

## OPEN PROBLEMS

- Does Euclid's proof yield all primes?  
Choices: take the smallest; take all.
- Pascal's triangle: how many 3-fold occurrences? 4-fold?
- Does every closed curve in the plane contain the vertices of a square?  
Notes: Differentiable, piecewise analytic, non-standard analysis and shrinking
- $2^n$  problems.  
Erdős - Gyárfás conjecture: Every graph with minimum degree 3 contains a (simple) cycle length  $2^n$  for some  $n$ .  
Stone's conjecture: if (square) matrix  $M$  is  $INS_k$  — # of solutions is  $k \forall q \in$  the relative interior of the union of the complementary cones of  $-M, I$  — then  $k = 2^n$  for some  $n$ .
- Anti-induction problems. Can affirmative action simplex pivoting rules take exponentially many steps? Norm Zadeh; Fathi&Tovey.

Steinberg's conjecture: Every planar graph without 4 or 5 cycles is 3-colorable.

Notes: Counterpart to Grötzsch's theorem. Sanders 4-9. 4-7 (Borodin et al).

- Determine the complexity or find a good algorithm for finding the uncovered set in the 2-D spatial voting model. Point  $x$  covers  $y$  if  $x \succ y$  and  $x \succ z \forall z \prec y$ . Notes: definitions vary in case of ties. Idea is that uncovered points can defeat anything else in 2 steps. Nick Miller, "In search of the uncovered set."
- Let  $c_{ij}$  be iid normal. As  $n \rightarrow \infty$  what is the probability that TSP 2-matching with subtour elimination polytope has integer solution? Same question with combs added. Hunsaker, Johnson, & Tovey.
- Candle in a mirrored polygon. Is there a point in the interior of a mirrored polygon from which light reaches all points in the polygon? Do all points have this property?

Note: vertices absorb light.

- Hirsch conjecture – diameter of graph of an  $m$ -facet polytope in  $\mathfrak{R}^d$  is  $\leq m - d$  – conjectured false by Vic Klee, perhaps in 7 or 8 dimensions.
- Coloring the plane.  $(x, y) \in E$  iff  $\|x - y\| = 1$ . Hadwiger & Nelson. Notes:  $4 \leq \chi \leq 7$ . Axiom of choice? Attained by a finite subgraph (Erdős & de Bruijn).  $6 \leq \chi \leq 15$  in 3-D.
- Small Ramsey numbers.  $R(4,6)$ .  $R(5,5)$ . Graph theorists needed!
- Conjecture (Tovey, 1984)  $R(k - 1, m + 1) > R(k, m)$  for  $k \geq m + 2$   
Notes: Obvious generalization to  $R(k_1, k_2, \dots, k_m)$ .  
Weaker form of conjecture suggested by Erdős:  
 $R(2k, 2k) \geq R(3k, k)$ .
- Lovasz's doubly critical conjecture. If  $\chi(G - v - w) = \chi(G) - 2 \forall (v, w) \in E$ , then  $G$  is complete.

- Graceful problems: Rosa-Kotzig-Ringel: are all trees graceful, i.e. vertex labeled  $1 \dots |V|$  to give distinct edge values  $1, 2, \dots |V| - 1$ . Avoid flora and fauna. What is the complexity of the graceful numbering problem for arbitrary graphs? Related, preceding and weaker: Ringel's conjecture: every tree on  $n$  vertices can partition  $K_{2n-1}$ .
- McKelvey Schofield Banks problem.
- Conjecture (Tovey 1986): region of optimality of planar TSP is simple.
- Problem: find a 2-optimal TSP tour in the plane in polynomial time.