

Homework 10, ISyE 2027 Spring 2006

Problem 1: Suppose X is a continuous random variable with pdf

$$f(x) = \begin{cases} 1 - |x|, & -1 \leq x \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

Compute the moment generating function of X .

Problem 2: Suppose N is a geometric random variable with parameter p .

(a) Compute the moment generating function (mgf) of N .

(b) Use the mgf to conclude that $E[X] = 1/p$.

(c) Now suppose B is a negative binomial random variable with parameters r and p (r of course is an integer). Compute the mgf of B (**Hint:** Use the result from part (a)).

Problem 3: Suppose Z is a standard normal random variable. Compute the mgf of Z^2 . What kind of random variable is this? (**Hint:** When computing the integral, try to make the integrand look like a density function that you're familiar with. It will require multiplying and dividing by a specific constant).

Problem 4: Suppose X is a random variable with the following mgf:

$$M_X(t) = (.5) + (.2)e^{2t} + (.1)e^{3t} + (.2)e^{5t}.$$

(a) Compute $E[X]$

(b) Compute $Var(X)$.

(c) Compute $E[X^4]$.

Problem 5: A baseball player claims that he is a .300 hitter (i.e. roughly 30 percent of all his at-bats result in a hit), yet at the end of the season he has gotten 42 hits in 168 at-bats. Is this just bad luck? To decide, approximate the probability that he would get 42 hits or less if his probability of getting a hit were $p = 0.3$ (**Hint:** think CLT).

Problem 6: An airplane contains 36 men whose weights are normally distributed with mean 175 pounds and standard deviation 16 pounds. (a) Estimate the probability that their total weight will be more than 6,500 pounds (assume the weight of each man is independent of the weight of each of the others). (b) Find a number t_0 so that with probability 0.95 the sum of the weights will be less than t_0 (**Hint:** look at Problem 5.1.4 of Hayter's book).

Problem 7: A basketball player makes 80% of his free throws on the average. Use the Central Limit Theorem to compute the probability that in 25 attempts he will make at least 23 (don't forget to adjust the probabilities using the "continuity correction" values).