Solutions to Homework 1, ISyE 2027 Spring 2006

Problem 1

Let A denote the event that a person is male, B that the person is under 30, and C that a person speaks French. Then the event that a male is over 30 is just $A \cap B^c$, the event that a female is under 30 and speaks French is $A^c \cap B \cap C$, and the event that a male who either is under 30 or speaks French is $A \cap (B \cup C)$.

Problem 2

Let $\Omega = \{(i, j) : 1 \leq i \leq 6, 1 \leq j \leq 6\}$ be our sample space, where (i, j) represents the red die being i and the green die being j. Each outcome is equally likely, so $P(\{(i, j)\}) = 1/36$ for any choice of i and j. Now, there are 15 outcomes in the event A = "the number on the red die is strictly larger than the number on the green die" $= \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)\}$, so it follows that

$$P(A) = 15/36.$$

The complement of A is just A^c = "the number on the red die is less than or equal to the number on the green die".

Problem 3

Suppose we toss a fair coin three times. Then our sample space is $\Omega = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (T, T, H), (T, H, T), (H, T, T), (T, T, T)\}$. The event that we see two heads in succession is just $A = \{(H, H, T), (T, H, H), (H, H, H)\}$, so the probability of this event is just

$$P(A) = 3/8.$$

Problem 4

Clearly there are 216 outcomes in this sample space, which is $\Omega = \{(i, j, k) : 1 \leq i \leq 6, 1 \leq j \leq 6, 1 \leq k \leq 6\}$, and the probability of each outcome is equally likely. There are six outcomes corresponding to the sum 1+2+6, six outcomes for 1+3+5, three for 1+4+4, three for 2+2+5, six for 2+3+4, and only one for 3+3+3. Hence, the probability that the sum is 9 is just 25/216. Furthermore, there are six outcomes corresponding to the sum 1+3+6, six for 1+4+5, 3 for 2+2+6, 6 for 2+3+5, 3 for 2+4+4, and 3 for 3+3+4, so the probability that the sum is 10 is 27/216. Therefore, 10 is a more likely total than 9.

Problem 5

From property (i) of our probability axioms, we know that $P(\Omega) = 1$. Using axiom (iii), we see that

$$1 = P(\Omega) = P(\{1, 2, 3, 4, 5, 6, 7\})$$

$$= P(\{1\}) + P(\{2\}) + P(\{3\}) + P(\{4\}) + P(\{5\}) + P(\{6\}) + P(\{7\})$$

$$= (1 + 8 + 27 + 64 + 125 + 216 + 343)c$$

so it follows that $c = 1/784 = 0.00128$.

Problem 6

Notice that $\{a, b\}^c = \{c\}$, so

$$P(\{c\}) = 1 - P(\{a, b\}) = 1 - 0.75 = 0.25.$$

Similarly, we see that $P(\{a\}) = 0.5$. Therefore,

$$P(\{b\}) = 1 - P(\{a\}) - P(\{c\}) = 0.25.$$

Problem 7

We know that $P(A) = 0.3$, $P(B) = 0.5$, and the two events are disjoint. From axiom (iii), we have that

$$P(A \cup B) = P(A) + P(B) = 0.8.$$

Also, from DeMorgan's law we see that

$$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 0.2.$$

Problem 8

Suppose we know that $P(A) = 0.4$ and $P(A \cap B) = 0.3$. Then we know that $P(B) \geq P(A \cap B) = 0.3$, so $P(B) \geq 0.3$. Furthermore,

$$1 \geq P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.1 + P(B)$$

so $P(B) \leq 1 - 0.1 = 0.9$. Therefore, $0.3 \leq P(B) \leq 0.9$, so $P(B)$ could possibly take any value in $[0.3, 0.9]$.

Problem 9

We're choosing a ball at random, so each ball in the bag is equally likely to be chosen. Let S denote the event that the ball chosen is shiny, and let R denote the event that the ball chosen is red. We know that $P(S \cap R) = 55/200 = 0.275$, $P(S) = 91/200 = 0.455$, and $P(R) = 79/200 = 0.395$. Then the probability that it is either a shiny ball or a red ball is just

$$P(S \cup R) = P(S) + P(R) - P(S \cap R) = 0.575$$

and the probability that it is a dull, blue ball is just

$$P(S^c \cap R^c) = 1 - P(S \cup R) = 0.425.$$