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DESIGN OF EFFICIENT BIN-NUMBERING SCHEMES FOR WAREHOUSES

John J. Bartholdi, III and Loren K. Platzman

Material Handling Research Center, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332 (U.S.A.)

ABSTRACT

We show how to number the storage locations (bins) in a warehouse so that retrieval by bin-number produces fast retrieval routes. The numbering scheme is suggested by a "spacefilling curve" and can be custom-designed for any facility.

1. FAST RETRIEVAL BY SORTING ON BIN NUMBER

In any warehouse bins are numbered to give them identifying labels. However, numbers can give more information: in particular they determine a sequence of the locations. We can use this additional information to retrieve items quickly.

The problem of warehouse retrieval has this general form: a picker (human or robotic) is given a list of *n* items (a "batch") together with the locations of those items. The picker must leave an initial point, visit each of the appropriate storage locations to retrieve the requested items, and travel to a final point as quickly as possible. In this model the initial point may correspond to some administrative center where pick lists are issued, or simply to the current location of the picker in the warehouse when the pick list is issued. The final point corresponds to the place where the items are delivered by the picker. We allow for the possibility that there may be alternative initial or final points. We assume that the warehouse uses "dedicated storage" so that each item is in a unique, fixed storage location.

The problem of picking items as quickly as possible is a form of the "travelling salesman problem", which is notoriously difficult to solve exactly within reasonable time. In fact, complexity theory strongly suggests that there does not exist a fast optimum-finding algorithm [1]. Accordingly, one must resort to heuristics—decision techniques that are fast enough to be useful, and whose solutions are good (though not the best possible).

A heuristic in common use for the retrieval problem is the **Bin Number Heuristic**:

Visit all of the necessary locations from smallest bin number to largest bin number.

This heuristic has several advantages over more complex procedures. Most importantly

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it produces a retrieval route very quickly. It is only necessary to sort the list of the required storage locations from smallest to largest bin number. Sorting can be done very efficiently and does not even require a computer.

The appeal of the Bin Number Heuristic is clear when alternative solution techniques are considered. Most other common techniques for generating retrieval routes explicitly account in some way for the time to travel between locations (i.e. bins, initial or final points). This consideration is a burden since the number of distances to be accounted for grows proportionally to n^2, where n is the number of locations. Moreover there are the additional complications of accounting for the speed, acceleration, and method of travel of the retrieval device. Since the Bin Number Heuristic ignores such information (yet can perform well, as we shall see) the user is freed from the expense of collecting it.

The Bin Number Heuristic has another important advantage over other techniques in that it is naturally suited for use in dynamic environments wherein the retrieval problems may change even as they are being solved. (For example, additional items may be added to the picking list after a retrieval sequence has been computed.) It is easy for the Bin Number Heuristic to adapt to such changes since these entail only insertion into or deletion from a sorted list. The effort to make such modifications is negligible. In contrast, retrieval sequences produced by other techniques are not so easy to modify, and indeed, may even require an entire recomputation of the retrieval sequence.

The Bin Number Heuristic has these admirable properties because it ignores much of the complicating structure of the retrieval problem (such as travel times!). To be effective such a method must nevertheless capture enough of the essential structure of the problem to give satisfactorily short retrieval routes. In effect we are trading some possible quality of the solution (i.e. travel time) for the speed, convenience, and versatility of a simple technique.

2. HOW GOOD IS A BIN-NUMBERING SCHEME?

Note that some bin-numbering schemes are terrible. (As an extreme situation, consider randomly numbered bins, where there is no pattern to the numbering.) Numbering bins row-by-row, as in Figs. 1 or 2, is better, and is frequently used. However, it is possible to do even better.

