frac{a+c}{2}$; $Q^* = \frac{a-c}{2b}$; $K^* = \frac{m^2}{2b\rho}$; $\Pi^M = \frac{(a-c)^2}{4b} - \frac{m^2}{2b\rho}$.

We summarize the SPNE in Table 13.

Note that, $\Pi_R^M = \frac{(a-m-c)^2}{4b}$. We can show that the manufacturer is better off with the retailer incentive than the customer rebate where the equilibrium for the latter is stated in Theorem 1(iii).

$\Pi_I^M - \Pi_R^M = \frac{m(-2\rho c + 2a\rho - m\rho - 2m)}{4b\rho} \geq 0$, when $\rho a \geq 2m + \rho c$, since $\rho a \geq 2m + \rho c \Rightarrow 2a\rho \geq 4m + 2\rho c \geq (2+\rho)m + 2\rho c$.

$\Pi_I^M - \Pi_R^M \geq 0$, when $\rho a \leq 2m + \rho c$, since $0 < \rho \leq 1$.

Table 13: The SPNE for the deterministic demand model with retailer incentive when the incentive pass-through rate is $0 < \rho \leq 1$

	$\rho a \leq 2m + \rho c$	$\rho a \geq 2m + \rho c$
w_I	$\frac{a-m-(\rho-1)c}{2-\rho}$	$\frac{a+c}{2}$
Q_I	$\frac{a-m-c}{(2-\rho)b}$	$\frac{a-c}{2b}$
K	$\frac{\rho(a-m-c)^2}{(\rho-2)^2 2b}$	$\frac{m^2}{2b\rho}$
Π_I^M	$\frac{(a-m-c)^2}{2b(2-\rho)}$	$\frac{(a-c)^2}{4b} - \frac{m^2}{2b\rho}$
Π_I^D	$\frac{(a-m-c)((1-\rho)(a-c)-m(\rho-3))}{b}$	$\frac{\rho(a-c)^2+4m^2}{8\rho b}$

Proof of Theorem 2(i).

No Promotion

Retailer's best response in "high" and "low" state is: $Q^{j*} = \left(\frac{a^j-w-m}{b}\right)^+; \quad j = l, h.$

Case 1) $w \leq a^l - m \Rightarrow Q^h = \frac{a^h-w-m}{b}; Q^l = \frac{a^l-w-m}{b}$

Case 2) $a^l - m \leq w \leq a^h - m \Rightarrow Q^h = \frac{a^h-w-m}{b}; Q^l = 0$

Case 3) $w \geq a^h - m \Rightarrow Q^h = 0; Q^l = 0$

Note that we can omit Case 3 since the manufacturer has the feasible solution of setting $w = c$ and receive zero profit. We analyze Cases 1 and 2 and find the manufacturer's optimal wholesale price in each case.

Case 1) $w \leq a^l - m \Rightarrow Q^h = \frac{a^h-w-m}{b}; Q^l = \frac{a^l-w-m}{b}$

$$\begin{aligned} \text{max} \quad & \beta(w-c)Q^h + (1-\beta)(w-c)Q^m \\ \text{s.t} \quad & w \leq a^l - m \end{aligned}$$

$w^* = min\{a^l - m, \frac{\beta a^h+(1-\beta)a^l-m+c}{2}\}$

Case 1.a) (NP.1) $\beta \leq \frac{a^l-m-c}{a^h-a^l} \Rightarrow w^* = \frac{\beta a^h+(1-\beta)a^l-m+c}{2}; Q^h = \frac{(2-\beta)a^h-(1-\beta)a^l-m-c}{2b}; Q^l = \frac{(1+\beta)a^l-\beta a^h-m-c}{2b}; \Pi^M = \frac{(\beta a^h+(1-\beta)a^l-m-c)^2}{4b}$

Case 1.b) (NP.2) $\beta \geq \frac{a^l-m-c}{a^h-a^l} \Rightarrow w^* = a^l-m; Q^h = \frac{a^h-a^l}{b}; Q^l = \frac{a^h-a^l}{b}; \Pi^M = \frac{\beta(a^l-m-c)(a^h-a^l)}{b}$

Case 2) $a^l - m \leq w \leq a^h - m \Rightarrow Q^h = \frac{a^h-w-m}{b}; Q^l = 0$

$$\begin{aligned} \text{max} \quad & \beta(w-c)Q^h \\ \text{s.t} \quad & a^l - m \leq w \leq a^h - m \end{aligned}$$

$w^* = max\{a^l - m, min\{a^h - m, \frac{a^h-m+c}{2}\}\}$

Case 2.a) (NP.3) $a^h - a^l \leq a^l - m - c \Rightarrow w^* = a^l - m$; $Q^h = \frac{a^h-a^l}{b}$; $\Pi^M = \frac{\beta(a^l-m-c)(a^h-a^l)}{b}$

Case 2.b) (NP.4) $a^h - a^l \geq a^l - m - c \Rightarrow w^* = \frac{a^h-m+c}{2}$; $Q^h = \frac{a^h-m-c}{2b}$; $\Pi^M = \frac{\beta(a^h-m-c)^2}{4b}$

We summarize the feasible solutions obtained in different regions in Table 14.

