Homework 9, ISyE 2027 Spring 2006

Due on Wednesday, April 12

Problem 1: Suppose X, Y, U, and V are random variables.
(i) If a and b are constants, show that $Cov(aX, bY) = abCov(X, Y)$.
(ii) Show that $Cov(X + Y, U + V) = Cov(X, U) + Cov(X, V) + Cov(Y, U) + Cov(Y, V)$

Problem 2: Suppose that the random variables X, Y, and Z are independent with $E(X) = 3$, $Var(X) = 4$, $E(Y) = -2$, $Var(Y) = 4$, $E(Z) = 10$, and $Var(Z) = 2$. Calculate the expectation and variance of the following random variables: (a) $3X + 7$, (b) $2X + 6Y$, (c) $5X - 9Z + 8$, (d) $6X + 2Y - Z + 16$.

Problem 3: Hayter, Problem 2.6.6

Problem 4: Hayter, Problem 2.6.8

Problem 5: Hayter, Problem 2.6.3

Problem 6: Hayter, Problem 2.6.10

Problem 7: Hayter, Problem 2.7.10

Problem 8: Suppose I flip a fair coin four times. Let X equal the number of heads observed in the first three tosses, and let Y equal the number of heads observed in the last two tosses.
(a) Is $Z = X^2 - Y$ discrete or continuous? If it's discrete, compute its pmf. Otherwise, compute its pdf.
(b) Compute $E[X - Y^2]$.
(c) Compute $Var(X - Y)$