Cost-Sharing Mechanism Design for Freight Consolidation among Small Suppliers

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A fair cost allocation scheme is critical for forming and sustaining horizontal cooperation that leads to cost reductions. In order to incentivize suppliers to cooperate in freight consolidation, we design a cost-sharing mechanism to solve a cost allocation problem arising in freight consolidation among small suppliers: that is, suppliers whose total demand fits into one truckload. This cost-sharing mechanism determines the set of suppliers who will participate in the consolidation and their corresponding cost shares. We design this cost-sharing mechanism to ensure that every supplier is willing to reveal their preference truthfully and the total consolidation cost incurred is recovered by the allocated costs. Moreover, our analytical and numerical study shows that our cost-sharing mechanism often yields cost shares that maximize the social welfare of all the suppliers.

Key words: freight consolidation; cost-sharing mechanism; cost allocation

History:

1. Introduction
Transportation costs have increased in the last decades for various reasons, such as the mismatch of supply and demand for freight transportation services (Russell et al. 2014). Competitive transportation costs are especially critical for the success of various industries. For instance, transportation costs are often a large percentage of the product costs in the agriculture industry (Nguyen et al. 2013). Furthermore, transportation costs, the single largest logistics cost element, usually account for more than 50% of the total logistics costs (Thomas and Griffin 1996). As a result, it is important for suppliers to reduce their transportation costs in order to be competitive.

In terms of transportation costs, suppliers with low market shares, which we call small suppliers, are at a competitive disadvantage compared to suppliers with high market shares, which we call...
large suppliers, because small suppliers have greater difficulty negotiating favorable transportation rates with carriers due to their smaller shipping volumes. Transportation costs for such suppliers can be reduced by freight consolidation, which is the process of assembling smaller shipments together from different locations; the resulting large shipping volumes allow for a reduction in transportation rates. A survey of 53 United States companies revealed that freight consolidation, which takes advantage of economies of scale, has contributed the most to reducing transportation costs (Jackson 1985). Significant cost savings have also been reported through freight consolidation in various industries (e.g. Bausch et al. (1995), Brown et al. (2001)). Freight consolidation often takes place among businesses that produce similar products or departments within the same company with a central planner to organize and implement the consolidation. Self-interested businesses are often willing to consolidate because third-party carriers usually charge cheaper shipping rates when the shipment volumes are large enough.

One example that shows the importance of freight consolidation is the plight of the California cut flower industry. Currently, this industry has been facing increasing competition from the cut flower growers in South America, especially Colombia. California’s share of the United States cut flower market has decreased from 64% to 20% in the last two decades, while South America’s share reached approximately 70% in 2007 (Arbeláez et al. 2007). A shared cross-docking and distribution facility located in Miami, Florida has enabled South American growers to compete effectively with Californian growers. The central planners in Miami organize and consolidate the products of South American growers in the distribution facility before sending them by truck to the rest of the United States. The resulting large volume shipments allow them to obtain the cheaper full-truckload (FTL) rates and the corresponding cost savings on transportation provide them with a huge competitive advantage. In contrast, most of the California cut flower growers, who currently send their products individually using more expensive less-than-truckload (LTL) rates, are often of small to medium size and have no power to negotiate favorable transportation rates on their own. Nguyen et al. (2013) evaluated the current transportation practices in the California cut flower industry and explored the possibility of building a consolidation center in Oxnard, California. They concluded that a shipping consolidation center could reduce transportation costs by 35%, saving $20 million per year if all the California cut flower growers were to participate in the consolidation.

Although establishing an alliance to consolidate is a feasible approach to improve the competitiveness of small suppliers, it is essential to know under what circumstances the individual suppliers will have the incentive to participate in the consolidation. A survey based on approximately 1500 representative logistics service providers in Belgium reported that designing a fair cost sharing scheme is the major impediment for forming horizontal cooperation among logistics service providers even though the profitability of cooperation is firmly and widely believed (Cruijssen et al.
Therefore, providing a way to fairly allocate the cost of consolidation becomes critical for facilitating cooperation among the companies.

In this paper, we design a cost-sharing mechanism to incentivize a group of small suppliers to participate in freight consolidation. In the environment we consider, there is a group of small suppliers that could cooperate by using a nearby consolidation center to group their demands to ship to a common faraway destination. Our proposed cost-sharing mechanism decides both the set of suppliers who participate in consolidation and their corresponding cost shares. We design our proposed cost-sharing mechanism to possess certain desirable properties. First, it is important for our cost-sharing mechanism to incentivize suppliers to reveal their willingness to pay truthfully: we want to make sure that no individual supplier or a group of suppliers can benefit from submitting false bids. Second, our cost-sharing mechanism should recover the cost incurred by consolidation as much as possible with the prices charged. Finally, the outcome of the proposed cost-sharing mechanism should maximize social welfare as much as possible.

Our focus in this paper is the case of small suppliers – that is, the total demand of all the suppliers fits into one truckload. This is often true in many applications since small suppliers have small shipping volumes on a regular basis. For instance, the demand of many California cut flower growers in 2010 according to the data provided by the California Cut Flower Commission (CCFC) shows that in more than 95% of the cases, the aggregated shipping volumes to a single destination of these growers are less than one truckload on a daily basis. Most of these growers are from small farms and are willing to participate in consolidation. Our study in this paper offers a cost-sharing mechanism design strategy for this particular situation.

The rest of the paper is organized as follows. In Section 2, we review the research on freight consolidation, cost-sharing mechanisms and cost allocation approaches in transportation collaborations. We formally define our problem in Section 3. In Section 4, we present our proposed cost-sharing mechanism and show the properties of the mechanism and its outcomes for different demand profiles. In Section 5, we investigate the social welfare resulting from our proposed cost-sharing mechanism both analytically and numerically. We conclude our work in Section 6.

2. Literature Review

The passage of the Motor Carrier Act of 1980 made more transportation options available to companies, enabling them to improve their logistics efficiency and customer service levels. Freight consolidation is one strategy that has been applied by many companies. Jackson (1985) surveyed 53 firms on freight consolidation practice. All the firms regarded freight consolidation as an important strategy to remain competitive in terms of cost and 77% of them indicated that freight consolidation also helped provide better service. Different freight consolidation strategies have been studied
in Blumenfeld et al. (1985), Campbell (1990), Daganzo (1988), Hall (1987). Quantity-based, time-based, and quantity-time-based shipment-release policies have been examined to leverage lower transportation rates with large volumes by aggregating shipment quantities in various ways (Abdelwahab and Sargious 1990, Bookbinder and Higginson 2002, Çetinkaya and Bookbinder 2003, Higginson 1995, Higginson and Bookbinder 1994). Efficient consolidation operations have also been studied in vendor managed inventory systems by Çetinkaya and Lee (2000) and Çetinkaya et al. (2006). Most of the literature on freight consolidation has focused on improving the efficiency of consolidation. However, little effort has been devoted to investigating cost allocation methods that encourage freight consolidation practices, which usually require cooperation among a group of companies.

Generally, there are two approaches to solve cost allocation problems. Cooperative game theory provides a framework to allocate costs among participants so that certain “fairness” criteria are satisfied. This approach focuses on what a group can achieve and whether it is possible to coordinate the group to achieve the goal by properly allocating costs. For example, the core (Gillies 1959) – one of the most well-studied solution concepts in cooperative game theory – consists of allocations that recover the cost incurred by all of the players and ensure that no individual or a group of players can benefit by defecting. The emptiness or nonemptiness of the core is often studied as a proxy for the possibility of cooperation. Another approach, which is our focus in this paper, is to design a cost-sharing mechanism that solves the cost allocation problem. Rather than investigating what can be achieved by cooperation, cost-sharing mechanism design focuses more on incentivizing each individual to participate voluntarily. A cost-sharing mechanism uses bids from potential participants to determine who should participate and how the costs are shared.

A cost-sharing setting consists of a set of players who are interested in receiving service from a provider. A binary demand setting restricts the decision of the service provider to either serve the player or not at all, whereas a general demand setting allows the provider to offer service at various levels. Each player has a private valuation of the service. The objective of the service provider is to decide who to serve, at what levels, and how to share the cost among the selected players. The algorithm that service providers apply to make these decisions is called a cost-sharing mechanism. In a cost-sharing mechanism, these decisions are made based on bids that players submit to the service provider. The bids of the players express their maximum willingness to pay for the service. The study of cost sharing mechanisms mainly focuses on three desired properties: (i) *truthfulness*, the idea that it is optimal for individual players or groups of players to bid their true valuations, (ii) *budget balance*, the notion that the mechanism charges the players the cost they incur, and (iii) *economic efficiency*, the idea that the welfare for all the players is maximized. Unfortunately, almost 40 years ago, Green et al. (1976) and Roberts (1979) proved that it is not possible for a cost-sharing
mechanism to possess these three desired properties simultaneously. This has led to a cost-sharing mechanism design paradigm that relaxes either the constraint on budget balance or economic efficiency. Furthermore, the impossibility results also motivate approximation measures on budget-balance and economic efficiency. Roughgarden and Sundararajan (2009) introduced a measure called social cost to quantify economic inefficiency in cost-sharing mechanisms. Mechanisms, which could yield zero or negative social welfare, always have nonnegative social costs. As a result, using social cost, we can identify with increased fidelity the relatively more efficient mechanisms.

Without the constraint of economic efficiency, Moulin (1999), and Moulin and Shenker (2001) proposed a framework, now known as the Moulin mechanism, that allows the design of truthful and approximately budget-balanced cost-sharing mechanisms. A Moulin mechanism decides on the players to be served and the cost shares through an iterative process with the help from a cost-sharing method, which provides the cost shares for any given set of players to be served. The mechanism starts with all players being considered. In each iteration, cost shares are calculated and offered to the considered players simultaneously, and only the players who accept the cost shares remain to be considered in the next iteration. The iterations continue until all remaining players accept the cost shares offered or the set of considered players become empty. Using a so-called cross-monotonic cost-sharing method, a Moulin mechanism offers a nondecreasing sequence of costs to the players to guarantee that no individual or coalition of players can be better off by submitting false bids. Meanwhile, approximate budget-balance is achieved by offering costs at each iteration that would in total approximately cover the cost incurred if the current iteration were to be the last. Due to its flexibility and reasonable economic efficiency, approximately budget-balanced Moulin mechanisms have been designed for a wide range of cost-sharing applications arising in scheduling (Brenner and Schäfer 2007, Bleischwitz and Monien 2009), network design (Jain and Vazirani 2001, Archer et al. 2004, Gupta et al. 2004, 2007), facility location (Devanur et al. 2005, Könemann et al. 2005, Leonardi and Schäfer 2004, Pál and Tardos 2003), and logistics (Xu and Yang 2009).