Table 14: All feasible solutions for the uncertain market potential model with no promotion

	NP.1	NP.2	NP.3	NP.4
w^*	$\frac{\bar{a}-m+c}{2}$	$a^l - m$		$\frac{a^h-m+c}{2}$
Q^{l*}	$\frac{(1+\beta)a^l-\beta a^h-m-c}{2b}$	0		0
Q^{h*}	$\frac{(2-\beta)a^h-(1-\beta)a^l-m-c}{2b}$	$\frac{a^h-a^l}{b}$		$\frac{a^h-m-c}{2b}$
Π^M	$\frac{(\bar{a}-m-c)^2}{4b}$	$\frac{\beta(a^l-m-c)(a^h-a^l)}{b}$		$\frac{\beta(a^h-m-c)^2}{4b}$
F.R:	$\beta \leq \frac{(a^l-m-c)}{(a^h-a^l)}$	$\beta \geq \frac{(a^l-m-c)}{(a^h-a^l)}$	$2a^l \geq a^h + m + c$	$2a^l \leq a^h + m + c$

Next, we show that the SPNE in Table 3 (Section 3.2) is the dominating solution in the region it is feasible.

- When $a^l - m - c \geq a^h - a^l$, it follows that $\beta \leq \frac{a^l-m-c}{a^h-a^l}$, and feasible solutions are NP.3 and NP.1. The manufacturer chooses NP.1 since $\Pi^M(NP.1) - \Pi^M(NP.3) = \frac{(\beta a^h-(1+\beta)a^l+m+c)^2}{4b} \geq 0$.
- When $a^l - m - c \leq a^h - a^l$, either $\beta \leq \frac{a^l-m-c}{a^h-a^l}$ or $\beta \geq \frac{a^l-m-c}{a^h-a^l}$ may hold. NP.4 is feasible in this region independent of the value of β. In addition to this, NP.1 is feasible when $\beta \leq \frac{a^l-m-c}{a^h-a^l}$ and NP.2 is feasible when $\beta \geq \frac{a^l-m-c}{a^h-a^l}$. Note the following:

  $\Pi^M(NP.4) - \Pi^M(NP.2) = \frac{\beta(a^h-2a^l+m+c)^2}{4b} \geq 0.$

  $\Pi^M(NP.1) - \Pi^M(NP.4) = \frac{(1-\beta)(\beta(a^h-a^l)^2-(a^l-m-c)^2)}{4b}$

  In the second equality, when $\beta \leq \frac{(a^l-m-c)^2}{(a^h-a^l)^2}$ $\Pi^M(NP.1) \geq \Pi^M(NP.4)$; otherwise $\Pi^M(NP.4) \geq \Pi^M(NP.1)$.

Proof of Proposition 1

Note that, a necessary condition for β_1 to exist is $(a^h - a^l - 2m)^2 + 4m(3m + 2c - 2a^l) \geq 0$.

$\frac{\partial \beta^*}{\partial a^h} = \frac{2(a^h-a^l)(2m^2-(a^l-c)^2)}{(a^h-a^l)^4} \leq 0$, since $a^l - c \geq 2m \Rightarrow (a^l - c)^2 \geq 4m^2 \geq 2m^2$.

$\frac{\partial \beta^*}{\partial a^l} = \frac{2((a^l-c)(a^h-c)-2m^2)}{(a^h-a^l)^3} \geq 0$, since $a^h - c \geq a^l - c \geq 2m \Rightarrow (a^h - c)(a^l - c) \geq 2m^2$.

$\frac{\partial\beta_1}{\partial a^h} = \frac{(a^h-a^l-2m-A)(a^l-a^h-A)}{2A(a^h-a^l)^2} \leq 0$, where $A = \sqrt{(a^h-a^l-2m)^2+4m(3m+2c-2a^l)}$. The inequality follows since $a^l \geq 2m+c \Rightarrow 3m+2c-2a^l \leq 0$, $(a^h-a^l-2m-A) \geq 0$, and $a^l-a^h-A \leq 0$.

$\frac{\partial\beta_1}{\partial a^l} = \frac{m(3a^h+a^l-8m-4c-A)}{A(a^h-a^l)^2} \geq \frac{m(3a^h+a^l-8m-4c-a^h+a^l+2m)}{A(a^h-a^l)^2} = \frac{m(2a^h+2a^l-6m-4c)}{A(a^h-a^l)^2} \geq 0$, where $A = \sqrt{(a^h-a^l-2m)^2+4m(3m+2c-2a^l)}$. The inequalities follow since $a^l \geq 2m+c \Rightarrow 2a^l \geq 3m+2c$, and $A \leq (a^h-a^l-2m)$.

$\frac{\partial\beta_2}{\partial a^h} = \frac{-2m(a^l-m-c)^2}{((a^l-c)^2+2m(a^h-a^l-m))^2} \leq 0$.

$\frac{\partial\beta_2}{\partial a^l} = \frac{2m(a^l-m-c)(2a^h-3m-2c)}{((a^l-c)^2+2m(a^h-a^l-m))^2} \geq 0$, since $a^h \geq 2m+c \Rightarrow 2a^h \geq 3m+2c$.