Quality Assured Setup Planning Based on the Stream-of-Variation Model for Multistage Machining Processes

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ABSTRACT

Setup planning is a set of activities used to arrange manufacturing features into an appropriate sequence for processing. It has significant impacts on the product quality, which is often measured in terms of dimensional variation in the Key Product Characteristics (KPC). Current approaches to setup planning are experience-based and tend to be conservative by selecting unnecessarily precise machines and fixtures to ensure final product quality. This is especially true in multistage machining processes (MMP’s) because it has been difficult to predict the variation propagation and its impact on the KPC quality of final product. In this paper, a new methodology is proposed to realize cost-effective, quality assured setup planning for MMP’s. Setup planning is formulated as an optimization problem based on quantitative evaluation of variation propagations. The optimal setup plan minimizes the cost related to process precision (CRPP) and satisfies the quality specifications. The proposed approach can significantly improve the effectiveness as well as the efficiency of the setup planning for MMP’s.

Key words: Setup Planning, Multistage Machining Processes, Variation Propagation
1. Introduction

Process planning is the systematic determination of the steps by which a product is manufactured. It is a key element that bridges activities in design and manufacturing. In the past decades, process planning and its automation enablers have received extensive study and made significant progress (Maropoulos, 1995). Many reported approaches of process planning include conceptual process planning, setup planning and detailed process planning, as shown in Fig. 1. Conceptual process planning includes engineering feature recognition, process selection, and machine/tooling selection. Detailed process planning includes fixture design, quality-assurance-strategy selection, and cost analysis.

![Fig. 1. The existing commonly used setup planning approaches](image)

Setup planning constitutes a critical component that connects conceptual process planning and detailed process planning. Conceptual process planning provides qualitative information to setup planning, including designated features, selected processes and datum scheme constraints. The purpose of setup planning is to arrange manufacturing features into an appropriate sequence of setups in order to ensure product
quality and productivity (Huang and Liu, 2003). A setup plan is comprised of setup formation, datum scheme selection and setup sequencing (Huang, 1998). It defines a series of datum/fixturing schemes for a multi-stage machining process (MMP), as shown in Fig. 1. However, the setup plan obtained from those traditional methods provides limited instructive information to subsequent planning activities in detailed process planning.

Product quality is one of the main concerns of setup planning. A well defined setup plan should be able to satisfy quality specifications under normal manufacturing conditions. Product quality is affected by the outcome of setup planning since the series of datum and fixtures defined by a specific setup plan may introduce errors which will propagate along the machining stages and accumulate in the final product. Different setup plans specify different datum/fixturing schemes, lead to different variation propagation scenario, and result in different product quality. Thus, one of the major tasks in setup planning is to identify the optimal setup from multiple alternatives to ensure product quality.

Some research has been conducted in quality assured setup planning, addressing issues in setup formation, datum scheme selection and setup sequencing. Zhang et al. (1996) proposed principles for achieving tolerance control proactively via appropriately grouping and sequencing features according to their tolerance relationships. Mantripragada and Whitney (1998) presented the “datum flow chain” concept to relate datum logic explicitly with product KPC tolerances and assembly sequences. Quantitative approaches were also developed to evaluate variation stack-up associated to different process design. Rong and Bai (1996) presented a method to verify machining
accuracy corresponding to fixture design. Song et al. (2005) developed a Monte Carlo simulation-based method to analyze the quality impact of production planning. Xu and Huang (2006) modeled the simulated quality distributions in multiple attributes utility (MAU) function. Besides the simulation-based approaches, analytical methods were also studied to investigate the interactions between product quality and process variability. For a given setup plan, Stream of Variation (SoV) methodologies (Shi, 2006) and state space modeling techniques were developed to model the dimensional variation propagation along different setups (Hu, 1997; Jin and Shi, 1999; Ding et al, 2002; Zhou et al, 2003; Huang et al, 2007; Huang et al, 2007).

Cost-effectiveness is another critical concern in setup planning. It can be evaluated in terms of the cost related to process precision (CRPP), such as the cost to achieve necessary fixture precision to satisfy product quality requirements. The precision refers to the inherent variability in an MMP and CRPP is the cost for achieving necessary precision level to ensure the satisfaction of product quality requirements. CRPP is assumed to be inversely proportional to the necessary process precision. Corresponding to different setup plans, different process precision is required and thus different costs are incurred. Therefore, setup planning should be a discrete constrained optimization procedure. Ong et al (2002) considered various cost factors in the optimization index, including the cost of machines and fixtures. However, these cost factors are not directly linked with process precision.

It is desirable that the optimal setup plan is the one that satisfies product quality specification with relatively imprecise fixtures and machines to minimize the CRPP. However, the setup plans developed solely based on principles and experiences could be
very conservative. Although they are generally feasible with respect to the quality consideration, cost-effectiveness may not be optimal. For instance, in order to ensure the final product quality, engineers tend to conservatively select unnecessarily precise fixtures and thus cause unnecessary CRPP. This is especially true for the upstream stages of an MMP because of the lack of the capabilities for variation propagation evaluation. Furthermore, in order to automate the process planning, the outcomes of setup planning should be effectively integrated with other activities of detailed process planning, e.g. fixture design. Fixture layout design for a particular setup is critical input information for setup planning, whereas the setup planning results determine the MMP whose fixture system should be optimized on process level. However, although the fixture layout design has been successfully investigated on both single stage level (Cai et al., 1997) and process level (Kim and Ding, 2004), effective setup(fixture planning study is still primitive. This is because that the qualitative-principle based setup planning provides limited potential for specifying quantitative precision requirements of fixture design. In addition, conservative process precision requirements will make the designed fixture unnecessarily expensive. This functional limitation of conventional setup planning significantly hinders the implementation of the process planning automation.

