## Solutions to Extra Counting Problems

### Problem 1

Notice that there are 5! ways that the players over 6 feet tall can stand in a row. Once they are in position, there are 6! ways for the shorter players to be arranged. Therefore, the number of ways they can have their picture taken is just $5!6! = (120)(720) = 86{,}400$.

### Problem 2

(a) Suppose that the committees must be disjoint. The number of ways we can form the 8-member committee is just $\binom{50}{8}$. Once these people have been chosen, the number of ways I can form the 7-member committee is just $\binom{42}{7}$. Hence, the number of ways I can form these two committees is just

$$\binom{50}{8}\binom{42}{7}.$$

(b) Now suppose that the committees can overlap. Again, the number of ways we can form the 8-member committee is just $\binom{50}{8}$. However, the number of ways I can form the 7-member committee is $\binom{50}{7}$ (since the committees can overlap). Therefore, the number of ways I can form these two committees is just

$$\binom{50}{8}\binom{50}{7}.$$

### Problem 3

We want to find the probability that we get three of the same number. First of all, the number of ways we can pick 4 numbers is just $\binom{40}{4}$. To count the number of ways we get three of the same number, let's reason as follows: first, there are (10) possible values for our triple. Once we select this number, there are $\binom{4}{3}$ ways to pick colors for these cards. Once this is fixed, there are $\binom{36}{1}$ ways to pick the remaining card in our sample. Thus, the probability we get three of the same number is just

$$\frac{(10)\binom{4}{3}\binom{36}{1}}{\binom{40}{4}}.$$

### Problem 4

First of all, there are 3! ways we can arrange the types of books on the shelf (i.e. History, Math, Novels, or Math, Novels, History, or Novels, History, Math, etc.). Once that is fixed, there are 5! ways to arrange the history books, 3! ways to arrange the math books, and 4! ways to arrange the novels. Thus, the number of ways the books can be arranged on the shelf is just

$$(3!)(5!)(3!)(4!)$$

### Problem 5

(a) This is equivalent to finding the number of ways we can make three sets of size 3 from a set of size 9 (recall your class notes), which is just equal to

$$\frac{9!}{3!3!3!} = 1680.$$

(b) The probability that line $A$ gets both used wrenches is just

$$\frac{\binom{7}{1}}{\binom{9}{3}} = 1/12.$$

**Problem 6**

(a) The number of ways that the assembly operation can be performed is just 4!.
(b) Let's count the number of ways that the soldering operation can be either first or second. Notice that there are 2 possible places for the soldering operation. Once this has been fixed, there are 3 ways to put an operation in the position not occupied by the soldering operation. Once these have been fixed, there are 2 ways to assign an operation to the third position, and only 1 way to assign an operation to the last position. Hence, the probability that the soldering operation comes first or second is

$$\frac{(2)(3)(2)(1)}{4!} = 1/2.$$

**Problem 7**

The probability that none of the three bulbs selected is defective is just

$$\frac{\binom{4}{3}}{\binom{6}{3}} = 1/5.$$