When economic efficiency is the primary concern together with truthfulness, the Vickrey-Clarke-Groves (VCG) mechanism (Clarke 1971, Groves 1973, Vickrey 1961) is a powerful framework. As a special case of VCG mechanisms, the marginal cost mechanism is often used to achieve efficient cost allocations. The cost shares in the marginal cost mechanism are defined so that the welfare each player obtains is its marginal contribution to the overall social welfare. However, this class of mechanisms usually has no budget-balance guarantee and sometimes raises zero revenue (Moulin and Shenker 2001).

In terms of applications of mechanism design in transportation collaborations, Furuhata et al. (2015) designed an online cost-sharing mechanism to provide quotes to passengers who shared
a door-to-door transportation service provided by a demand-responsive transport system. They proposed a novel cost sharing mechanism that satisfies a number of desired properties – such as online fairness, immediate response, and ex-post incentive compatibility – that specifically address the issues involved with sharing costs without knowing future demand. Mechanism design has rarely been used to solve cost allocation problems in transportation collaborations; cooperative game theory has been the more common approach.

Different cost allocation methods have been proposed in the literature. Rule-based cost allocation methods have been widely applied to share costs in transportation cooperation. For instance, the Shapley value (Shapley 1953) was applied by Krajewska et al. (2008) to allocate the profit achieved by a group of freight carriers, who cooperate by pooling transportation resources. The nucleolus (Schmeidler 1969) was considered by Liu et al. (2010) and Frisk et al. (2010) to allocate profits or costs in collaborations of using trucks. Besides the rule-based cost allocation methods, linear programming duality has also been employed to provide cost sharing solutions. Sánchez-Soriano et al. (2001) introduced a class of transportation games and showed that the non-negative optimal dual solutions of the underlying linear programs characterized the cost allocations in the core that also assign less cost to each player than its stand-alone cost. Some studies (e.g. Agarwal and Ergun (2008), Houghtalen et al. (2011)) have used inverse optimization to calculate the capacity exchange prices that induce optimal coordinated capacity usage in a network, e.g. in an air cargo alliance. Other studies, especially those that are more application-based have proposed cost allocation methods that address additional notions of fairness. For example, Liu et al. (2010) introduced the Weighted Relative Savings Model (WRSM), which considers individual contributions to the coalition and the balance of savings for each player. Frisk et al. (2010) proposed the Equal Profit Method (EPM), which applied a linear program to obtain a cost allocation in which the profit ratios of the participants were kept as close as possible, for the cooperation of eight forest companies in southern Sweden. They argued that the guaranteed similar relative savings provided greater incentives for players to join the coalition in the negotiation phase.

3. Problem Definition
We study a freight consolidation system that consists of a group of suppliers who produce similar products, are located in a certain geographical region, and ship to a common destination. All the suppliers in the group are interested in cost reduction through freight consolidation. There is a central planner who operates a center that provides consolidation service in the same region. Both the suppliers and the consolidation center use trucks to ship their products.

We formally define our cost-sharing problem as follows. Let $N$ denote the set of suppliers who are interested in consolidating their shipments. Each supplier $i \in N$ has a positive shipping demand $d_i$.
in ft\(^3\) and a valuation \(v_i\) for the service provided by the consolidation center. The total demand of the suppliers fits into one truckload, i.e. \(\sum_{i \in N} d_i \leq k_F\), where \(k_F\) is the capacity of one truck. The valuations reflect the suppliers’ opinions on how much the consolidation center’s service is worth.

Figure 1 shows the structure of the consolidation system. Suppliers in \(N\) have two shipping options. They can ship their demand either directly to the destination or through the consolidation center. Suppliers express their willingness to consolidate by submitting a bid for service at the beginning of the consolidation process. We denote supplier \(i\)’s bid by \(q_i\). Based on these bids, the consolidation center selects a set of suppliers \(S \subseteq N\) to serve. Selected suppliers have their products consolidated first and then shipped to the common destination. We call the shipment from the suppliers to the consolidation center “inbound shipping”, and the corresponding cost incurred by each supplier the “inbound shipping cost”. We call the shipment from the consolidation center to the destination “outbound shipping”, and the corresponding cost incurred by the consolidation center the “outbound shipping cost”. We call the shipment from the suppliers to the destination “direct shipping”, and the corresponding cost for each supplier the “stand-alone cost”. Suppliers are independent and do not coordinate shipping among themselves. (For example, two suppliers could be paying the same local transportation company for the inbound shipping, and this company could very well be routing a single truck to both locations, but would still charge the suppliers per-volume rates.)

There are two important parameters in the trucking cost structure. One is the less-than-truckload (LTL) rate, or the cost for shipping each cubic foot when the shipping demand is less than some threshold value. The other is the full-truckload (FTL) rate, or the fixed cost for using the entire
truck when the shipping demand is greater than the threshold value. Let \( b \) denote this threshold value, which we call the full-truckload (FTL) equivalent volume. Shipping demand \( b \) or more in one truck costs the same as shipping a full truckload. The FTL rate is usually priced per mile while the LTL rate is usually priced based on other factors besides distance, such as density, freight class, weight per cubic foot, etc. However, with similar products in the shipment, we can assume that these factors influence the price in the same way across suppliers and thus the LTL rate and FTL rate only depend on the mileage between the origin and the destination. Given the distance between the origin and the destination, we denote the corresponding LTL rate and FTL rate by \( c_L \) and \( c_F \), respectively. The transportation cost \( c \) is a function of the shipping volume \( d \) and it is illustrated in Figure 2. In mathematical terms,

\[
c(d) = \begin{cases} 
    c_L d & \text{if } 0 < d \leq b, \\
    c_F & \text{if } d \geq b.
\end{cases}
\]

The transportation cost increases linearly with the LTL rate \( c_L \) as the demand increases until the demand reaches the FTL equivalent volume \( b \); then the cost \( c_F \) remains the same for any demand volume beyond \( b \) but less than \( k_F \). Note that \( c_F = c_L b \).

We assume that the suppliers and the consolidation center face the same trucking cost structure but not necessarily the same rates or FTL equivalent volume. We define the shipping cost functions for the suppliers and the consolidation center based on the following assumptions:

**Consolidation center location assumption:** The suppliers are all close to the consolidation center and approximately the same distance away. Consequently, we assume all the suppliers have the same LTL rate \( g_{L0} \) and FTL rate \( g_{F0} \) for inbound shipping.

**Destination location assumption 1:** The suppliers are all far away from the destination and approximately the same distance away. Consequently, we assume all the suppliers have the same LTL rate \( g_{L1} \) and FTL rate \( g_{F1} \) for direct shipping.

**Destination location assumption 2:** The distances between the suppliers and the destination are larger than the distances between the suppliers and the consolidation center. As a consequence, \( g_{L1} > g_{L0} \) and \( g_{F1} > g_{F0} \).
**Location assumption:** The suppliers, consolidation center and destination are located such that the inbound shipping distances, outbound shipping distance and the direct shipping distances satisfy the strict triangle inequality, i.e. $g_{L1} < g_{L0} + c_{L1}$, where $c_{L1}$ is the LTL rate for the consolidation center.

**Threshold value assumption:** The inbound and direct shipping costs of the suppliers have the same FTL equivalent volume $b_G$ (ft$^3$).

In general, the above assumptions represent the situation where the suppliers and the consolidation center are located in the same region and the destination is sufficiently far away such that outbound shipping costs dominate the inbound shipping costs if suppliers send demand via the consolidation center. For instance, the CCFC proposes Oxnard as the location for flower consolidation in California because half of the flower production originates from Oxnard and its vicinity. Most demand destinations outside California will lead to dominant outbound shipping costs compared to inbound shipping costs. In addition, we consider the group of suppliers as a small community in which each supplier is able to obtain the same transportation rate through negotiation with the carriers. For example, if carriers charge suppliers based on shipping zones, flower growers in the Oxnard area share the same transportation rate for the same destination even though there may be small differences in distances.

Following the definition of the cost structure and the assumptions above, the inbound shipping cost for supplier $i$ is
\[
G^0_i = \begin{cases} 
g_{L0}d_i & \text{if } 0 < d_i \leq b_G, 
g_F0 & \text{if } d_i \geq b_G. 
\end{cases}
\]
The stand-alone shipping cost for supplier $i$ is
\[
G^1_i = \begin{cases} 
g_{L1}d_i & \text{if } 0 < d_i \leq b_G, 
g_F1 & \text{if } d_i \geq b_G. 
\end{cases}
\]
We assume that suppliers are responsible for their own inbound shipping costs if selected. We consider the outbound shipping cost as the only cost incurred by the consolidation center while providing the service and therefore only the outbound shipping cost will be shared among the selected suppliers. Given the destination, let $c_{F1}$ denote the FTL rate for outbound shipping at the consolidation center. We denote the FTL equivalent volume by $b_C = \frac{c_{F1}}{c_{L1}}$. In mathematical terms, the cost $\phi$ of shipping demand $d$ at the consolidation center is
\[
\phi(d) = \begin{cases} 
c_{L1}d & \text{if } 0 \leq d \leq b_C, 
c_{F1} & \text{if } d \geq b_C. 
\end{cases}
\]
Since the cost function $\phi : \mathbb{R} \rightarrow \mathbb{R}$ has nonincreasing incremental value as the size of input set increases, $\phi(\sum_{i \in S} d_i)$ is submodular in $S \subseteq N$. The total cost $C(S)$ incurred when we consolidate and ship the demand of suppliers in $S$ is
\[
C(S) = \sum_{i \in S} G^0_i + \phi \left( \sum_{i \in S} d_i \right).
\]
The function $\sum_{i \in S} G_i^0$ is submodular in $S \subseteq N$ as well. Because the class of submodular functions is closed under non-negative linear combinations, $C(S)$ is submodular in $S$.

We assume that the consolidation center only has partial information about the transportation costs of the suppliers. In particular, the consolidation center knows that it has the same trucking cost structure as the suppliers, but it does not know the exact parameters of the cost functions for the suppliers. The information that the consolidation center solicits from the suppliers is their bids for their shipping demand. Therefore, the suppliers’ shipping demands are known to the consolidation center as well.

We further restrict the service provided by the consolidation center to be binary: either a supplier is not served at all or its full shipping demand is served. Note that under the assumption that the total demand is less than or equal to one truckload, it is straightforward to show that a binary service policy is optimal from the suppliers' perspective. However, this might not be true when the total demand requires multiple trucks. Later in Section 4, we will show that the binary service policy is also optimal from the social welfare perspective when the total demand is less than or equal to one truckload.

4. Cost-Sharing Mechanism Design

In this section, we design cost-sharing mechanisms for the cost-sharing problem defined above. Recall that the total demand of all the suppliers fits into one truck, i.e. $\sum_{k \in N} d_k \leq k_F$, and so the demand of each supplier is less than one truckload. In this section, we will first introduce the Moulin mechanism framework, which we use to design our cost-sharing mechanism. We then present our cost-sharing mechanism and show that this mechanism is both truthful and budget-balanced. Finally, we will discuss the outcomes of our cost-sharing mechanism for different demand profiles by providing some properties of the mechanism.

