Statistical Modeling by Wavelets

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Preface

> Just two months ago astronomers did not know about it. But now they are giving good odds that Hyakutake will be the most impressive comet since the invention of telescope 400 years ago. *(Herald Sun, Durham, NC, March 24, 1996.)*

One can trace the origins of wavelets back to the beginning of this century; however, wavelets, understood as a systematic way of producing local orthogonal bases, are a recent unification of existing theories in various fields and some important "discoveries." They are mathematical objects that have interpretation and application in many scientific fields, most notably in the fields of signal processing, nonparametric function estimation, and data compression. In the early 1990s, a series of papers by Donoho and Johnstone and their coauthors demonstrated that wavelets are appropriate tools in problems of denoising, regression, and density estimation. The subsequent burgeoning wavelet research broadened to a wide range of statistical problems.

This book is aimed at graduate students in statistics and mathematics, practicing statisticians, and statistically curious engineers. It can serve as a text for an introductory wavelet course concerned with an interface of wavelet methods and statistical inference. The necessary mathematical background is proficiency in advanced calculus and algebra; consequently, this book should be useful to advanced undergraduate students as well as to graduate students in statistics, mathematics, and engineering.

This book originated from the class notes supporting the Special Topics Course on Multiscale Methods at Duke University. The content can be divided into two parts:

an introduction to wavelets (Chapters 1–5) and statistical modeling (Chapters 6–11). An introduction and some mathematical prerequisites are presented in Chapters 1 and 2. Continuous and discrete wavelet transformations are covered in Chapters 3 and 4. Some important generalizations (coiflets, biorthogonal wavelets, wavelet packets, stationary, periodized and multivariate wavelets) are covered in Chapter 5.

Chapters 6–11 are data-oriented. Chapter 6 is the crux of the book, covering the theory and practice of wavelet shrinkage. Important theoretical aspects of wavelet density estimation are covered in Chapter 7. Chapter 8 discusses Bayesian modeling in the wavelet domain. Time series are covered in Chapter 9, while Chapter 10 contains several probabilistic and simulational properties of wavelet-based random functions and densities. Chapter 11 gives some novel and important wavelet applications in statistics.

Instead of providing appendices with data sets and programs used in the book, I opted for a more modern style. The web page:

`http://www.isds.duke.edu/˜brani/wiley.html`

is associated with the book. This page contains all data sets, functions, and programs referred to.

I hope the reader will find this book useful. All comments, suggestions, updates, and critiques will be appreciated.

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Many colleagues contributed to this project in different ways: Anestis Antoniadis, Tony Cai, Merlise Clyde, Lubo Dechevsky, Iain Johnstone, Gabriel Katul, Eric Kolaczyk, Pedro Morettin, Peter Müller, Giovanni Parmigiani, Marianna Pensky, David Rios, Fabrizio Ruggeri, Rainer von-Sacks, Naoki Saito, Yazhen Wang, and Gilbert Walter, to list a few. Collaboration with software gurus Hong-Ye Gao [TeraLogic Inc.] and Andrew Bruce [MathSoft Inc.] was fruitful. The S+Wavelets module (for S-Plus) was used for almost all of the computer examples, figures, and calculations. I am grateful to Alison Bory, Angioline Loredo, and Steve Quigley from Wiley, for their enthusiastic assistance, and to Courtney Johnson, Michael Kozdron, and Kathy Zhou, doctoral students at Duke University, for their help in proofreading the manuscript.

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