Existing setup planning approaches are summarized in Table 1. As can be seen, most reported research has focused on the evaluation of setup plan alternatives. Some work exists for conducting optimal setup planning, based on qualitative or simulation-based evaluation of product quality. Although the simulation provides an effective strategy to compare alternative setup plans regarding their output product quality, it consumes a substantial amount of time and computational resources.
This paper adopts an integrated setup/fixture planning strategy in process planning. It focuses on the systematic development of a cost-effective, quality assured setup planning, which is a fundamental enabler of the integrated setup/fixture planning. Because of the complexity of the integrated problem and the overwhelming computational requirements, an iterative approach is appropriate. As illustrated in Fig. 2, the stage/setup level optimal fixture layouts for all candidate datum schemes are first determined and fixed. In each stage, different datum scheme options may be assigned with different fixture layouts. These stage/setup level fixture layouts are the inputs to the setup planning, together with the information on feature representation, design specification, constraints on datum scheme and setup sequence. As shown in Fig. 2, the development of the proposed setup planning consists of three steps: (i) Candidate setup formations and datum schemes are formulated based on input information. Their potential variation stack-up can be analytically predicted by the SoV model. (ii) Based on those candidate setups defined in step (i), the setup planning is formulated as a sequential decision making on an optimal series of setups that cost-effectively satisfies product quality specifications. A cost criterion is defined to evaluate the optimality of candidate setup plans under the constraints of product quality specifications. (iii) Dynamic
Programming (DP) is used to solve the optimal sequential decision making problem and generate the optimal setup plan, which provides setup information for subsequent activities in process planning. Based on analytical quality evaluation strategy, the proposed optimal setup planning methodology will be effective and efficient. When the optimal setup plan is determined, Kim and Ding (2004)’s approach can be applied to achieve process-level optimal fixture layout, which will be used to update the stage/setup level fixture layouts for repeating the iterative optimization procedure.

2. Quality Assured, Cost Effective Setup Planning

The design specifications of a machined product are often satisfied by machining operations performed on a series of stages. In each stage, a set of features will be
generated with a specific setup. Due to the variation of the machining operations, the dimensional precision of the final product is affected by three major variation sources:

(i) *Machine and cutting tool*, which refers to the random deviation of the cutting tools from their nominal paths.

(ii) *Fixture*, which refers to the random deviation of the fixture locators from their nominal positions.

(iii) *Datum*, which refers to the random deviation of the datum features, generated in previous stages, from their nominal positions and/or dimensions.

Both (i) and (ii) are treated as random process deviation. The third source exists because some features generated in the upstream stages are used as the datum features in the downstream stages according to the setup plan. Thus, the dimensional variation, which is introduced by fixtures and/or machine and cutting tools in the upstream stages, will be propagated through datum features and accumulated in the features generated in the downstream stages through datum features. Different setup plans, i.e., different datum schemes and different setup sequences, lead to different variation propagation scenarios, and thus, result in different final product quality. In order to compare candidate setup plans, an effective method is needed to evaluate the impacts of potential datum schemes and setup sequences on the quality of final product.

2.1 Variation propagation model for setup planning

One effective tool to model the variation propagation in MMP’s is the state space modeling technique (Shi, 2006). Zhou *et al.* (2003) presented a detailed derivation and validation of the model with given process/product design, including the information of setup formation, datum scheme selection and setup sequence. However, some additions
are necessary due to the following unique characteristics in setup planning:

(i) *Multiple datum scheme options:* In setup planning, every stage has a set of candidate datum schemes. Different datum schemes support different operations that generate different features, which further constrain the pool of candidate datum schemes for downstream stages. Also, datum scheme selection is directly related to the fixture design and thus significantly affects the CRPP. Thus, there is a need for explicit representation of the selected datum scheme for every stage.

(ii) *Setup precedence requirements:* According to the design specifications, some features must be fabricated in a stage with comparatively precise datum features, which may be machined in an upstream stage. This kind of precedence relationships is not often straightforward to determine, especially when the tolerance interdependences among features are complicated. Therefore, the capability to explicitly represent the sequence of setups and the chain of datum schemes is needed to evaluate different setup precedence options.

(iii) *Tracing the setup chain:* Since the CRPP is inversely proportional to the precision of fixtures, process planners tend to select less precise fixtures to reduce the cost. However, this will increase the dimensional variation of the generated features and increase the datum variation if some of them are used as datum in the downstream stages. As a result, the datum features with large variation force the downstream fixtures to be very precise to satisfy quality specifications. In other words, due to the complex variation propagation, relaxing the upstream process precision may result in the need for tighter tolerances in the downstream processes and thus increase the total CRPP. Therefore, to achieve an overall cost-effectiveness, the variation propagation of the setup
chain must be traced and explicitly modeled.

Fig. 3 illustrates variation propagation scenario of the setup plan of an MMP. The nomenclature is explained as follows:

- The datum scheme (DS) of stage $k$ ($k = 1, 2, \ldots, N$) is denoted as DS $d_k$ ($d_k = 1, 2, \ldots, D_k$, where $D_k$ is the total number of feasible datum scheme options for stage $k$). A datum scheme refers to the coordinate system specified by a group of datum surfaces, within which the machining process can be performed. Datum scheme is very important to the variation propagation modeling since all those three aforementioned variation sources affect the quality of newly generated features through datum, as shown in Fig. 3.

- Corresponding to a selected datum scheme $d_k$ in stage $k$, the quality of all features are denoted by a state vector $x^d_k$, with each element represents the dimensional deviation from its nominal value.