4.1. The Moulin Mechanism

The Moulin mechanism (Moulin 1999, Moulin and Shenker 2001) affords a lot of flexibility in designing truthful while budget-balanced or approximately budget-balanced cost-sharing mechanisms. It simulates an iterative ascending auction to determine which subset of players to serve by using a cost-sharing method, a function $\chi$ that assigns a nonnegative cost share for each player $i \in S$ for every subset $S \subseteq N$. The cost shares for the selected subset of players indicates the cost allocation strategy, or alternatively, the price charged to each player. The Moulin mechanism operates as follows:

1. Collect a bid $q_i$ from each player $i \in N$.
2. Initialize $S := N$. 
3. If $q_i \geq \chi(i,S)$ for every $i \in S$, then stop. Return the set $S$. Each player $i \in S$ is charged the price $p_i = \chi(i,S)$.

4. If $q_j < \chi(j,S)$ for a player $j \in S$, then set $S := S \setminus \{j\}$ and return to Step 3.

Because only the players whose bids are greater than or equal to their cost share stay in the subset $S$, the players selected by the Moulin mechanism are never charged more than what they bid.

The cost-sharing method $\chi$ plays a very important role in a Moulin mechanism design. It is often required to be cross-monotonic, which means that the cost share of each player only increases as other players are removed, i.e. for all $S \subseteq T \subseteq N$ and $i \in S$, $\chi(i,S) \geq \chi(i,T)$. This implies that each player in $S$ is offered a sequence of nondecreasing cost shares through the iterations. When the cost-sharing method $\chi$ is cross-monotonic, the Moulin mechanism is group strategyproof. Group strategyproofness is a strong notion of truthfulness: an individual player cannot be better off by false bidding, and a subset of players can never strictly increase the utility of one of its members without decreasing the utility of some other member by coordinating false bids.

We split the possible outcomes of this mechanism – the set of players served $S$ – into three categories:

**Total participation:** All the players in $N$ are served.

**Zero participation:** None of the players in $N$ are served.

**Partial participation:** A non-empty proper subset of $N$ is served.

**Observation 1:** Any Moulin mechanism yields total participation if and only if $\chi(i,N) \leq q_i$ for all $i \in N$.

**Observation 2:** Any Moulin mechanism yields zero participation if and only if in every iteration $k = 1, 2, \ldots, n$, there exists at least one player $i$ such that $\chi(i,S^k) > q_i$, where $S^k$ denotes the remaining set of players at the beginning of iteration $k$. If the Moulin mechanism yields zero participation, then one player is removed from $S^k$ in Step 4 of iteration $k$ of the mechanism. In other words, in each iteration $k$ of the mechanism there exists at least one player $i$ that has $\chi(i,S^k) > q_i$. On the other hand, if in each iteration $k$, there exists a player $i$ that satisfies $\chi(i,S^k) > q_i$, then a player will be removed from $S^k$ until there are no more players left.

4.2. Cost-Sharing Mechanism Proportional to Effective Demand for Sharing (PEDS)

Given the shipping volumes and the corresponding bids, the most intuitive way of sharing the outbound shipping cost is to share it proportional to each supplier’s actual demand. In other words, the cost share for supplier $i \in S$ would be $\frac{d_i}{\sum_{j \in S} d_j} \phi(\sum_{j \in S} d_j)$, where $S$ is the selected set of suppliers.

This cost sharing method tends to allocate more cost to the suppliers with larger demand and thus such suppliers, without whom consolidation may not be beneficial, may not have the incentive to bid for consolidation.
To illustrate, consider an example where there are three suppliers who want to consolidate their demand and their transportation costs of shipping directly and shipping through the consolidation center are given in Table 1. We assume $b_C = b_G = 5000$, $c_{F1} = $1000, $c_{L1} = $0.2/ft$^3$, and each supplier bids their direct shipping costs. Supplier 3 has larger demand than Supplier 1 and Supplier 2. If every supplier ships its demand individually, the total transportation cost is $1400$. However, if we consolidate all of their demand, the total transportation cost is $C(N) = $1300. If we share the outbound shipping cost proportional to actual demand in the Moulin mechanism, the cost share for each supplier in each iteration is shown in Table 2.

Table 1  Cost-sharing example

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (1000 ft$^3$)</td>
<td>1</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Direct shipping cost ($)</td>
<td>200</td>
<td>200</td>
<td>1000</td>
</tr>
<tr>
<td>Inbound shipping cost ($)</td>
<td>43</td>
<td>43</td>
<td>214</td>
</tr>
</tbody>
</table>

Table 2  Cost shares proportional to actual demand

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iteration 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outbound cost share ($)</td>
<td>100</td>
<td>100</td>
<td>800</td>
</tr>
<tr>
<td>Inbound shipping cost ($)</td>
<td>43</td>
<td>43</td>
<td>214</td>
</tr>
<tr>
<td>Total cost ($)</td>
<td>143</td>
<td>143</td>
<td>1014</td>
</tr>
<tr>
<td>Decision</td>
<td>Accept</td>
<td>Accept</td>
<td>Decline</td>
</tr>
<tr>
<td>Iteration 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outbound cost share ($)</td>
<td>200</td>
<td>200</td>
<td>N/A</td>
</tr>
<tr>
<td>Inbound shipping cost ($)</td>
<td>43</td>
<td>43</td>
<td>N/A</td>
</tr>
<tr>
<td>Total cost ($)</td>
<td>243</td>
<td>243</td>
<td>N/A</td>
</tr>
<tr>
<td>Decision</td>
<td>Decline</td>
<td>Decline</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 2 shows that sharing the outbound shipping cost proportional to actual demand leads to a total cost share for Supplier 3 that is greater than its direct shipping cost. As a result, Supplier 3 declines to use the service. This leaves only Supplier 1 and Supplier 2 under consideration in the next iteration. However, without Supplier 3, Supplier 1 and Supplier 2 end up with cost shares that are higher than their direct shipping costs. Consequently, none of the suppliers are able to benefit from consolidation, which could have saved a total of $100 if implemented properly. The example above shows the deficiency of sharing the outbound shipping cost proportional to actual demand and reveals the importance of the participation of the suppliers with larger demand.

Therefore, in order to encourage the suppliers with larger demand to participate, we set a maximum demand volume $b_E$ that a supplier needs to be responsible for in the cost share. This implies that a supplier with a demand greater than $b_E$ only shares the outbound shipping cost
proportional to $b_E$ and thus is allocated a smaller cost. In other words, each supplier has an effective demand for sharing defined by the maximum demand volume $b_E$.

**Effective demand for sharing:** For each supplier $i \in N$, its effective demand for sharing is $d_i^* = \min\{d_i, b_E\}$.

We propose a cost-sharing method that shares the outbound shipping cost proportional to effective demand for sharing (PEDS). Let $D_S = \sum_{i \in S} d_i$ and $D_S' = \sum_{i \in S} d_i^*$ for any $S \subseteq N$. The cost share offered to supplier $i \in S$ is equal to its inbound shipping cost plus the share of the outbound shipping cost for set $S$, proportional to supplier $i$’s effective demand for sharing; that is, for $S \subseteq N$, $i \in S$, we define the cost share $\chi(i, S)$ as

$$\chi(i, S) = G_0^i + \frac{d_i^*}{D_S'} \phi(D_S) = \begin{cases} 
G_0^i + c_{L1} D_S \cdot \frac{d_i}{D_S} & \text{if } 0 \leq D_S \leq b_C, \ 0 < d_i < b_E; \\
G_0^i + c_{L1} D_S \cdot \frac{b_E}{D_S} & \text{if } 0 \leq D_S \leq b_C, \ d_i \geq b_E; \\
G_0^i + \frac{d_i}{D_S} c_F & \text{if } b_C \leq D_S, \ 0 < d_i < b_E; \\
G_0^i + \frac{b_E}{D_S} c_F & \text{if } b_C \leq D_S, \ d_i \geq b_E.
\end{cases}$$

Now let $b_E = 5000$. If we apply cost-sharing method PEDS in the example mentioned earlier, the cost shares we obtain for each supplier are summarized in Table 3.

<table>
<thead>
<tr>
<th>Table 3 Cost shares proportional to effective demand for sharing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Iteration 1</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Outbound cost share ($)</td>
</tr>
<tr>
<td>Inbound shipping cost ($)</td>
</tr>
<tr>
<td>Total cost ($)</td>
</tr>
<tr>
<td>Decision</td>
</tr>
</tbody>
</table>

Cost-sharing method PEDS charges less to Supplier 3 than when costs are shared proportional to actual demand. The total cost for each supplier from cost-sharing method PEDS is less than its direct shipping cost. Thus, all suppliers are willing to participate in the consolidation. Additionally, with the participation of Supplier 3, each supplier of this coalition is able to reduce its transportation costs.

If one views $b_E$ as the consolidation center’s estimation of the suppliers’ FTL equivalent volume, cost-sharing mechanism PEDS shares the costs proportional to the consolidation center’s estimate of each supplier’s stand-alone cost. When the estimated FTL equivalent volume equals the true FTL equivalent volume of the suppliers, PEDS shares the cost proportional to the actual stand-alone cost of each supplier. Therefore, cost-sharing method PEDS is a more favorable cost allocation method because the estimated stand-alone cost is a better reflection of the true costs of the suppliers. The cost of shipping a large demand $d > b_C$ of a supplier is not proportional to the demand volume because $G_{F1} < G_{L1} d$ is the true cost.
As mentioned above, cross-monotonic cost-sharing methods lead to truthful Moulin mechanisms. We will next explore the cross-monotonicity of cost-sharing method PEDS and the properties of the corresponding cost-sharing mechanism PEDS.

**Proposition 1.** If \( b_E \geq b_C \), cost-sharing method PEDS is cross-monotonic.

**Proof.** Let \( i \) be an arbitrary supplier whose cost share we observe and compare in different subsets. Since the inbound shipping cost \( G^0_i \) is always included in supplier \( i \)'s cost share in the cost-sharing method PEDS, we only focus on supplier \( i \)'s cost share of the outbound shipping cost to prove cross-monotonicity.

Let \( S \) be an arbitrary set such that \( i \in S \) and \( S \subseteq N \setminus \{j\} \), where \( i \neq j \). We obtain \( T \) by augmenting \( S \) with supplier \( j \), i.e. \( T = S \cup \{j\} \). Let \( \Gamma(S,T) \) denote the total cost share of the outbound shipping cost for suppliers in \( S \) while serving \( T \).