The effectiveness of the Bin Number Heuristic depends directly on the bin-numbering scheme, i.e. on what numbers are assigned to what storage locations. A bin-numbering is good if it enables the Bin Number Heuristic to produce retrieval sequences that, on the average, result in short retrieval times. It is possible to compare bin-numbering schemes through simulation. Consider two candidate schemes, bin-numbering No. 1 and bin-num-

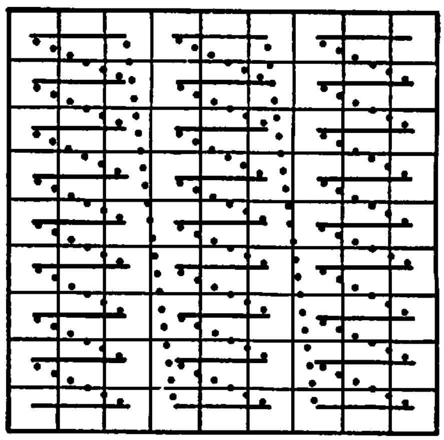

Fig. 1. The small squares represent bins along the wall of a warehouse. The bin-numbering is represented by the (continuous and dotted) line that begins at the bin in the lower left. Following this line, the bin numbers increase to the upper right of the wall. The dotted portions show where the numbering "jumps" to a new region of the wall.

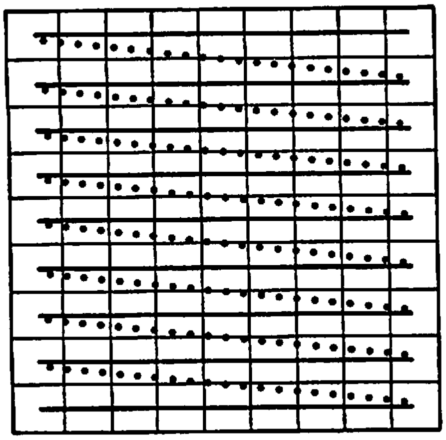

Fig. 2. A wall of bins numbered row by row, beginning at the lower left. The dotted portions show where the numbering jumps to a new region of the wall.

bering No. 2. For each scheme, retrieval sequences will be produced by the Bin Number Heuristic. Now subject each scheme to a typical stream of retrieval requests and measure the average retrieval time for each scheme. (This step of the comparison requires the travel times between storage locations and initial and final points. However, this information is not needed by the Bin Number Heuristic.) The better scheme is that one for which the average retrieval times are faster.

We conducted such tests on some simple models of warehouses, and observed interesting qualitative behavior of different bin-numbering schemes. The resulting observations can help enhance the performance of retrieval by bin-number and therefore increase warehouse throughput and reduce costs.

3. BIN-NUMBERING BASED ON SPACEFILLING CURVES

In our tests we identified a novel type of bin-numbering that is particularly effective. It is based on a mathematical construction known as a "spacefilling curve", the use of which in abstract sequencing problems was introduced by us in Refs. [2,3]. A brief look at spacefilling curves yields qualitative insight into effective bin-numbering sequences, and provides a simple theoretical venue for mathematical analysis.

A spacefilling curve is a continuous mapping of the unit interval onto the unit square. It may be thought of as a sequence in which to visit every point within the unit square. (A finite approximation of a spacefilling curve is shown in Fig. 3.) A property of spacefilling curves that is of special interest is that they tend to "preserve nearness": if two points are close on the curve, then they are close in the square; conversely, if two points are close in the square, then they tend to be close on the curve. This tendency to preserve nearness is due to the highly convoluted shape of a spacefilling curve. It tends to visit all of the points of one region before travelling to a new region.

These properties are precisely what is required of a bin-numbering scheme to enable the Bin Number Heuristic to work well. Consequently, we tested the performance of the

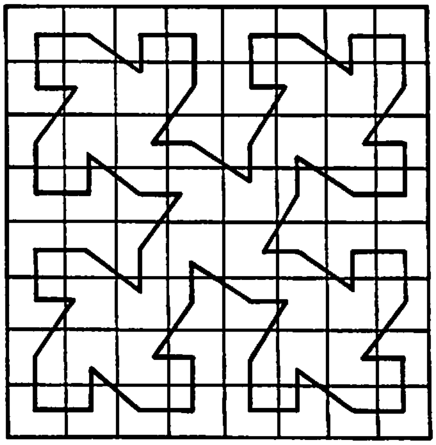

Fig. 3. A wall of bins numbered by a (finite approximation to a) spacefilling curve. The numbers begin at the lower left and increase around the curve, returning to the lower left. This numbering is unlike those of Figs. 1 and 2 in that it proceeds from bin to neighboring bin. It does not make "jumps" to new regions of the wall.