- The random deviation of process variables associated with a selected datum scheme $d_k$ in stage $k$ is denoted as $u^{d_k}$. Corresponding to the major variation sources, $u^{d_k}$ models the random process deviations of both machine/cutting tools and fixture locators, as defined in (Zhou et al, 2003). Represented as deviations of the
tool path from its nominal path, $u^d_k$ models many types of sources, including geometric and kinematics errors, thermal errors, cutting force induced errors and tool-wear induced errors (Zhou et al., 2003). The elements in $u^d_k$ are called process variables and are treated as independent system input following multivariate normal distribution.

- The un-modeled system noises due to the model linearization are represented by $w_k$. Compared to the deviations modeled in $u^d_k$ and $x^d_k$, the elements in $w_k$ are higher order small values. $w_k$ are assumed to be independent of any component of $u^d_k (k = 1, 2, \ldots, N; d_k = 1, 2, \ldots, D_k)$. And, the elements of $w_k$ are assumed to be independent of each other and have zero mean.

- Since the features are measured in the coordinate system defined by the selected datum scheme $d_k$, the measurements of quality are denoted as $y^d_k$. In this paper, the measurements are assumed to be multivariate normal.

- The measurement noise is denoted by a random vector $v_k$, which is independent of $x^d_k$, $u^d_k$ and $w_k (k = 1, 2, \ldots, N; d_k = 1, 2, \ldots, D_k)$. The components of $v_k$ are assumed to be independent of each other and have zero mean. And the magnitudes of $v_k$ components are determined by the accuracy/precision of the measurement device, which are usually on the level of $\mu m$.

Adopting the assumptions of rigid part and small error, a linear state space model can be constructed to associate the product quality with a sequence of setups according to the setup plan, as shown in Eq. (1),
\[
\begin{align*}
\mathbf{x}^{d_k}_k &= \mathbf{A}^{d_k}_{k-1} \mathbf{x}^{d_{k-1}}_{k-1} + \mathbf{B}^{d_k}_{k} \mathbf{u}^{d_k}_k + \mathbf{w}_k, \\
\mathbf{y}^{d_k}_k &= \mathbf{C}^{d_k}_{k} \mathbf{x}^{d_k}_k + \mathbf{v}_k, \quad k = 1, 2, \ldots, N,
\end{align*}
\]

where \( \mathbf{A}^{d_k}_{k-1} \mathbf{x}^{d_{k-1}}_{k-1} \) represents the datum induced random deviation corresponding to the selected datum scheme \( d_k \) in stage \( k \), and \( \mathbf{x}^{d_{k-1}}_{k-1} \) is the quality, in terms of dimensional deviation, transmitted from upstream stages. \( \mathbf{B}^{d_k}_{k} \mathbf{u}^{d_k}_k \) describes the impact of deviation from the process variables, corresponding to the selected datum scheme \( d_k \), in the quality of features generated in stage \( k \). \( \mathbf{C}^{d_k}_{k} \) is the observation matrix mapping features’ quality to the measurements. A validation of this SoV modeling in (Zhou et al, 2003) demonstrates that the SoV model can adequately represents the process errors and their propagations in the multistage machining process. Ren et al (2006) further demonstrated that the model linearization is valid when number of stage is moderate.

As aforementioned, setup planning is a series of decisions based on alternative datum schemes for multiple stages, as illustrated in Fig. 4. For the optimal datum scheme selected for stage \( k \), Eq. (1) can be reformulated as:

\[
\begin{align*}
\mathbf{x}^{d'_k}_k &= \mathbf{A}^{d'_k}_{k-1} \mathbf{x}^{d'_{k-1}}_{k-1} + \mathbf{B}^{d'_k}_{k} \mathbf{u}^{d'_k}_k + \mathbf{w}_k, \\
\mathbf{y}^{d'_k}_k &= \mathbf{C}^{d'_k}_{k} \mathbf{x}^{d'_k}_k + \mathbf{v}_k,
\end{align*}
\]

where \( d'_k \in \{ d_k \mid d_k = 1, 2, \ldots, D_k \} \), for \( k = 1, 2, \ldots, N \), represents the index of the selected datum scheme for stage \( k \).
optimal datum scheme in stage $k$. Please note that $d_k^*$ is one link of the optimal datum scheme chain $(d_1^* \ d_2^* \ ... \ d_N^*)$ that is determined through considering all the stages in the entire processes. Thus, $d_k^*$ may not necessarily be optimal for a single stage $k$.

The state space model in Eq. (1) can be transformed into a linear input-output model as:

$$ y_k^{d_k} = \sum_{i=1}^{k} C_k^{d_k} \Phi_{k,i}^{(*)} B_i^{d_i} u_i^{d_i} + C_k^{d_k} \Phi_{k,0}^{(*)} x_0 + \sum_{i=1}^{k} C_k^{d_k} \Phi_{k,i}^{(*)} w_i + v_k, \quad (3) $$

where $\Phi_{k,i}^{(*)}$ is the state transition matrix tracing the datum schemes transformation from stage $i$ to $k-1$; and $\Phi_{k,i}^{(*)} = A_{k-1}^{d_k} A_{k-2}^{d_i} \ldots A_i^{d_i}$ for $i < k$, and $\Phi_{k,k}^{(*)} = I$. Initial state vector $x_0$ represents the original quality of the part that enters the first stage of the process. Without loss of generality, $x_0$ is set to 0. Then Eq. (3) changes to

$$ y_k^{d_k} = \sum_{i=1}^{k} C_k^{d_k} \Phi_{k,i}^{(*)} B_i^{d_i} u_i^{d_i} + \sum_{i=1}^{k} C_k^{d_k} \Phi_{k,i}^{(*)} w_i + v_k, \quad (4) $$