If \( D_S \leq b_C \) and \( D_T \leq b_C \), then \( d_i \leq b_C \leq b_E \) and \( d'_i = d_i \) for all \( i \in T \). We have

\[
\Gamma(S,T) = \frac{D'_S}{D_T} \cdot c_{L1}D_T = \frac{D_S}{D_T} \cdot c_{L1}D_T = c_{L1} \cdot D_S = \phi(D_S).
\]

If \( D_S \leq b_C \) and \( D_T > b_C \), then \( d_i \leq b_C \leq b_E, d'_i = d_i \) for all \( i \in S \) and \( \phi(D_T) = c_{F1} \). We consider 2 cases. First, if \( d_j < b_E \), then \( d'_j = d_j \). We have

\[
\Gamma(S,T) = \frac{D'_S}{D_T}c_{F1} = \frac{D_S}{D_T}c_{F1} < \frac{c_{F1}}{b_C} D_S = c_{L1}D_S = \phi(D_S).
\]

Second, If \( d_j \geq b_E \), then \( d'_j = b_E \). We have

\[
\Gamma(S,T) = \frac{D_S}{D_S + b_E}c_{F1} < \frac{c_{F1}}{b_C} D_S = \phi(D_S),
\]

where the inequality holds because \( b_E \geq b_C \).

Finally, if \( D_S \geq b_C \) and \( D_T > b_C \), then \( \phi(D_T) = \phi(D_S) = c_{F1} \). Adding one more supplier does not increase the outbound shipping cost but results in one more supplier sharing the unchanged cost. Consequently, \( \Gamma(S,T) \leq \phi(D_T) = \phi(D_S) \).

The three cases above show that the total cost share of the outbound shipping cost does not increase when one more supplier is served, i.e. \( \Gamma(S,T) \leq \phi(D_S) \). Since the total cost share \( \Gamma(S,T) \) does not increase, the share of the outbound shipping cost for supplier \( i \) does not increase as well. Together with the inbound shipping cost, \( \chi(i,T) \) does not increase either compared to \( \chi(i,S) \). Therefore, when \( i \in S, S \subseteq N \setminus \{j\}, T = S \cup \{j\} \), and \( i \neq j \), we have \( \chi(i,S) \geq \chi(i,T) \). This implies that for arbitrary \( S \subseteq T \subseteq N \), \( \chi(i,S) \geq \chi(i,T) \).

Since cost-sharing mechanism PEDS is cross-monotonic when \( b_E \geq b_C \), the Moulin mechanism that applies our cost-sharing method PEDS with \( b_E \geq b_C \) is group strategyproof. We call this cost-sharing mechanism PEDS. In addition, cost-sharing method PEDS, which shares the total incurred
cost of consolidation among the selected suppliers, guarantees that the cost-sharing mechanism PEDS is also budget-balanced.

Note that since the total cost \( C(S) \) is submodular in \( S \), we can also find a cross-monotonic cost sharing method by considering extensions of the marginal vectors or the Shapley value of \( C \) (Sprumont (1990), Moulin and Shenker (2001)). However, the resulting cost sharing methods and mechanisms arguably lose the intuitive and practical appeal of PEDS.

4.3. Properties of Cost-Sharing Mechanism PEDS

In this section, we examine the conditions for cost-sharing mechanism PEDS to yield total or zero participation. We consider different demand profiles and estimations of suppliers’ FTL equivalent volume to provide managerial insights for the consolidation center. From now on, we assume that the valuation of the consolidation service \( v_i \) of supplier \( i \) is its stand-alone cost and thus submits a bid \( q_i = v_i \) to cost-sharing mechanism PEDS. We can make this assumption due to the group strategyproofness of cost-sharing mechanism PEDS. The stand-alone cost of supplier \( i \) is the truthful valuation of the consolidation center’s service for supplier \( i \) because that is the cost supplier \( i \) has to pay for if it does not participate in consolidation. Note that from this point on, whenever we discuss cost-sharing method PEDS or cost-sharing mechanism PEDS, we assume \( b_E \geq b_C \).

In Lemma 1, we give a sufficient condition for cost sharing mechanism PEDS to yield zero participation.

**Lemma 1.** If \( \chi(i, N) > q_i \) for all \( i \in N \), cost-sharing mechanism PEDS yields zero participation.

**Proof.** Assume \( \chi(i, N) > q_i \) for all \( i \in N \). Suppose supplier \( j \in N \) is removed in the first iteration of the mechanism, resulting in \( S := N \setminus \{j\} \) in the second iteration. Because cost-sharing method PEDS is cross-monotonic, \( \chi(i, S) \geq \chi(i, N) > q_i \) for all \( i \in S \). As a result, another supplier will be removed from \( S \) in the second iteration. By the same argument, there exists at least one supplier \( i \) such that \( \chi(i, S) > q_i \) in each iteration. According to Observation 2, cost-sharing mechanism yields zero participation. \( \square \)

Suppose we have a set of suppliers \( N \), whose total demand is less than the FTL equivalent volume of the consolidation center, i.e. \( D_N < b_C \). Cost-sharing mechanism PEDS always yields zero participation for such a set of suppliers. Equivalently, it is not beneficial for such a set of suppliers to ship their demand via the consolidation center. Because \( D_N < b_C \leq b_E \), the demand of each supplier \( i \in N \) is \( d_i < b_E \). Therefore, the effective demand for sharing of each supplier \( i \in N \), \( d'_i = d_i \). As a result, suppliers share the outbound shipping cost by paying the LTL rate no matter how many suppliers participate in the consolidation, i.e. \( \frac{d_i}{D_N} c_{L1} D_N = d_i c_{L1} \). Thus, any supplier \( i \) with \( d_i < b_G \) has a cost share \( \chi(i, N) = d_i g_{L0} + d_i c_{L1} \) in the first iteration of the mechanism. The location assumption \( g_{L1} < g_{L0} + c_{L1} \) implies that \( \chi(i, N) = d_i (g_{L0} + c_{L1}) > d_i g_{L1} = q_i \). Any supplier
i with \(d_i \geq b_G\) has a cost share \(\chi(i, N) = g_{F0} + d_i c_{L1}\), which is also greater than its corresponding stand-alone cost because \(\chi(i, N) = b_G g_{L0} + d_i c_{L1} \geq b_G (g_{L0} + c_{L1}) > b_G g_{L1} = g_{F1} = q_i\). Consequently, every supplier \(i\) has a cost share \(\chi(i, N) > q_i\). Therefore, according to Lemma 1, cost-sharing mechanism PEDS yields zero participation when the set of suppliers \(N\) has total demand less than \(b_C\). Intuitively, because \(g_{L1} < g_{L0} + c_{L1}\), these suppliers cannot reduce their costs by consolidating: sending shipments to the consolidation center adds extra cost by increasing the total distance traveled, and these suppliers cannot benefit from the volume discount. For this reason, from here on we restrict our attention to sets of suppliers \(N\) whose total demand is greater than the FTL equivalent volume at the consolidation center, i.e. \(D_N \geq b_C\).

The consolidation center decides the value of \(b_E\) before collecting bids. Without knowing the exact value of \(b_G\), the consolidation center’s estimate can be above, below, or equal to the true \(b_G\). Note that \(b_E \geq b_G\) and \(D_N \geq b_C\), so \(D_N \geq b_C\). When the consolidation center overestimates the suppliers’ FTL equivalent volume, i.e. \(b_E > b_G\), the conditions for cost-sharing mechanism PEDS to yield total and zero participation are summarized in Proposition 2 and Proposition 3. (For their proofs, see Appendix A.)

**Proposition 2.** When \(b_E > b_G\), cost-sharing mechanism PEDS yields total participation if and only if \(D'_N \geq \frac{b_E - c_{F1}}{b_G g_{L1} - g_{L0}}\).

**Proposition 3.** When \(b_E > b_G\), if \(D'_N < \frac{c_{F1}}{g_{L1} - g_{L0}}\), cost-sharing mechanism PEDS yields zero participation.

Note that the smallest possible value of \(b_E\) is \(b_G\). If \(b_G < b_C\), then any possible \(b_E\) is an overestimate of \(b_G\). Given such a condition, if the consolidation center’s goal is to incentivize consolidation, its strategy should be to have \(b_E = b_G\) so that the demand condition on \(D'_N\) for total participation given in Proposition 2 is as easy to meet as possible.

When the consolidation center underestimates the suppliers’ FTL equivalent volume, i.e. \(b_E < b_G\) (implying \(b_G \geq b_C\)), the conditions for cost-sharing mechanism PEDS to yield total and zero participation are summarized in Proposition 4 and Proposition 5 (For their proofs, see Appendix A).

**Proposition 4.** When \(b_E < b_G\), cost-sharing mechanism PEDS yields total participation if and only if \(D'_N \geq \frac{c_{F1}}{g_{L1} - g_{L0}}\).

**Proposition 5.** When \(\frac{g_{L1} - g_{L0}}{c_{L1}} b_G < b_E < b_G\), if \(D'_N < \frac{b_E - c_{F1}}{b_G g_{L1} - g_{L0}}\), cost-sharing mechanism PEDS yields zero participation.

The lower bound \(\frac{g_{L1} - g_{L0}}{c_{L1}} b_G\) for \(b_E\) in Proposition 5 is necessary for \(\frac{b_E - c_{F1}}{b_G g_{L1} - g_{L0}}\) to be a valid upper bound for \(D'_N\). Since \(D'_N \geq b_C\), \(b_E > \frac{g_{L1} - g_{L0}}{c_{L1}} b_G\) guarantees that \(\frac{b_E - c_{F1}}{b_G g_{L1} - g_{L0}} \geq b_C\). When \(b_E\) is below
\[ \frac{gL_1-gL_0}{cL_1} b_G, \] the outcome of cost-sharing mechanism PEDS when \( D'_N < \frac{cF_1}{gL_1-gL_0} \) can either be partial or zero participation.

Whether the consolidation center underestimates or overestimates the suppliers’ FTL equivalent, there is a range of \( D'_N \) – for example \( \left[ \frac{cF_1}{gL_1-gL_0}, \frac{b_E}{b_G} \frac{cF_1}{gL_1-gL_0} \right] \) with overestimation – under which the outcome of cost-sharing mechanism PEDS is unclear. This ambiguity no longer exists when the consolidation center correctly estimates the suppliers’ FTL equivalent volume.

**Corollary 1.** When \( b_E = b_G \), cost-sharing mechanism PEDS yields either zero or total participation.

**Proof.** In Propositions 2, 3, 4 and 5, when \( b_E = b_G \), the conditions for total and zero participation depend on the same critical value \( \frac{cF_1}{gL_1-gL_0} \). Thus, the conditions for total and zero participation complement each other. Therefore, given any demand profile, the outcome of cost-sharing mechanism PEDS is either zero participation or total participation. \( \Box \)

Based on the above analysis of the outcomes of cost-sharing mechanism PEDS, if the consolidation center aims to promote consolidation, it should set \( b_E = b_C \) if it knows that \( b_G < b_C \). If it knows that \( b_G \geq b_C \), although estimating \( b_G \) correctly can reduce the ambiguity of the outcome, the choice of \( b_E \) does not affect the condition on \( D'_N \) that yields total participation for underestimation.