Bin Number Heuristic when bin-numbering was done according to a spacefilling curve.

We studied in detail the performance of the Bin Number Heuristic in the following special case. Assume that the retrieval device travels in a straight line from point to point, and that the initial and final points are the same. Assume that the storage locations are at the evenly spaced grid points within the unit square. We numbered these "bins" by the spacefilling curve of Fig. 3 and assumed that in a typical batch all storage locations were equally likely to be visited, independently of the others. In this case the theoretical results of Ref. [3] apply, and we interpret them here in the context of warehouse retrieval:

Result 1. If n locations are to be visited throughout a warehouse of area A, then the length of the retrieval route is at most $\sqrt{2nA}$.

Result 2. If every location is equally likely to be visited, then on the average the retrieval route produced by the Bin Number Heuristic will be 25% longer than the shortest possible route length.

The sense of Result 1 is to guarantee the performance of the Bin Number Heuristic. It constitutes an assurance that, even in the worst case, it will not take "too long" to retrieve any set of items. Result 2 says that the average quality of performance of the heuristic is fairly good. (In contrast, it is impossible to make such guarantees for most commonly implemented bin-numberings, such as those of Figs. 1 and 2.)

These results are robust in that they also hold (with slight changes in the constants of proportionality) for bin-numbering schemes modelled after other spacefilling curves [4]. In addition, the same is true for more general models of travel time: all that is required is that the measure of travel time be such that bins that are "close" by the Euclidean metric also be "close" by the metric of travel time. The latter is not really a restriction since even if it is not strictly true, the heuristic continues to perform fairly well, just not as well as otherwise.

For comparison we also tested the quality of solutions when the bins were numbered according to other schemes. Figure 4 represents the finite approximation to another spacefilling curve, but it did not produce as short retrieval routes in this case. The bin-numbering of Figs. 1 and 2 performed even worse. When the picker must return to the common initial/final point, the bin-numbering scheme of Fig. 3 worked best. This is for two reasons. First the schemes of Figs. 1, 2 and 4 begin numbering at one end of the wall and end at the other and this is not well-suited to the requirement that the picker return to the common initial/final point. These numbering schemes might be preferred if the picker had to start at one end of the wall and finish at the other end. More seriously, the scheme of Fig. 1 (the most typical bin-numbering) is "discontinuous" since, as indicated by the dotted lines, the numbering schemes occasionally make large jumps to number the

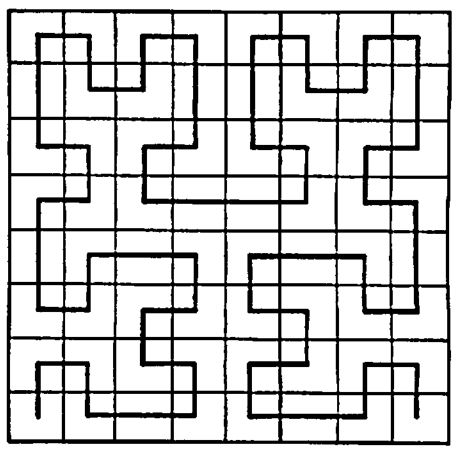

Fig. 4. A wall of bins numbered according to another spacefilling curve. The numbers begin at the lower left and increase along the curve to the lower right. This numbering, like that of Fig. 3, proceeds from bin to adjacent bin.

bins in a new region of the wall. The differences in the performance of these numbering schemes suggest that bin-numbering should avoid large jumps if possible. This corresponds to the property of continuity of spacefilling curves, which is achieved because the curve is convoluted to avoid jumps. Convolution also enables the numberings of Figs. 3 and 4 to avoid the excessive back-and-forth travel produced by the numbering of Fig. 2. Recognizing this, we studied how convolution can be important in the design of an effective bin-numbering scheme.

4. HOW TO DESIGN THE BEST BIN-NUMBERING SCHEME

It is obvious that the performance of a bin-numbering scheme depends heavily on the shape of the warehouse (the presence of barriers, etc), the locations of initial and final points, as well as on the travel of the picker. Perhaps less obviously it also depends (as we shall show) on the statistical description of the requests to which the warehouse is subjected.