For a selected datum scheme, $d_k$, and the decisions on datum schemes for upstream stages $\{d_1 \ d_2 \ ... \ d_{k-1}\}$, the coefficient matrices, $A_k^{d_k}$, $B_k^{d_k}$, $C_k^{d_k}$ and $\Phi_{k,i}^{(*)}$ ($i = 1,2,\ldots,k$), can be derived following the same procedure as presented in (Zhou et al, 2003). This variation propagation modeling technique provides the setup planner a tool to predict the product quality of candidate datum schemes and alternative setup sequences of an MMP. Compared to the method, proposed by Xu and Huang (2006), that can only assess the quality after the whole setup plan is defined, state space modeling provides the capability to assess product quality for each intermediate setup. This modeling technique can be effectively incorporated into the decision making process for the
optimal setup plan determination.

2.2 Setup plan evaluation strategy

Different setup plan will result in different product quality in terms of KPC variation and incur different CRPP. From the optimization point of view, setup planning can be formulated as a discrete constrained optimization problem.

2.2.1 Optimization of the setup planning

In this paper, the objective of setup planning is to minimize the CRPP while satisfying the KPC quality constraints. The mathematical representation is defined as:

$$\min_{T_u} \{C_{T_u}(T_u)\}$$

s.t. \( \frac{USL_i - LSL_i}{\sigma_{y_i}} \geq \tau_i, \quad i = 1, 2, ..., M. \) (5)

where \( T_u = [T_{u_1}, T_{u_2}, ..., T_{u_P}]^T \) is a \( P \times 1 \) vector with each element \( T_{u_i} \), represents the tolerance of a corresponding process variable \( u_i \) defined in \( u \), and

\( u = [u_1^T, u_2^T, ..., u_N^T]^T \), with \( u_k (k = 1, 2, ..., N) \) as a \( p_k \times 1 \) vector representing the process variables (i.e., fixture locator deviations) in stage \( k \). Please note that

\( P = \sum_{k=1}^{N} p_k \). \( M \) is the total number of KPC and \( P \) is the total number of process variables. \( USL_i \) and \( LSL_i \) are the predefined Upper Specification Limit and Lower Specification Limit of KPC \( y_i \), respectively. \( \sigma_{y_i} \) is the standard deviation of KPC \( y_i \) and \( \tau_j \) is a constant, \( i = 1, 2, ..., M \). \( C_{T_u}(T_u) \) is the CRPP function of process tolerance. Various cost functions have been proposed for different tolerance synthesis. Considering the structural simplicity, a reciprocal function is adopted in this paper:
where $w_j$'s, $j = 1, 2, \ldots, P$, are weighting coefficients. These weighting coefficients should be determined according to practical situation. For instance, coefficients assigned to the fixtures used in the same stage can be equal to each other; fixtures or machine tools manufactured by the same supplier or used in the same stage may be assigned with the same value. More discussions on the selection of those weighting coefficients are provided in the case studies in Section 3.

For a complicated MMP, there always exist multiple quality characteristics. It is desirable to define a multivariate process capability index for process quality control. However, at the setup planning stage, there is no a priori information of the correlations between quality characteristics. A scalar multivariate process capability index may be misleading if it is defined without appropriate consideration of correlations between quality characteristics. Thus, in industrial applications, for the sake of convenience, most of the tolerance regions are specified as a collection of individual specifications for each variable, as defined in Eq. (5). The intersection of these specifications would form a rectangular solid zone (Jackson, 1991). Chen (1994) proposed a multivariate process capability index over a rectangular solid tolerance zone $V = \{ \mathbf{y} \in R^M: \max(\{|y_i-\mu_i|/r_i, i=1,2,\ldots,M\}) \leq 1 \}$. Based on this definition, a necessary condition for a process to be capable over a rectangular solid zone is that each individual univariate process is capable with respect to the corresponding specification limits. In addition, according to the discussion of Chen (1994), correlations between quality characteristics make the process more capable over a rectangular tolerance zone. Therefore, in this paper, individual
process capability constraints are adopted to conservatively ensure that the setup plan is capable to satisfy to the specifications on all quality characteristics.

Ding et al. (2005) studied the relationship between tolerance and variation of process variables through examining the clearance of the pin-hole locating pair. In this paper, the process capability ratio, \( \eta_j = T_{u_j} / \sigma_{u_j} \), are assumed to be constants. Therefore, the tolerance of a process variable can be replaced by its standard deviation. Recall that the elements in \( d_k \) are defined as the deviations of fixture locators with zero mean, thus their variances \( \sigma^2_{u_j} = E(u_j^2) \), \( j = 1, 2, \ldots, P \). Let \( \Xi_u = [\sigma_{u_1}, \sigma_{u_2}, \ldots, \sigma_{u_P}]^T \), the tolerance of process variables can be defined by

\[
T_u = \begin{bmatrix}
T_{u_1} & T_{u_2} & \ldots & T_{u_P}
\end{bmatrix}^T = \text{diag} \{\eta_1, \eta_2, \ldots, \eta_P\}: \Xi_u.
\]

Then the objective function, \( C_{T_u}(T_u) \), in Eq. (5) can be transformed to:

\[
C_u(u) = \sum_{j=1}^P \frac{w_j}{\eta_j \cdot \sigma_{u_j}}.
\]

2.2.2 Dynamic Programming formulation

Previous sections present the techniques that enable: (i) the description of the impacts of datum scheme selection and setup sequencing on the variation of product quality, (ii) the modeling of the variation propagation, and (iii) the quantitative evaluation of the candidate setup plans. Based on these enablers, setup planning can be formulated as a sequential decision making on the selections of datum schemes in all stages to satisfy quality specifications with overall cost-effectiveness. In this sequential decision making problem, the datum scheme selected for stage \( k \) is affected by that selected for the upstream stages and will affect that selected for the downstream stages. This
characteristic is identical to that of dynamic programming (DP) problem. Therefore, DP methodology is adopted to solve the optimization problem. Fig. 5 illustrates a sequential decision process for a chain of datum scheme selection.