### 5. Economic Efficiency of Cost-Sharing Mechanism PEDS

In this section, we examine the resulting social welfare of cost-sharing mechanism PEDS. We first introduce an optimization model that finds an economically efficient solution for any given demand profile. By comparing the outcomes of cost-sharing mechanism PEDS with the economically efficient solutions, we analytically show that the total and zero participation outcomes guaranteed above for certain demand profiles are economically efficient. Then we numerically study the economic efficiency for the demand profiles whose outcomes of cost-sharing mechanism PEDS are unknown.

Typically, the economic efficiency of a cost-sharing mechanism is measured by social welfare. An economically efficient solution is the one that maximizes the social welfare: \( W(S) = V(S) - C(S) \), where \( S \) is the set of suppliers selected by the mechanism, \( V(S) \) is the total valuation of the suppliers in \( S \) and \( C(S) \) is the total cost to serve the suppliers in \( S \). Unfortunately, Feigenbaum et al. (2002) showed that truthful and approximately budget-balanced cost-sharing mechanisms often yield outcomes with zero or negative social welfare even though outcomes with strictly positive social welfare exist. This makes it difficult to compare the relative economic efficiency of cost-sharing mechanisms with the same budget-balance guarantee.

To sidestep this issue, Roughgarden and Sundararajan (2009) introduced social cost, a measure of economic efficiency. The social cost \( \pi(S) \) is defined as the sum of the cost incurred by serving \( S \)
and the total valuations of the suppliers not in $S$. In mathematical terms, $\pi(S) = C(S) + V(N \setminus S)$, where $V(N \setminus S)$ is the total valuation of the suppliers who are excluded from $S$. In fact, social cost can be constructed by an affine transformation from social welfare: $\pi(S) = -W(S) + V(N)$. The relationship between social welfare and social cost implies that minimizing social cost is equivalent to maximizing social welfare, although social cost is always nonnegative. As a result, we use social cost as the measure of economic efficiency and determine outcomes with the maximum social welfare by minimizing social cost.

In our problem, the social cost is equal to the total shipping cost of all the suppliers in $N$. We consider the following optimization model to minimize the total shipping cost of all the suppliers. Suppliers now can choose how much demand to ship directly and how much demand to ship via the consolidation center. Therefore, the service provided by the consolidation center in this model is not restricted to be binary. However, we show that the solution which minimizes the social cost corresponds to binary service provided by the consolidation center. The decision variables and the model are presented below.

**Decision variables:**

- $x^i_{F0}$: Binary. If supplier $i$’s inbound shipping uses the FTL rate, then $x^i_{F0} = 1$, otherwise $0 \forall i \in N$.
- $x^i_{L0}$: Binary. If supplier $i$’s inbound shipping uses the LTL rate, then $x^i_{L0} = 1$, otherwise $0 \forall i \in N$.
- $x^i_{F1}$: Binary. If supplier $i$’s direct shipping uses the FTL rate, then $x^i_{F1} = 1$, otherwise $0 \forall i \in N$.
- $x^i_{L1}$: Binary. If supplier $i$’s direct shipping uses the LTL rate, then $x^i_{L1} = 1$, otherwise $0 \forall i \in N$.
- $x_{CF}$: Binary. If outbound shipping uses the FTL rate, then $x_{CF} = 1$, otherwise $0$.
- $x_{CL}$: Binary. If outbound shipping uses the LTL rate, then $x_{CL} = 1$, otherwise $0$.
- $y^i_{F0}$: Amount of supplier $i$’s demand sent by the FTL rate to the consolidation center $\forall i \in N$.
- $y^i_{L0}$: Amount of supplier $i$’s demand sent by the LTL rate to the consolidation center $\forall i \in N$.
- $y^i_{F1}$: Amount of supplier $i$’s demand sent by the FTL rate to the destination $\forall i \in N$.
- $y^i_{L1}$: Amount of supplier $i$’s demand sent by the LTL rate to the destination $\forall i \in N$.
- $y_{CF}$: Amount of demand sent by the FTL rate from the consolidation center to the destination.
- $y_{CL}$: Amount of demand sent by the LTL rate from the consolidation center to the destination.

**Model:**

$$\min \sum_{i \in N} \left( g_{F0}x^i_{F0} + g_{F1}x^i_{F1} + g_{L0}y^i_{L0} + g_{L1}y^i_{L1} \right) + c_{F1}x_{CF} + c_{L1}y_{CL}$$

s.t.

$$y^i_{F0} \leq k_{F}x^i_{F0} \forall i \in N,$$  

$$y^i_{L0} \leq b_{G}x^i_{L0} \forall i \in N,$$  

$$y^i_{F1} \leq k_{F}x^i_{F1} \forall i \in N,$$
Constraints (1), (3), and (5) ensure that the shipping volumes do not exceed the truckload when shipping with the FTL rates. Constraints (2), (4), and (6) ensure that the shipping volumes do not exceed the FTL equivalent volumes when shipping with the LTL rates. Constraint (7) makes sure that each supplier ships all of its demand. Constraint (8) enforces that what ships into the consolidation center will ship out.

An optimal solution of this model provides each supplier $i \in N$ with a shipping plan that minimizes the total shipping cost of all the suppliers. The total cost consists of two parts: one part is the shipping cost of each supplier, the other part is the shipping cost of the consolidation center. In this model, if a shipping volume $d$ from a supplier to the consolidation center or the destination is smaller than $b_G$, then LTL is the optimal shipping method; if $d$ is greater than or equal to $b_G$, then FTL is the optimal shipping method. Increasing the shipment volume when the total shipping demand exceeds $b_G$ does not incur extra cost. Therefore, a supplier uses either the FTL rate or the LTL rate to ship demand to the consolidation center or the destination in an optimal solution since the shipping volume for this supplier, $d$, is assumed to be less than the capacity of a full truckload $k_F$. In mathematical terms, for every supplier $i$, $\tilde{x}_{iL0} \cdot \tilde{x}_{iL0} = 0$ and $\tilde{x}_{iF1} \cdot \tilde{x}_{iL1} = 0$ where $\tilde{x}$ is in the optimal solution. The same logic applies to the consolidation center as well, i.e. $\tilde{x}_{CL} \cdot \tilde{x}_{CL} = 0$ where $\tilde{x}$ is in the optimal solution. We analyze the structure of the optimal solutions to this model to understand how the minimum social cost is achieved. We show in Proposition 6 and Corollary 2 that in an optimal solution of the above optimization model each supplier has its entire demand shipped either directly or through the consolidation center. In Proposition 7 we show the minimum social cost solution is either zero participation or total participation. (For their proofs, see Appendix B).

**PROPOSITION 6.** There exists an optimal solution to the model in which each supplier ships all its demand either to the consolidation center or directly to the destination.
Following the above proposition, we can show a stronger result on the structure of the optimal solution.

**Corollary 2.** Every optimal solution to the model shares the same structure: $\tilde{x}_{F_0} + \tilde{x}_{L_0} + \tilde{x}_{F_1} + \tilde{x}_{L_1} = 1$ where $\tilde{x}$ is any optimal solution. In other words, in every optimal solution to the model, each supplier's entire demand is shipped either to the consolidation center or directly to the destination.

So far, we have shown the best practice for each supplier in $N$. Although suppliers have two shipping options, shipping the entire demand of one supplier using one option leads to the minimum social cost. The optimal system-wide shipping plan is given next.

**Proposition 7.** Every optimal solution to the model yields either zero participation or total participation. A solution in which a proper subset of suppliers $S \subset N$, $S \neq \emptyset$ ships their demand to the consolidation center first while the rest of the suppliers ship their demand directly to the destination is not optimal.

Since the economically efficient solution is either zero participation or total participation, we can easily verify if an outcome of cost-sharing mechanism PEDS is economically efficient or not. If the outcome is partial participation, it is not an economically efficient solution. If the outcome is total or zero participation, we can compare its total shipping cost to the zero or total participation total shipping cost to see if the outcome is economically efficient or not. When the total shipping cost of total participation equals to that of zero participation, we assume that total participation is the solution of the optimization model. We examine the economic efficiency of the outcomes guaranteed under the conditions given in Propositions 2, 3, 4, and 5, and Corollary 1. The results are summarized in the next proposition.

**Proposition 8.** Cost-sharing mechanism PEDS is economically efficient under each of the following conditions:

- a. $b_E > b_G$ and $D_N' \geq \frac{b_E}{b_G} \frac{c_{F_1}}{g_{L_1} - g_{L_0}}$
- b. $b_E > b_G$ and $D_N' < \frac{c_{F_1}}{g_{L_1} - g_{L_0}}$
- c. $b_E < b_G$ and $D_N' \geq \frac{c_{F_1}}{g_{L_1} - g_{L_0}}$
- d. $\frac{g_{L_1} - g_{L_0}}{c_{L_1}} b_G < b_E < b_G$ and $D_N' < \frac{b_E}{b_G} \frac{c_{F_1}}{g_{L_1} - g_{L_0}}$

**Proof.** Under conditions a and c, cost-sharing mechanism PEDS yields total participation, which means that each participant pays no more than its stand-alone cost. Then, the social cost of total participation is no more than the social cost of zero participation. Given the assumption that the minimum social cost solution is total participation when the social costs of total participation and zero participation are the same, cost-sharing mechanism PEDS produces economically efficient solutions under conditions a and c.
Under conditions b and d, cost-sharing mechanism PEDS yields zero participation induced by Lemma 1, which means that each participant pays strictly more than its stand-alone cost if total participation is enforced. Then, the social cost of total participation is strictly more than that of zero participation. Therefore, cost-sharing mechanism PEDS produces economically efficient solutions under conditions b and d.

**Proposition 9.** When \( b_E = b_G \), cost-sharing mechanism PEDS is economically efficient under any demand profile.

**Proof.** When \( b_E = b_G \), then \( b_C \leq b_E = b_G \). According to Propositions 2, 3, 4, and 5 and Corollary 1, cost-sharing mechanism PEDS yields zero participation when \( D_N' < \frac{c_{F1}}{g_{L1} - g_{L0}} \) and total participation when \( D_N' \geq \frac{c_{F1}}{g_{L1} - g_{L0}} \). In the optimization model, the cost of total participation is \( n g_{F0} + \delta g_{L0} + c_{F1} \) and the cost of zero participation is \( n g_{F1} + \delta g_{L1} \), where \( n \) denotes the number of suppliers whose demands are greater than or equal to \( b_G \) and \( \delta \) denotes the total demand of the suppliers whose demands are smaller than \( b_G \).

Suppose \( D_N' < \frac{c_{F1}}{g_{L1} - g_{L0}} \). The cost difference between zero participation and total participation is

\[
ng_{F1} + \delta g_{L1} - (ng_{F0} + \delta g_{L0} + c_{F1}) = nb_G g_{L1} + \delta g_{L1} - nb_G g_{L0} - \delta g_{L0} - c_{F1}
\]

\[
= (nb_G + \delta)(g_{L1} - g_{L0}) - c_{F1}
\]

\[
= D_N'(g_{L1} - g_{L0}) - c_{F1}
\]

\[
< 0,
\]

where the last equality holds because \( b_E = b_G \). Therefore, the optimization model yields zero participation when \( D_N' < \frac{c_{F1}}{g_{L1} - g_{L0}} \). Similarly, the optimization model yields total participation when \( D_N' \geq \frac{c_{F1}}{g_{L1} - g_{L0}} \). As a result, cost-sharing mechanism PEDS yields an economically efficient solution for any demand profile.  

