INSTRUCTIONS

• Work alone. Do not collaborate with or copy from anyone else.

• You may NOT use a calculator for this exam; all computations should be easy enough to do by hand.

• Answer all seven questions. The extra credit questions are optional. Show your work, so that I can give you partial credit if you don’t get the correct answer.

• When asked to explain “why”, you may keep your answers brief – the questions are designed so that you do not need to provide long answers.

• The exam is “closed-book”. You may not refer to any books, notes, or other materials during class, aside from the exam materials themselves.

• Good luck!

NOTE FOR PROCTORS: The December 3rd date is only approximate; please allow your students some flexibility in order to give them enough time to prepare. Thanks!
1. (16 points) For each network problem (i) through (iv), choose the network diagram (a) through (d) that most accurately represents the problem description. Some network diagrams may not be used, and some may be used more than once.

(i) Given several warehouses and factories, find the least-expensive way of supplying the warehouses from the factories so that each warehouse gets what it needs without exceeding the production capacity of any factory.

(ii) Given the time lags on each connection of the internet, find the quickest route for your messages to a colleague across the country.

(iii) Given a network of roads and a number of police officers, decide which police officer should patrol each road for speeders.

(iv) Given the size of each part of the national power grid, determine how much power can be sent to California in the event of an emergency.
2. (25 points) A manufacturer of voting equipment is planning to build warehouses across the United States, and has narrowed its location choices to each of the state capitals. In each state capital, it will cost $h_i$ dollars per year to maintain a warehouse, and each warehouse will be able to store $c$ units of voting equipment. The manufacturer anticipates a yearly demand of $p_i$ products in each state $i$. All demand in a state is delivered to the state capital, at a cost of $t_{ij}$ dollars per unit per mile (where $i$ is the warehouse location and $j$ is the demand state). Let $d_{ij}$ be the distance between the capitals of states $i$ and $j$.

(a) (25 points) Formulate an integer program that the manufacturer can use to decide in which states it should build warehouses in order to minimize its total costs.

(b) (Extra Credit: 5 points) Suppose the decision of where to build the warehouses has already been made. What type of network flow problem is the manufacturer left with? Why?
3. (10 points) Model each of the following using integer programming constraints. Make your constraints as tight as possible.

(a) (5 points) To avoid fraud, two insurance companies would like to exchange data once each day. However, they can only do so if both companies purchase expensive computer hardware. Let $y_A$ and $y_B$ be binary variables describing whether company A and company B purchase the hardware. Write a constraint or constraints to restrict the number of yearly data exchanges unless the hardware is purchased by both companies.

(b) (Extra Credit: 5 points) A group of friends are browsing through the local Blockbuster Video store, trying to decide which movies to rent. The friends, all ISyE professors, would like to plan their movie-watching schedule using integer programming. Let $x_i = 1$ if movie $i$ is rented, and 0 if not. Write a constraint or constraints to say “If we rent both The Truman Show and Pleasantville, then we must rent exactly two of The Matrix, Jurassic Park, and Sixty One.”

(c) (5 points) A hardware factory produces screws. The current machinery takes $t_0$ seconds to produce each screw, and can be run for a total of $s_0$ seconds per day. The factory owner has the option of replacing the machinery with an updated version that can produce screws in $t_1$ seconds each, but can only be run for $s_1$ seconds per day without overheating. Write a constraint or constraints that will enforce the current screw-making capacity if the machinery is not replaced, and the new capacity if it is.
4. (15 points) In the following branch-and-bound tree, answer the following questions:

(a) (8 points) Which nodes do you still need to branch from? Why?

(b) (7 points) What is the gap between the best solution and the best bound found so far? Show your calculations.

5. (8 points) For the question below, select all of the correct answers from the choices given. You may need to select more than one answer.

If problem Y is NP-hard, to show that problem X is not NP-hard you need to:

(i) Transform problem Y into problem X in polynomial time.
(ii) Transform problem X into problem Y in polynomial time.
(iii) Show how to solve problem Y in polynomial time.
(iv) Show how to solve problem X in polynomial time.
6. (10 points) Answer **EITHER** question (a) **OR** question (b).

(a) (10 points) Write the constraint corresponding to the LINGO statement

\[ \sum_{i \in \text{origins}} \sum_{j \in \text{sinks}} f(i, j) \geq \text{level}(k); \]

(b) (10 points) Write the constraint corresponding to the AMPL statement

subject to \( \sum_{i \in \text{origins}} \sum_{j \in \text{sinks}} f[i,j] \geq \text{level}[k]; \)

7. (16 points) Consider the following network flow problem in which the dashed arcs are the basic arcs. (The numbers shown in the figure are the costs on each arc and the supplies/demands at each node.)

![Network Flow Diagram](image-url)

(a) (8 points) What are the flows on each arc at this basic solution?

(1,2)_______ (1,3)_______ (2,4)_______ (2,5)_______ (3,2)_____

(3,5)_______ (4,5)_______ (4,6)_______ (5,6)_______

(b) (4 points) Explain why this is not a feasible solution.

(c) (4 points) What primal simplex pivot (entering variable, leaving variable) would create a feasible basis from (a)?