![DP network of setup planning decision sequence](image)

**Fig. 5.** DP network of setup planning decision sequence

In Fig. 5, there are \( N+1 \) columns in the diagram, representing the \( N \) stages of the machining processes, and an initial DP state \((\bullet, x_0)\). Each column \( k \) \((k = 1, 2, \ldots, N)\) consists of \( D_k \) nodes, corresponding to \( D_k \) feasible datum schemes. A node \((Q_k, x_{k_d})\), \( d_k = 1, 2, \ldots, D_k \), in Fig. 5 is a DP state that represents the datum scheme selection in stage \( k \), where \( Q_k \) defines the in-process quality specifications for the features generated from stage 1 to stage \( k \). Since the quality specifications for the incoming part is not related to the quality consideration of the machining process, it is set to “•” in the initial DP state, i.e., not specified. According to Eq. (5), \( Q_k \) is an \( M \times M \) matrix with the diagonal elements

\[
q_{k,i,i} = \left( \frac{[USL_{k,i} - LSL_{k,i}]}{\tau_i} \right)^2, \quad i = 1, 2, \ldots, M; \quad k = 1, 2, \ldots, N.
\]

USL_{k,i} and
\( LSL_{k,i} \) are the given in-process specification limits for KPC \( i \) in stage \( k \). The off-diagonal elements of \( Q_k \) can also be specified regarding to the covariance matrix structure of \( y_k^d \) for a given \( d_k \). The connections linking nodes in column \( k-1 \) to those in column \( k \) reflect state transitions. Given the datum scheme and setup sequence selected for upstream stages, different nodes from two neighbor stages are connected or disconnected, according to the pre-defined datum scheme constraints. Although there are \( D_k \) potential DP states for each stage, the process planner observes only the one that is finally selected. Therefore, the concept of “DP-stage” \((Q_k, x_k)\) is defined to “contain” all the possible states, \((Q_k, x_k^d), d_k = 1, 2, \ldots, D_k\), in a column \( k \) (Denardo, 2003). As shown in the bottom portion of Fig. 5, the \( u_k \) “contains” all the possible \( u_k^d \)'s, \( d_k = 1, 2, \ldots, D_k \). Associated with each DP-stage is a set of decisions \( \Theta_k \) on datum scheme selection.

Selecting datum scheme, \( d_k \), incurs cost \( V_k(u_k, d_k) \) and implements transition from DP-stage \((Q_{k-1}, x_{k-1})\) to DP-stage \((Q_k, x_k)\). Let \( q_k(u_k, d_k) \) be the constraints on the KPC variations generated in stage \( k \) if datum scheme \( d_k \) is selected. In other words, \( q_k(u_k, d_k) \) is the maximum KPC variations that can be allowed after the fabrication performed in stages 1 through \( k \). Also let \( t((Q_k, x_k), d_k, d_{k-1}) \) be the state transition function linking \( x_{k-1}^{d_{k-1}} \) and \( x_k^{d_k} \), then Eq. (1) can be of the form

\[
x_k^{d_k} = t((Q_k, x_k), d_k, d_{k-1}) = A_k^{d_k} x_{k-1}^{d_{k-1}} + B_k^{d_k} u_k^{d_k} + w_k .
\]

The decision-making on \( d_k \)'s, \( k = 1, 2, \ldots, N \) repeats itself for all stages, following \( t((Q_k, x_k), d_k, d_{k-1}) \). The cost of decision \( d_k \) in stage \( k \) is defined as
\[ V_k(u_k, d_k) = C_{u_k^{d_k}}(u_k^{d_k}) = \sum_{j=1}^{Q_k} \frac{w_j}{\eta_j \cdot \sigma_{u_j^{d_k}}} , \]  

where \( p_k \) (\( k = 1, 2, \ldots, N \)) is the dimension of \( u_k^{d_k} \) and \( P = \sum_{k=1}^{N} p_k \cdot \sigma_{u_j^{d_k}} \) is the standard deviation of the \( j \)-th element of \( u_k^{d_k} \), \( j = 1, 2, \ldots, p_k \). This cost can be interpreted as the cost consumed to provide enough process precision for stage \( k \), corresponding to the selected datum scheme \( d_k \). Let \( L(Q_k, x_k) \) be the minimum CRPP that is consumed from stage 1 to stage \( k \) by selecting datum schemes \( d_1, d_2, \ldots, d_k \), and generating quality variation at most \( Q_k \), the DP function can be defined as:

\[
L(Q_k, x_k) = \begin{cases} 
\min_{d_k \in \mathcal{d}_k, q_k(u_k, d_k) \in \mathcal{Q}_k} \{L(Q_k - q_k(u_k, d_k), t((Q_k, x_k), d_k, d_{k-1})) + V_k(u_k, d_k)\}, & \text{for } k = 1, \ldots, N, \\
0, & \text{for } k = 0,
\end{cases}
\]  