### 5.1. Experimental Results of the Unknown Economic Efficiency Cases

So far we have showed analytically in Propositions 8 and 9 that under certain conditions, cost-sharing mechanism PEDS yields economically efficient solutions. However, the economic efficiency of cost-sharing mechanism PEDS remains unknown for what we call unknown economic efficiency cases; in particular:

1. \( b_E > b_G \) and \( \frac{c_{F1}}{g_{L1} - g_{L0}} \leq D_N' < \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}} \),

2. \( \frac{g_{L1} - g_{L0}}{c_{L1}} b_G < b_E < b_G \) and \( \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}} \leq D_N' < \frac{c_{F1}}{g_{L1} - g_{L0}} \),

3. \( b_E < \frac{g_{L1} - g_{L0}}{c_{L1}} b_G < b_G \) and \( D_N' < \frac{c_{F1}}{g_{L1} - g_{L0}} \).
We use numerical experiments to investigate the economic efficiency of cost-sharing mechanism PEDS for these cases. In particular, we want to know how often and how much the outcomes of cost-sharing mechanism PEDS deviate from minimum social cost solutions, how overestimation and underestimation influence the economic efficiency of the mechanism, and how the variations in distances affect the mechanism’s economic efficiency.

We fix the values of most of the parameters and change the values of $b_G$, $b_E$ and $g_{L0}$ to study the influences of overestimation, underestimation and variations in distances. The fixed value parameters are shown in Table 4. The value of $b_G$ is varied so that sometimes it is less than $b_C$ and sometimes it is greater than $b_C$. The value of $b_E$ is chosen to be either 10%, 20%, 50%, or 100% away from $b_C$ while being in its valid range, i.e. $b_C \leq b_E \leq k_F$. The selected values of $b_G$ and $b_E$ are displayed in Tables 5 and 6 for overestimation and underestimation, respectively. We adjust $g_{L0}$ to change the ratio between the distance for direct shipping and the distance for inbound shipping, i.e. $\frac{g_{L1}}{g_{L0}}$. A smaller $g_{L0}$ indicates a farther destination compared to the location of the consolidation center. Similarly, a larger $g_{L0}$ indicates a closer destination. The values we use for $g_{L0}$ are $0.05/\text{ft}^3$, $0.1/\text{ft}^3$, and $0.3/\text{ft}^3$.

### Table 4  Fixed parameters

<table>
<thead>
<tr>
<th>$k_F$ ($\text{ft}^3$)</th>
<th>$b_C$ ($\text{ft}^3$)</th>
<th>$g_{L1}$ ($$/\text{ft}^3$$)</th>
<th>$c_{L1}$ ($$/\text{ft}^3$$)</th>
<th>$c_{F1}$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>2000</td>
<td>1.28</td>
<td>1.25</td>
<td>2500</td>
</tr>
</tbody>
</table>

### Table 5  $b_G$ and $b_E$ for overestimation

<table>
<thead>
<tr>
<th>$b_G$ ($\text{ft}^3$)</th>
<th>$b_E$ ($\text{ft}^3$)</th>
<th>off percentage</th>
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<tbody>
<tr>
<td>1500</td>
<td>3000</td>
<td>↑100%</td>
</tr>
<tr>
<td>2250</td>
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<td></td>
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<tr>
<td>1800</td>
<td>3600</td>
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<tr>
<td>2700</td>
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<tr>
<td>2160</td>
<td>↑20%</td>
<td></td>
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<tr>
<td>2000</td>
<td>4000</td>
<td>↑100%</td>
</tr>
<tr>
<td>3000</td>
<td>↑50%</td>
<td></td>
</tr>
<tr>
<td>2400</td>
<td>↑20%</td>
<td></td>
</tr>
<tr>
<td>2200</td>
<td>↑10%</td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td>3750</td>
<td>↑50%</td>
</tr>
<tr>
<td>3000</td>
<td>↑20%</td>
<td></td>
</tr>
<tr>
<td>2750</td>
<td>↑10%</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>3600</td>
<td>↑20%</td>
</tr>
<tr>
<td>3300</td>
<td>↑10%</td>
<td></td>
</tr>
</tbody>
</table>

*↑* means overestimate

In order to obtain instances of the unknown economic efficiency cases with 5 suppliers for each given combination of $b_G$, $b_E$, and $g_{L0}$, we first generate a demand profile with 5 suppliers and
then check if it satisfies the conditions to be an unknown economic efficiency case. For each given combination of $b_G$, $b_E$, and $g_{L0}$, we generate demand profiles until we obtain 10,000 unknown economic efficiency cases. The demand profiles with 5 suppliers are generated in the following way: the total demand $D_N$ is first generated from a uniform distribution on $(b_C, k_F)$. Then 5 more random numbers $x_i$, $i = 1, 2, 3, 4, 5$ are generated from a uniform distribution on $(0, 1)$ to determine the demand for each supplier. The demand for supplier $i$ is $x_i rac{D_n}{\sum_{i \in N} x_i} D_N$.

For each given combination of $b_G$, $b_E$, and $g_{L0}$, we regard the ratio between 10,000 and the number of required demand profiles to obtain 10,000 unknown economic efficient cases as the frequency such cases. Empirically, we see that the frequency of such cases ranges from 0.016 to 0.984. In particular, when $b_G = 1500$, $b_E = 3000$, and $g_{L0} = 0.05$, the frequency is as high as 0.984. When $g_{L0} = 0.05$ and 0.1, the frequencies of unknown economic efficiency cases are much lower for underestimation than overestimation. This indicates that accurate estimation and underestimation may help decrease the occurrences of unknown economic efficiency cases. The fact that the frequencies for $g_{L0} = 0.3$ are always higher than that for $g_{L0} = 0.05$ and 0.1 when $b_G > b_C$ indicates that the unknown economic efficiency cases are more likely to happen with closer destinations when $b_G > b_C$.

In terms of the economic efficiency of cost-sharing mechanism PEDS for the generated instances, only 217 instances were found to have outcomes different from the minimum social cost solutions among the 690,000 sampled instances of unknown economic efficiency cases. The overall average gap in social cost for these 217 instances is 5.45%. The details are presented in Table 7.

In Table 7, the “not econ. eff.” columns show the number of instances whose mechanism outcome is not economically efficient and the “avg. gap” columns show the average gap in social cost for these instances. The results in Table 7 suggest that the outcomes of cost-sharing mechanism PEDS for the unknown economic efficiency cases are rarely economically inefficient. For instance, although the frequency of unknown economic efficiency cases is 0.984 when $b_G = 1500$, $b_E = 3000$, and $g_{L0} = 0.05$, in only 6 out of the 10,000 instances did cost-sharing mechanism PEDS yield
Table 7  Social cost gaps for unknown economic efficiency cases

<table>
<thead>
<tr>
<th></th>
<th>$b_G$</th>
<th>$b_E$</th>
<th>over percentage</th>
<th>$g_{L0} = 0.05$</th>
<th></th>
<th>$g_{L0} = 0.1$</th>
<th></th>
<th>$g_{L0} = 0.3$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>not econ. eff.</td>
<td>avg. gap</td>
<td>not econ. eff.</td>
<td>avg. gap</td>
<td>not econ. eff.</td>
<td>avg. gap</td>
<td></td>
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<tr>
<td>Overestimation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>3000</td>
<td>↑100%</td>
<td>6</td>
<td>12.45%</td>
<td>8</td>
<td>8.07%</td>
<td>67</td>
<td>5.72%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2250</td>
<td>↑50%</td>
<td>9</td>
<td>5.86%</td>
<td>17</td>
<td>6.68%</td>
<td>57</td>
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<td></td>
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<td>0</td>
<td>0</td>
<td>11</td>
<td>3.46%</td>
<td></td>
</tr>
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<td>↑50%</td>
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<td>0</td>
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<td>0</td>
<td>10</td>
<td>4.68%</td>
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</tr>
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<td></td>
<td>2100</td>
<td>↑20%</td>
<td>1</td>
<td>0.07%</td>
<td>4</td>
<td>1.85%</td>
<td>15</td>
<td>2.04%</td>
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<td>3</td>
<td>1.97%</td>
<td></td>
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<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
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<td>2750</td>
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<td>↑20%</td>
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<td>0</td>
<td>0</td>
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<tr>
<td></td>
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Underestimation

<table>
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<tr>
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<th>$b_E$</th>
<th>over percentage</th>
<th>$g_{L0} = 0.05$</th>
<th></th>
<th>$g_{L0} = 0.1$</th>
<th></th>
<th>$g_{L0} = 0.3$</th>
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</thead>
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<tr>
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<td>not econ. eff.</td>
<td>avg. gap</td>
<td>not econ. eff.</td>
<td>avg. gap</td>
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<td>0</td>
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<tr>
<td></td>
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<td>↓20%</td>
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<tr>
<td>4000</td>
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<td></td>
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<td>0</td>
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<tr>
<td>2000</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>8.47%</td>
<td></td>
</tr>
</tbody>
</table>

sub-optimal social cost solutions. A social cost gap occurs most frequently when $b_G = 1500$ and 1800. However, it is uncommon for suppliers to have a FTL equivalent volume that is smaller than the FTL equivalent volume at the consolidation center. With larger aggregated volumes, the consolidation center should be able to negotiate a lower FTL equivalent volume, which leads to cheaper FTL rates with fixed LTL rates. Another observation is that the social cost gaps occur more frequently when $g_{L0} = 0.3$. This indicates that cost-sharing mechanism PEDS is more likely to yield minimum social cost solutions for the unknown economic efficiency cases with supplier destinations that are farther away.

In summary, our experimental results show that cost-sharing mechanism PEDS seldom yields sub-optimal social cost solutions for the unknown economic efficiency cases. Combined with the results in Propositions 8 and 9, we can conclude that the outcomes of cost-sharing mechanism PEDS often closely resemble the solution of a social cost minimizing central planner.
6. Conclusion

In this paper, we designed cost-sharing mechanism PEDS, which shares the cost incurred by consolidation and incentivizes suppliers to truthfully participate. We designed cost-sharing method PEDS underlying this mechanism to encourage the participation of the suppliers with larger demand by limiting the maximum demand that they are responsible for. Since the cost-sharing method PEDS is cross-monotonic and fully shares the total cost incurred by consolidation, cost-sharing mechanism PEDS is truthful and budget-balanced.