In general we are willing to spend considerable effort to design an effective bin-numbering scheme for a warehouse since this is a design problem and needs to be solved only once. A good design will ensure that the Bin Number Heuristic will perform well, and this good performance will amortize the design costs.

Unfortunately it is computationally infeasible to construct the best bin-numbering. However it is possible to construct schemes that are good, if not optimal, by a heuristic procedure. An important additional result is qualitative insight into what a good scheme looks like.

We tested an "interchange" heuristic to custom-design bin-numbering schemes. It works by perturbing a current bin-numbering and testing whether the result is an improvement. (For a general discussion of interchange heuristics see Ref. [5]; for details of our implementation see Ref. [6].) Given a typical stream of requests and a matrix of travel times between storage locations, the design heuristic will produce a bin-numbering scheme that is "tuned" to perform well for the particular warehouse and its typical requests.

We tested the design heuristic on several simplified warehouse retrieval problems. The first type of problem was as follows: storage locations were modelled by a finite rectangular grid of points in the plane; to each item (and storage location) j there corresponded a probability $p(j)$ that the item in that storage location will be requested (independently of the other probabilities). (We emphasize that the assumption of independence is not necessary to apply our method; we adopted it simply as a convenient way of generating sample problems. Our method can be implemented with sample data.) We allowed different items to have different probabilities of being requested. To model the practical warehousing rule of "busiest items closest", we located all the items with high probabilities of being requested near the common initial/final point. The picker travelled by straight-line distance.

The bin-numbering scheme produced by the design heuristic for this model had a quite interesting structure (Fig. 5). In the region where the probabilities $p(j)$ were high, the bin-numbering scheme tended to have long straight segments. This makes sense since these bins are highly likely to be requested, and the shortest way of visiting these tends to be independent of the rest of the retrieval problem. On the other hand, in the region of low $p(j)$ the bin-numbering scheme tended to be highly convoluted. This is because it is not clear which of the bins will be requested, and the bin-numbering scheme attempts to keep all options open, to stay in a favorable position to visit any nearby bins that might

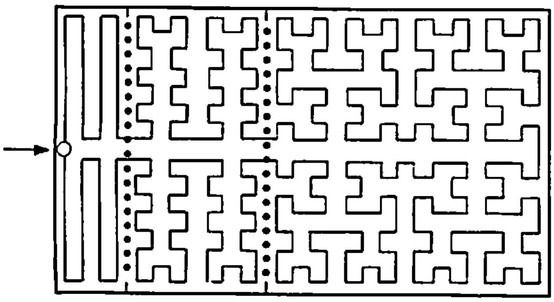

Fig. 5. A custom-designed bin-numbering for a warehouse wall. The bins at the left contain frequently requested items; bins in the middle contain occasionally requested items; bins at the right contain rarely requested items.

be requested. It avoids as much as possible commiting itself too much to any one area. The lower the $p(j)$'s in our simulation, the more convoluted the resulting bin-numbering scheme became. (Fig. 5 is idealized; our actual output was not so perfect due to edge effects, but the qualitative behavior was clear.)

A second type of problem for which we tested the design algorithm was as follows. Consider a single aisle of a warehouse, with bins on either side. The initial point and the final points are at opposite ends of the aisle. In this case an effective bin-numbering scheme will prevent too much back-and-forth travel across the aisle. In such a situation a commonly used bin-numbering scheme is the "Z"

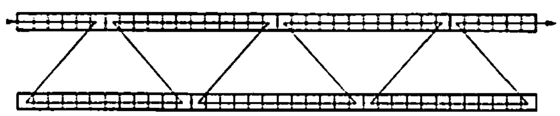

Fig. 6. Storage locations along both sides of a warehouse aisle are usually numbered in a "Z" pattern.

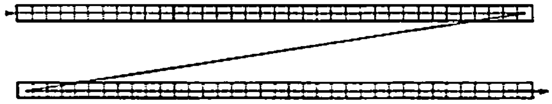

Fig. 7. A bin-numbering that works well when all storage locations are likely to be visited. In this extreme case the best bin-numbering has large-scale structure.