(9)

where \( Q_k \) is pre-defined, \( q_k(u_k, d_k) \) and \( V_k(u_k, d_k) \) can be derived based on the state space model (1). According to Eq. (4), the covariance matrix of \( y_k^{d_k} \) is:

\[
\Sigma_{y_k^{d_k}} = \sum_{i=1}^{k-1} \left( C_k^{d_i} \Phi_{k,i}^{(*)} B_k^{d_i} \right) \Sigma_{u_i^{d_i}} \left( C_k^{d_i} \Phi_{k,i}^{(*)} B_k^{d_i} \right)^T + \left( C_k^{d_k} \Phi_{k,k}^{(*)} B_k^{d_k} \right) \Sigma_{u_k^{d_k}} \left( C_k^{d_k} \Phi_{k,k}^{(*)} B_k^{d_k} \right)^T + \sum_{i=1}^{k} \left( C_k^{d_i} \Phi_{k,i}^{(*)} \right) \Sigma_{w_i} \left( C_k^{d_i} \Phi_{k,i}^{(*)} \right)^T + \Sigma_v ,
\]  

(10)

where \( \Sigma_\bullet \) is the covariance matrix for variable \( \bullet \). Eq. (10) shows that the KPC covariance can be treated as the accumulated covariance of all process variables used from stage 1 to stage \( k \), plus the covariance of the un-modeled process variations and the variance of measurement noise. In order to ensure that the product quality generated from stage 1 to stage \( k \) satisfies the specifications, \( \Sigma_{y_k^{d_k}} \) should satisfy the specification
\[
\sigma_{y_{k,i}}^2 \leq s \cdot q_{k,i,i}, \quad i = 1, 2, \ldots, M, \quad (11)
\]

where \( \sigma_{y_{k,i}}^2 \) is the \( i \)th diagonal element of matrix \( \Sigma_{y_{k,i}} \), \( q_{k,i,i} \) is the \( i \)th diagonal element of matrix \( Q_k \) and the scalar \( s \) is the safety-factor \((0 \leq s \leq 1)\). Since \( w_k \) and \( v_k \) contains second or higher order of small values whose magnitudes are much smaller than that of \( x^{d_k}_k \) and \( u^{d_k}_k \), their contribution to the \( \Sigma_{y^{d_k}_k} \) can be ignored. Thus, by eliminating the third and the fourth terms on the right-hand-side of Eq. (10), \( \Sigma_{y^{d_k}_k} \) can be approximated by

\[
\tilde{\Sigma}_{y^{d_k}_k} = \sum_{i=1}^{k-1} \left( C_k^{d_k} \Phi_{k,i}^{(*)} B_i^{d_k} \right) \Sigma_{u^{d_k}_k} \left( C_k^{d_k} \Phi_{k,i}^{(*)} B_i^{d_k} \right)^T + \left( C_k^{d_k} \Phi_{k,k}^{(*)} B_k^{d_k} \right) \Sigma_{u^{d_k}_k} \left( C_k^{d_k} \Phi_{k,k}^{(*)} B_k^{d_k} \right)^T. \quad (12)
\]

The first term on the right-hand-side of Eq. (12) stands for the quality covariance (measured based on datum scheme \( d_k \)) accumulated from stage 1 to stage \( k \), whereas the second term stands for the quality covariance generated in stage \( k \) by selecting datum scheme \( d_k \). Let

\[
\bar{\tilde{\Sigma}}_{y^{d_k}} = \sum_{i=1}^{k-1} \left(C_k^{d_k} \Phi_{k,i}^{(*)} B_i^{d_k}\right) \Sigma_{u^{d_k}_k} \left(C_k^{d_k} \Phi_{k,i}^{(*)} B_i^{d_k}\right)^T, \quad (13)
\]

be the quality covariance accumulated from stage 1 to stage \( k-1 \), the amount of newly generated quality covariance can be derived as:

\[
\Sigma_{d_k} = \tilde{\Sigma}_{y^{d_k}} - \bar{\tilde{\Sigma}}_{y^{d_k}} = \left(C_k^{d_k} \Phi_{k,k}^{(*)} B_k^{d_k}\right) \Sigma_{u^{d_k}_k} \left(C_k^{d_k} \Phi_{k,k}^{(*)} B_k^{d_k}\right)^T. \quad (14)
\]

Since the process cost modeled in Eq. (7) is inversely proportional to the process variations. In order to minimize the process cost, process variations, the diagonal elements in \( \Sigma_{u^{d_k}_k}, k=1, 2, \ldots, N \), should be relaxed as much as possible. This will lead to
the increase of the KPC variations defined by the diagonal elements in $\tilde{\Sigma}_{y_i}$. Considering the quality constraints specified by $Q_k$, $\tilde{\Sigma}_{y_i}$ should satisfy

$$\tilde{\Sigma}_{y_i} = sQ_k,$$

where $s$ is the same as that defined in Eq. (11). Given the $Q_k$'s, $k=1,2,...,N$, the constraints $q_k(u_k,d_k)$ has the form

$$q_k(u_k,d_k) = \left( sQ_k - \tilde{\Sigma}_{y_{i-1}} \right) = \left( C^d_k \Phi_k \hat{B}_k \right) \Sigma_{u_k} \left( C^d_k \Phi_k \hat{B}_k \right)^T.$$