We found that zero or total participation is guaranteed from cost-sharing mechanism PEDS for demand profiles that satisfy certain conditions. The conditions on the total effective demand for sharing $D'_N$ indicate that the value of $b_E$ does not affect whether total participation occurs when the consolidation center underestimates $b_G$ and that the consolidation center should set $b_E = b_C$ when it knows that any value of $b_E$ is an overestimation of $b_G$. When the consolidation center correctly estimates $b_G$, cost-sharing mechanism PEDS only yields zero or total participation for any given demand profile.

We examined the economic efficiency of cost-sharing mechanism PEDS using the social cost as the measure. We obtained the economically efficient shipping solutions using an optimization model that minimizes the social cost for all suppliers. We analytically showed that the outcomes of zero or total participation guaranteed by the cost-sharing mechanism PEDS for certain demand profiles are economically efficient. We studied the economic efficiency of the proposed mechanism for the demand profiles whose mechanism outcomes are unknown with numerical experiments. The results of the experiments demonstrated that cost-sharing mechanism PEDS rarely yields sub-optimal social cost outcomes for the unknown economic efficiency cases. Overall, the proposed cost-sharing mechanism PEDS not only possesses truthfulness and budget-balancedness, but also yields economically efficient solutions for the vast majority of demand profiles.

Our future research will focus on extending the problem to the case when the total demand of suppliers requires more than one truck. This single extension greatly complicates the problem by introducing nonconvex and nonconcave piecewise linear cost functions and a number of properties that we developed will not hold beyond the case of small suppliers. Moreover, in order to reveal insights for cost-sharing strategies for real market cooperation, we will also consider variations among the suppliers, which can be caused by different distances to the consolidation center and the destination, different negotiation power due to business sizes, etc.

Acknowledgments

The research reported in this paper was partially supported by the National Science Foundation under grant CMMI-1265616.
Appendix A:

**Proposition 2.** When \( b_E > b_G \), cost-sharing mechanism PEDS yields total participation if and only if \( D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}} \).

**Proof.** Based on the relationships among \( d_i \), \( b_G \) and \( b_E \), we categorize the suppliers into three groups: supplier \( i \) with \( d_i < b_G \), supplier \( i \) with \( b_G \leq d_i < b_E \) and supplier \( i \) with \( d_i \geq b_E \).

\[ \implies \text{Suppose we have total participation. Based on Observation 1, for supplier } i \text{ with } d_i < b_G, \text{ we have} \]

\[ d_i g_{L1} \geq d_i g_{L0} + d_i \frac{c_{F1}}{D'_N} \iff g_{L1} - g_{L0} \geq \frac{c_{F1}}{D'_N} \iff D'_N \geq \frac{c_{F1}}{g_{L1} - g_{L0}}. \]

For supplier \( i \) with \( b_G \leq d_i < b_E \), we have

\[ g_{F1} \geq g_{F0} + \frac{d_i}{D'_N} c_{F1} \iff b_G (g_{L1} - g_{L0}) \geq \frac{d_i}{D'_N} c_{F1} \iff D'_N \geq \frac{d_i}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}}. \]

In order to have all suppliers in this group participate in the consolidation we need \( D'_N \geq \frac{d'_*}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}} \) where \( d'_* = \max\{d_i \mid b_G \leq d_i < b_E\} \). For supplier \( i \) with \( d_i \geq b_E \), we have

\[ g_{F1} \geq g_{F0} + \frac{b_E}{D'_N} c_{F1} \iff b_G (g_{L1} - g_{L0}) \geq \frac{b_E}{D'_N} c_{F1} \iff D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}}. \]

From the three conditions above, if we have total participation, then \( D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}} \).

\[ \implies \text{Suppose we have } D'_N \geq \frac{b_E}{b_G} \frac{c_{F1}}{g_{L1} - g_{L0}} \text{. For supplier } i \text{ with } d_i < b_G, \text{ the cost share in the first iteration of the mechanism is} \]

\[ \chi(i, N) = d_i g_{L0} + \frac{d_i}{D'_N} c_{F1} \leq d_i g_{L0} + \frac{b_G}{b_E} d_i (g_{L1} - g_{L0}) = \frac{b_G}{b_E} d_i g_{L1} + (1 - \frac{b_G}{b_E}) d_i g_{L0} < \frac{b_G}{b_E} d_i g_{L1} + (1 - \frac{b_G}{b_E}) d_i g_{L0} = d_i g_{L1} = q_i. \]

For supplier \( i \) with \( b_G \leq d_i < b_E \), the cost share in the first iteration of the mechanism is

\[ \chi(i, N) = g_{F0} + \frac{d_i}{D'_N} c_{F1} \leq g_{F0} + \frac{b_G}{b_E} d_i (g_{L1} - g_{L0}) = \frac{d_i}{b_E} g_{F1} + (1 - \frac{d_i}{b_E}) g_{F0} < \frac{d_i}{b_E} g_{F1} + (1 - \frac{d_i}{b_E}) g_{F1} = g_{F1} = q_i. \]

For supplier \( i \) with \( d_i \geq b_E \), the cost share in the first iteration of the mechanism is

\[ \chi(i, N) = g_{F0} + \frac{b_E}{D'_N} c_{F1} \leq g_{F0} + b_G (g_{L1} - g_{L0}) = g_{F1} = q_i. \]

Therefore, by Observation 1, cost-sharing mechanism PEDS yields total participation. \(

**Proposition 3.** When \( b_E > b_G \), if \( D'_N < \frac{c_{F1}}{g_{L1} - g_{L0}} \), cost-sharing mechanism PEDS yields zero participation.

**Proof.** Similar to the proof of Proposition 2, we categorize the suppliers into three groups: supplier \( i \) with \( d_i < b_G \), supplier \( i \) with \( b_G \leq d_i < b_E \) and supplier \( i \) with \( d_i \geq b_E \).
If $D_N' < \frac{cF_1}{g_{L1} - g_{L0}}$, for supplier $i$ with $d_i < b_G$, the cost share in the first iteration of the mechanism is

$$
\chi(i, N) = d_i g_{L0} + \frac{d_i}{D_N'} cF_1 > d_i g_{L0} + d_i (g_{L1} - g_{L0}) = d_i g_{L1} = q_i.
$$

For supplier $i$ with $b_G \leq d_i < b_E$, the cost share in the first iteration of the mechanism is

$$
\chi(i, N) = g_{F0} + \frac{d_i}{D_N'} cF_1 > g_{F0} + d_i (g_{L1} - g_{L0}) \geq g_{F0} + b_G (g_{L1} - g_{L0}) = g_{F1} = q_i.
$$

For supplier $i$ with $d_i \geq b_E$, the cost share in the first iteration of the mechanism is

$$
\chi(i, N) = g_{F0} + \frac{b_E}{D_N'} cF_1 > g_{F0} + b_E (g_{L1} - g_{L0}) \geq g_{F0} + b_G (g_{L1} - g_{L0}) = g_{F1} = q_i.
$$

Therefore, by Lemma 1, cost-sharing mechanism PEDS yields zero participation. □

**Proposition 4.** When $b_E < b_G$, cost-sharing mechanism PEDS yields total participation if and only if $D_N' \geq \frac{cF_1}{g_{L1} - g_{L0}}$.

**Proof.** Based on the relationships among $d_i$, $b_G$ and $b_E$, we again categorize the suppliers into three groups: supplier $i$ with $d_i < b_E$, supplier $i$ with $b_E \leq d_i < b_G$ and supplier $i$ with $d_i \geq b_G$.

“⇒” Suppose we have total participation. Based on Observation 1, for supplier $i$ with $d_i < b_E$, we have

$$
d_i g_{L1} \geq d_i g_{L0} + \frac{d_i}{D_N'} cF_1 \iff g_{L1} - g_{L0} \geq \frac{cF_1}{D_N'} \iff D_N' \geq \frac{cF_1}{g_{L1} - g_{L0}}.
$$

For supplier $i$ with $b_E \leq d_i < b_G$, we have

$$
d_i g_{L1} \geq d_i g_{L0} + \frac{b_E}{D_N'} cF_1 \iff d_i (g_{L1} - g_{L0}) \geq \frac{b_E}{D_N'} cF_1 \iff D_N' \geq \frac{b_E}{d_i} \frac{cF_1}{g_{L1} - g_{L0}}.
$$

In order to have all suppliers in this group participate in the consolidation we need $D_N' \geq \frac{b_E}{d_i} \frac{cF_1}{g_{L1} - g_{L0}}$ where $d_i = \min\{d_i | b_E \leq d_i < b_G\}$. For supplier $i$ with $d_i \geq b_G$, we have

$$
g_{F1} \geq g_{F0} + \frac{b_E}{D_N'} cF_1 \iff g_{F1} (g_{L1} - g_{L0}) \geq \frac{b_E}{D_N'} cF_1 \iff D_N' \geq \frac{b_E}{b_G} \frac{cF_1}{g_{L1} - g_{L0}}.
$$

From the three conditions above, if we have total participation, then $D_N' \geq \frac{cF_1}{g_{L1} - g_{L0}}$.

“⇐” Suppose we have $D_N' \geq \frac{cF_1}{g_{L1} - g_{L0}}$. For supplier $i$ with $d_i < b_E$, the cost share in the first iteration of the mechanism is

$$
\chi(i, N) = d_i g_{L0} + \frac{d_i}{D_N'} cF_1 \leq d_i g_{L0} + d_i (g_{L1} - g_{L0}) = d_i g_{L1} = q_i.
$$

For supplier $i$ with $b_E \leq d_i < b_G$, the cost share in the first iteration of the mechanism is

$$
\chi(i, N) = g_{F0} + \frac{b_E}{D_N'} cF_1 \leq g_{F0} + b_E (g_{L1} - g_{L0}) = g_{F1} = q_i.
$$

For supplier $i$ with $d_i \geq b_G$, the cost share in the first iteration of the mechanism is

$$
\chi(i, N) = g_{F0} + \frac{b_E}{D_N'} cF_1 \leq g_{F0} + b_E (g_{L1} - g_{L0}) \leq g_{F0} + b_G (g_{L1} - g_{L0}) = g_{F1} = q_i.
$$

Therefore, by Observation 1, cost-sharing mechanism PEDS yields total participation. □
PROPOSITION 5. When \( \frac{g_{l1} - g_{l0}}{c_{l1}} b_{G} < b_{E} < b_{C} \), if \( D'_{N} < \frac{b_{E} - c_{F1}}{b_{G} g_{l1} - g_{l0}} \), cost-sharing mechanism PEDS yields zero participation.

Proof. Similar to the proof of Proposition 4, we categorize the suppliers into three groups: supplier \( i \) with \( d_{i} < b_{E} \), supplier \( i \) with \( b_{E} \leq d_{i} < b_{C} \) and supplier \( i \) with \( d_{i} \geq b_{C} \).