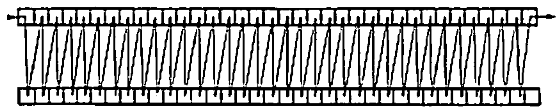

Fig. 8. A bin-numbering that works well when storage locations are *un*likely to be visited. In this extreme case the best bin-numbering is highly convoluted so that it has complex small-scale structure.

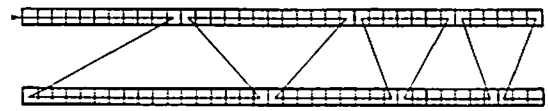

Fig. 9. The best bin-numbering when the more frequently requested items are stored to the left. Note that the numbering becomes more convoluted among the infrequently requested items on the right.

pattern of Fig. 6. Typically all of the Z's are of the same size. However our simulation suggests that this is unlikely to be best. When all bins were highly likely to be requested, the resulting bin-numbering tended to have decisive global structure as in Fig. 7. At the other extreme, when all bins were unlikely to be requested the resulting bin-numbering scheme was again convoluted as in Fig. 8. Finally, when the high-likelihood items were grouped at one end of the aisle, the resulting bin-numbering scheme was a hybrid, with long straight segments within the high-likelihood locations and much convolution within the low-likelihood items, as in Fig. 9.

5. BIN-NUMBERING IN COMPLICATED STRUCTURES

A general partitioning technique enables bin-numbering schemes for more complicated layouts to be constructed. One can independently construct the numberings for natural components of the warehouse, such as aisles, reserved areas, etc. Then a model can be constructed in which each component is represented as a single location with (normalized) probability computed from the probabilities of all the items contained therein. The design algorithm is then used to construct a sequence in which the *components* should be visited. Then any picker would visit the components of the warehouse in specified sequence, and within each component, would visit the storage locations in specified sequence.

As an example, we designed a bin-numbering scheme for a composite structure, a single aisle of a warehouse on each side of which are walls of bins. The items with a high probability of being requested are stored low on the wall, and the picker must begin at one end of the aisle and finish at the opposite end. In this case the resulting bin-numbering was a composite of Figs. 4 and 6. The numbers tended to group naturally into "regions" like Fig. 4, and the regions tended to be sequenced as in Fig. 6; see Fig. 10.

Since real warehouses are of course considerably more complicated than this, we emphasize again that the design heuristic will work for any structure of warehouse, for any assignment of items to storage locations, for any type of picker, and for any stream of requests.

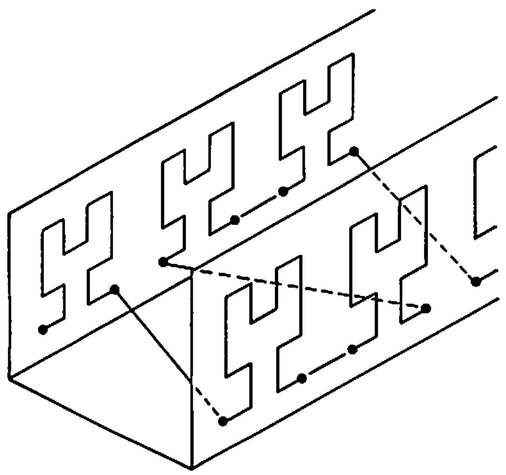

Fig. 10. Walls of bins along both sides of a warehouse aisle have been numbered in the sequence indicated by the curve. The numbering scheme is a composite of Figures 4 and 6, and was suggested by the design heuristic.

6. CONCLUSIONS

This work suggests several characteristics of an effective bin-numbering scheme. First, storage locations that are close together in travel time should have nearly equal bin numbers. This ensures that in retrieval all of the required storage locations in one region will be visited before travelling to another region. Conversely, storage locations that have nearly equal bin numbers should tend to be close together in travel time. This makes it easy to find storage locations since the name of each conveys information about its location.

Another important characteristic is that the bin-numbering scheme should be more convoluted in regions of the warehouse occupied by infrequently requested items. On the other hand, it should have a larger, more global structure within regions where the items are frequently requested.

Bin-numberings consistent with these conclusions can be implemented by simple qualitative judgment, or can be determined more exactly by using the interchange heuristic to custom-design the numbering for a particular warehouse environment.

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