From Eq. (16), the covariance matrix of $u_k^{d_i}$ can be derived as

$$\Sigma_{u_k} = \left( \Gamma_k^\ast \right)^\top \left( sQ_k - \tilde{\Sigma}_{y_{i-1}} \right) \left[ \left( \Gamma_k^\ast \right)^\top \right]^{-1},$$

where $\Gamma_k^\ast = C^d_k \Phi_k \hat{B}_k = C^d_k \hat{B}_k$, and $(\Gamma_k^\ast)^\top$ denotes the Moore-Penrose inverse of the rectangular matrix $\Gamma_k^\ast$. $\tilde{\Sigma}_{y_{i-1}}$ contains the variation propagation information and is determined by the datum scheme selection and sequencing decisions made for upstream stages. When $\Gamma_k^\ast$ is column-wise full rank, Eq. (17) can give a real solution of $\Sigma_{u_k}$. Assuming that the process variables are mutually independent, the tolerance specification for $u_k^{d_i}$ can be obtained by a $p_k \times 1$ vector

$$T_{u_k} = [\eta_k \sigma_1^{d_i}, \eta_k \sigma_2^{d_i},...,\eta_k \sigma_{p_k}^{d_i}]^T,$$

where $\left( \sigma_j^{d_i} \right)^2$ is the $j^{th}$ diagonal elements of $\Sigma_{u_k}$, $j=1,2,...,p_k$, $k=1,2,...,N$ and $d_i=1,2,...,D_k$. According to the definition of $u_k^{d_i}$, $T_{u_k}$ contains the tolerance of machining/cutting tools and fixture locators. In order to increase the exchangeability of
fixture locators, improve maintainability of the fixture system, and reduce the “Long-Run Overall Production Cost,” different locators on the same fixture are assigned with the same tolerance, as discussed by Chen et al (2006). Therefore, fixture locators’ tolerances can also be specified as $\eta_k \sigma^d_k$, where $\sigma^d_k = \min_{\eta \in J_f} \{\sigma^d_j\}$ and $J_f$ is a set containing all the index of fixture locators in $u^d_k$. With Equations from Eq. (8) to Eq. (18), setup planning can be formulated as solving a series of DP functional equations.

2.2.3 Optimization algorithm

Reaching algorithm (Denardo, 2003) is used to solve the dynamic programming problem defined in Eq. (9). According to Fig. 5, the value of each DP-state node $(Q, x^d_k)$ is denoted as $s_{k,d_k}$, which represents the minimum process precision cost incurred so far from stage 1 to stage $k$ by selecting datum scheme $d_k$ in stage $k$. Let $v^d_{k+1}$ denote the corresponding cost incurred in stage $k$ corresponding to datum selections of the upstream stage $k-1$ and that of the stage $k$, and $v^d_{k,d_k} = V_k(u^d_k, d_k)$. The pseudo code of the reaching algorithm is defined as:

(i) Set $s_{0,*} = 0$ and $s_{k,d_k} = +\infty$ for $k = 1, 2, \ldots, N$; $d_k = 1, 2, \ldots, D_k$,

(ii) DO for $k = 1, 2, \ldots, N$

(iii) DO for $d_k = 1, 2, \ldots, D_k$

$$s_{k,d_k} \leftarrow \min \left\{ s_{k,d_k} \right\} \inf_{d_{k-1},d_{k-1}} \{ s_{k-1,d_{k-1}} + v^d_{k,d_{k-1}} \} .$$

In this algorithm, $v^d_{k,d_k}$ will be set to $\infty$ for an infeasible datum scheme selection. This value indicates that, given the variation accumulated in upstream stages, the selected datum scheme at current stage cannot meet the quality specification. The final results
include (i) the minimized total CRPP, \( L(Q_N, \mathbf{x}_N) \); (ii) a sequence of decisions \( (d^*_1, d^*_2, \ldots, d^*_n) \) on datum schemes for a sequence of stages, which is the optimal setup plan; and (iii) the tolerance specifications, \( T_u \), of the fixtures used in all stages.

Fig. 6. Part drawing and KPC specifications

3. Case Study

A case study is conducted to demonstrate the SoV-model based, quality assured optimal setup planning for an MMP. The product KPC and their associated design specifications are defined in Fig. 6. Based on the analysis of features locations and tooling approaching directions, a 3-stage machining process is proposed. The candidate datum schemes for each stage are proposed and shown in Fig. 7. Correspondingly, stage/setup level fixture layouts are assumed as given. These include general 3-2-1 fixturing schemes (e.g., Setup Option 1_1) and pin-hole fixturing schemes (e.g., Setup Option 2_1), as discussed by Zhou et al. (2003).
Fig. 7. Setup options for a 3-stage machining process

Table 2 summarizes the alternative datum schemes (DS) and setup formations (SF) for each stage. Corresponding to these datum scheme candidates $d_k$'s ($k=1, 2, 3$), the coefficient matrices in state space models, $A_{d_k}$, $B_{d_k}$ and $C_{d_k}$, are generated. According to the constraints on datum scheme and datum sequence, the DP network is established, as shown in Fig. 8.

Table 2. Setup options for the 3-stage machining process

<table>
<thead>
<tr>
<th>Index</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SF$: BF, BF11, BF12</td>
<td>$SF$: FF, FF11, FF12, KF</td>
<td>$SF$: TF, TF11, LF, LF11, LF12, RF, RF11, RF12</td>
</tr>
<tr>
<td></td>
<td>$SF$: BF, BF11, BF12</td>
<td>$SF$: TF, TF11</td>
<td>$SF$: FF, FF11, FF12, KF, LF, LF11, LF12, RF, RF11, F12</td>
</tr>
<tr>
<td></td>
<td>$SF$: TF, TF11</td>
<td>$SF$: BF, BF11, BF12</td>
<td>$SF$: TF, TF11, LF, LF11, LF12, RF, RF11, RF12, KF</td>
</tr>
</tbody>
</table>
In this case study, without loss of generality, only the variation of fixture locators are included in the process variable vectors \( u^d_k \). Thus each one of the three \( u^d_k \)'s \((k=1,2,3)\) contains 6 process variables, corresponding to the six locators. The total number of process variable, \( P \), is 18. Safety factor \( s \) in Eq. (15) is set to 1.5 to account for the potential quality impacts of process variations contributed by machine/cutting tool. Coefficients \( \eta_j \) \((j = 1,2,\ldots,18)\) in Eq. (7) are set 6. The weighting coefficients \( w_j \) are set to 1/18, which means all fixture locators are treated equally concerning their CRPP. The costs for each state transition are also shown in Fig. 8. The notation “\( \infty \)” indicates that the variation accumulated from upstream stages has made it impossible to satisfy the quality specifications in the third stage.