If \( D'_{N} < \frac{b_{E} - c_{F1}}{b_{G} g_{l1} - g_{l0}} \), for supplier \( i \) with \( d_{i} < b_{E} \), the cost share in the first iteration of the mechanism is

\[
\chi(i, N) = d_{i} g_{l0} + \frac{d_{i}}{D'_{N}} c_{F1} > d_{i} g_{l0} + \frac{b_{G}}{b_{E}} d_{i} (g_{l1} - g_{l0}) = \frac{b_{G}}{b_{E}} d_{i} g_{l1} - \left( \frac{b_{G}}{b_{E}} - 1 \right) d_{i} g_{l0} > \frac{b_{G}}{b_{E}} d_{i} g_{l1} - \left( \frac{b_{G}}{b_{E}} - 1 \right) d_{i} g_{l0} = d_{i} g_{l1} = q_{i}.
\]

For supplier \( i \) with \( b_{E} \leq d_{i} < b_{C} \), the cost share in the first iteration of the mechanism is

\[
\chi(i, N) = d_{i} g_{l0} + \frac{b_{E}}{D'_{N}} c_{F1} > d_{i} g_{l0} + b_{G} (g_{l1} - g_{l0}) = d_{i} g_{l1} - g_{l0} = q_{i}.
\]

For supplier \( i \) with \( d_{i} \geq b_{C} \), the cost share in the first iteration of the mechanism is

\[
\chi(i, N) = g_{F0} + \frac{b_{E}}{D'_{N}} c_{F1} > g_{F0} + b_{G} (g_{l1} - g_{l0}) = g_{F1} = q_{i}.
\]

Therefore, by Lemma 1, cost-sharing mechanism PEDS yields zero participation. \( \Box \)

Appendix B:

PROPOSITION 6. There exists an optimal solution to the model in which each supplier ships all its demand either to the consolidation center or directly to the destination.

Proof. We prove this proposition by contradiction.

First of all, we show that \( 0 < y_{CF} + y_{CL} < b_{C} \) is not optimal. Suppose we have an optimal solution with \( 0 < y_{CF} + y_{CL} < b_{C} \). WLOG, we assume that \( y_{CF} = 0 \). Therefore, the consolidation center incurs a cost of \( y_{CL} c_{l1} \). If \( b_{C} \geq b_{C} \), then \( y_{CL} < b_{C} \). The total shipping cost is \( y_{CL} c_{l1} \). However, shipping the same demand directly to the destination instead costs only \( y_{CL} g_{l1} < y_{CL} c_{l1} \). If \( b_{C} < b_{C} \), it is possible that \( b_{C} < y_{CL} < b_{C} \). In this case, the total shipping cost is \( y_{CL} c_{l1} + (ng_{F0} + \delta g_{l0}) \), where \( n \) denotes the number of suppliers who send their demand by FTL rate and \( \delta \) denotes the demand volume sent by LTL rate. The cost of shipping the same demand directly to the destination is \( ng_{F1} + \delta g_{l1} \). Since

\[
ng_{F1} + \delta g_{l1} - y_{CL} c_{l1} - (ng_{F0} + \delta g_{l0}) = (nb_{C} + \delta)(g_{l1} - g_{l0}) - y_{CL} c_{l1}
\]

\[
\leq (nb_{C} + \delta)(g_{l1} - g_{l0}) - (nb_{C} + \delta)c_{l1}
\]

\[
= (nb_{C} + \delta)(g_{l1} - g_{l0} - c_{l1}) < 0,
\]

shipping directly is cheaper than consolidating. The first inequality is valid because the actual demand sent via FTL rate by each of the \( n \) suppliers should be greater than or equal to \( b_{C} \). The validity of the second
inequality lies in the assumption that \( g_{L1} < c_{L1} + g_{L0} \). Therefore, \( 0 < y_{CF} + y_{CL} < b_C \) is not optimal. This also indicates that either \( y_{CF} + y_{CL} = 0 \) or \( y_{CF} + y_{CL} \geq b_C \) is true in an optimal solution.

Next, we assume supplier \( i \) ships \( d^c_i > 0 \) to the consolidation center and \( d^D_i > 0 \) to the destination in an optimal solution, i.e. \( d^c_i + d^D_i = d_i \). Since it is optimal to use either LTL or FTL rate for each supplier’s inbound shipping and direct shipping, there are only four possible shipping plans for supplier \( i \).

1. \( y^L_{i0} = d^c_i, y^F_{i1} = d^D_i \). If supplier \( i \) ships by this plan, then \( d^c_i < b_C \) and \( d^D_i \geq b_C \). Since \( d^D_i \geq b_C \), if we also ship \( d^c_i \) directly to the destination, no extra cost of direct shipping is incurred. Therefore, shipping \( d_i > b_C \) directly to the destination reduces the shipping cost of supplier \( i \) by \( g_{L0}d^c_i \). Now we examine whether the shipping cost of consolidation is increased by shipping \( d^c_i \) directly instead of consolidating it first. If \( (y_{CF} + y_{CL}) \geq b_C \) and \( (y_{CF} + y_{CL} - d^c_i) \geq b_C \), then the optimal shipping cost of the consolidation center remains \( c_{F1} \). If \( (y_{CF} + y_{CL}) \geq b_C \) and \( (y_{CF} + y_{CL} - d^c_i) < b_C \), the cost of shipping decreases from \( c_{F1} \) to \( (y_{CF} + y_{CL} - d^c_i) \cdot c_{L1} \leq c_{L1}b_C = c_{F1} \). Therefore, shipping \( d_i \) directly to the destination instead yields a decrease in the total cost by at least \( g_{L0}d^c_i \), and so this plan cannot be optimal.

2. \( y^F_{i0} = d^c_i, y^F_{i1} = d^D_i \) and \( 3. \) \( y^F_{i0} = d^c_i, y^F_{i1} = d^D_i \). If supplier \( i \) ships by either of these two plans, then \( d^c_i \geq b_C \). Since \( d^c_i \geq b_C \), if we also ship \( d^D_i \) to the consolidation center first, no extra cost of inbound shipping is incurred. Therefore, shipping \( d_i > b_C \) to the consolidation center reduces the shipping cost of supplier \( i \) by \( g_{L1}d^D_i \) in plan (2) or \( g_{F1} \) in plan (3). As for the shipping cost of the consolidation center, since \( (y_{CF} + y_{CL}) \geq b_C \), the optimal shipping cost of the consolidation center remains \( c_{F1} \) if \( d^D_i \) is shipped to the consolidation center. To summarize, shipping \( d_i \) to the consolidation center ensures a decrease in the total cost by \( g_{L1}d^D_i \) or \( g_{F1} \), and so these plans cannot be optimal.

4. \( y^L_{i0} = d^c_i \) and \( y^L_{i1} = d^D_i \). If supplier \( i \) ships by this plan, then \( d^c_i < b_C \) and \( d^D_i < b_C \). Because \( y^L_{i0} = d^c_i > 0 \), we must have \( y_{CF} + y_{CL} \geq b_C \). Therefore, increasing the shipment volume of the consolidation center does not incur any extra cost. Now if we ship \( d^D_i \) to the consolidation center as well, the total cost is decreased by \( (g_{L1} - g_{L0})d^D_i \) if \( d^D_i < b_C \) or \( (g_{F1} - g_{F0})d^D_i \) if \( d^D_i \geq b_C \). As a consequence, shipping \( d_i \) to the consolidation center ensures a decrease in the total cost by \( (g_{L1} - g_{L0})d^D_i \) or \( (g_{F1} - g_{F0})d^D_i \), and so this plan cannot be optimal.

In the analysis above, we show that shipping \( d_i \) entirely either to the consolidation center or directly to the destination yields a smaller total cost than shipping \( d^c_i > 0 \) directly to the consolidation center and \( d^D_i > 0 \) to the destination. This contradicts the assumption that shipping \( d^c_i \) to the consolidation center and \( d^D_i \) to the destination is optimal. Therefore, there exists an optimal solution, in which each supplier ships all its demand either to the consolidation center or to the destination. □

**Corollary 2.** Every optimal solution \((\hat{x}, \hat{y})\) to the model satisfies: \( \hat{x}^F_{i0} + \hat{x}^i_{L0} + \hat{x}^F_{i1} + \hat{x}^i_{L1} = 1 \). In other words, in every optimal solution to the model, each supplier’s entire demand is shipped either to the consolidation center or directly to the destination.

**Proof.** In the proof of Proposition 6, we showed that by shipping each supplier’s entire demand to the consolidation center or directly to the destination, we are able to reduce the total cost at least by \( g_{L0}d^c_i \) in shipping plan (1), \( g_{L1}d^D_i \) in shipping plan (2), \( g_{F1} \) in shipping plan (3), and \( (g_{L1} - g_{L0})d^D_i \) or \( (g_{F1} - g_{F0})d^D_i \) in shipping plan (4). With shipping rates and demand being non-zero, \( g_{L1} > g_{L0} \) and \( g_{F1} > g_{F0} \), these cost
reductions are strictly positive. Consequently, the total cost of shipping each supplier’s entire demand either
to the consolidation center or directly to the destination is strictly less than shipping some of a supplier’s
demand to the consolidation center and the rest directly to the destination. Therefore, in every optimal
solution to the model, each supplier’s entire demand is shipped either to the consolidation center or directly
to the destination. Combined with the result that it is optimal for suppliers to ship either by the FTL rate
or the LTL rate, we can conclude that
\[ \tilde{x}^i_{F0} + \tilde{x}^i_{L0} + \tilde{x}^i_{F1} + \tilde{x}^i_{L1} = 1. \]

**Proposition 7.** Every optimal solution to the model yields either zero participation or total participation.

A solution in which a subset of suppliers \( S \subset N, S \neq \emptyset \) ships their demand to the consolidation center first
while the rest of the suppliers ship their demand directly to the destination is not optimal.

**Proof.** We prove this proposition by contradiction.

Based on Corollary 2, we assume that in an optimal solution, a subset of suppliers \( S \subset N \) ships their
demand to the consolidation center first and the rest of the suppliers ship their demand directly to the
destination. Suppose supplier \( i \in N \setminus S \) is one of the suppliers who ships its demand directly to the destination.

If \( y_{CF} + y_{CL} = 0 \) in the optimal solution, the optimal solution of the model is zero participation.

If \( y_{CF} + y_{CL} \geq b_C \) in the optimal solution, the optimal shipping method for the consolidation center is
FTL which costs \( c_{F1} \). If supplier \( i \) ships its demand to the consolidation center first, then its shipping cost
reduces from \( g_{L1}d_i \) to \( g_{L0}d_i \) or from \( g_{F1} \) to \( g_{F0} \). Also, adding \( d_i \) to the outbound shipping does not incur any
extra cost. Therefore, the total cost decreases by \( (g_{L1} - g_{L0})d_i \) or \( (g_{F1} - g_{F0}) \) if supplier \( i \) ships its demand
to the consolidation center first.

Both results contradict the assumption that a solution in which a subset of suppliers \( S \subset N, S \neq \emptyset \) ships
their demand to the consolidation center first while the rest of the suppliers ship their demand directly to the
destination is optimal. As a consequence, every optimal solution must yield either zero participation or
total participation.

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