![Fig. 8](Image)

The intermediate results are summarized in Table 3. The optimal setup plan is identified and highlighted, in Fig. 8, as the bold path. The optimal setup plan is: (i) in the 1st stage, the part is fixed with datum features BF, FF and LF, and features TF, TF11 are generated; (ii) in the 2nd stage, the part is fixed with datum features TF, FF and TF, and features BF, BF11 and BF12 are generated; (iii) The remaining features will be
generated in the 3rd stage with the part being fixed on datum features BF, BF11 and BF12. This optimal setup plan can be denoted as a DFC: \{{\text{BF-FF-LF}}, \{\text{TF-FF-TF}}, \{\text{BF-BF11-BF12}}\}, with the total CRPP of 79.983.

Table 3. Intermediate results of reaching algorithm

<table>
<thead>
<tr>
<th>$s_{k,d_k}$</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.275</td>
<td>60.095</td>
<td>87.791</td>
</tr>
<tr>
<td>Setup option $d_k$</td>
<td>2</td>
<td>47.244</td>
<td>68.744</td>
</tr>
<tr>
<td>3</td>
<td>17.408</td>
<td>40.742</td>
<td>79.983*</td>
</tr>
</tbody>
</table>

One of the by-products of the SoV-based setup planning methodology is the tolerance specifications for the fixture design. In this case study, based on the $\sigma_{u_i}$’s, $T_{u_i}$ ($i=1, 2, 3$) are given as $T_{u_1}=\begin{bmatrix} 0.086 & 0.086 & 0.086 & 0.086 & 0.086 & 0.086 \end{bmatrix}^T$, $T_{u_2}=\begin{bmatrix} 0.037 & 0.037 & 0.037 & 0.037 & 0.037 & 0.037 \end{bmatrix}^T$, and $T_{u_3}=\begin{bmatrix} 0.019 & 0.019 & 0.019 & 0.019 & 0.019 & 0.019 \end{bmatrix}^T$. The fixture design that meets these specifications will be cost-effective and sufficiently precise to ensure the product quality. The results show that the fixtures for upstream stages, i.e., stage 1 and stage 2, are not required to be as precise as that for the downstream operations, i.e. the optimal setup plan is not conservative.

Sensitivity analysis was also conducted to examine the impact of the assignments of weighting coefficients’ values on the optimization results. It is assumed that: (i) the weighting coefficients assigned to the locators belonging to the same fixture are the same; (ii) fixtures used at stage 1 will be assigned different weighting coefficients from that assigned to fixtures used at stage 2 and stage 3, and (iii) the weighting coefficients assigned to fixtures used in stage 2 and stage 3 are the same. For instance, if a sum of
weighting coefficients, 0.1 (0.1/6 for each locator) is assigned to stage 1 fixture, that for fixtures in stage 2 and stage 3 will be 0.45 (0.45/6 for each locator).

Table 4. Impact of sum weighing coefficients on optimization results

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Sum Weighting Coefficients for Stage 1</th>
<th>Optimal Setup Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>{FF-TF-RF}, {BF-BF11-BF12}, {BF-BF11-BF12}</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
<td>{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}</td>
</tr>
<tr>
<td>8</td>
<td>0.8</td>
<td>{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}</td>
</tr>
<tr>
<td>9</td>
<td>0.9</td>
<td>{BF-FF-LF}, {TF-FF-TF}, {BF-BF11-BF12}</td>
</tr>
</tbody>
</table>

Table 4 shows the optimization results associated to different combinations of the coefficients assignments. The optimal setup plans are consistent, except for case 1, where fixture in stage 1 is significantly under-weighted with a weighting coefficient 0.1. This indicates that the optimization result for this case study is not sensitive to the value of weighting coefficient. This is because the datum scheme option 3 for stage 1 significantly out-performs the other two options in terms of CRPP. The differences among those three options dominates the whole optimization of the three stages, as shown in Fig. 8.

4 Conclusions

This paper proposed a methodology for optimal setup planning for MMP’s. Based on the SoV concept, state space modeling technique is expanded to be applicable to datum selection and setup sequencing decisions. The SoV model provides the basis for quantitative, analytical evaluation of the quality impacts of candidate setup plans. This
evaluation capability enables the formulation of the setup planning as an optimization problem that minimizes the CRPP with the final product quality as constraints. DP is employed to solve this sequential optimal decision making problem.

In the proposed method, setup planning is formulated as a Dynamic Programming problem, which provides a nice representation of the sequential decision making procedure. However, one disadvantage of DP is that it needs intensive computational resources. When the number of stages and the number of alternative datum schemes are getting large, the cost for obtaining an optimal solution will be unaffordable. Potential solutions include: (i) Using different formulation, such as reinforcement learning, neurodynamic programming or approximate dynamic programming; (ii) Incorporating engineering domain knowledge to decouple an MMP into smaller segments of sub-processes and/or add more constraints to reduce the number of alternative datum schemes. These topics will be investigated in our future work.

Acknowledgement

The authors gratefully acknowledge the financial support of the Engineering Research Center for Reconfigurable Manufacturing Systems (NSF Grant EEC-9529125) at the University of Michigan. The authors would also like to thank the editors and reviewers for their insightful comments and suggestions, which have significantly improved the paper quality and readability.

Reference:


