ISyE 8872  Topics in Nonlinear Optimization

Fall 2001

Assignment 7

Issued: November 15, 2001

Due: November 27, 2001

Problem 1
Let $f_1, f_2 \in \text{Conv} \mathbb{R}^n$ be such that $\text{dom} (f_1) \cap \text{dom} (f_2) \neq \emptyset$.

1. Show that $\text{cl} (f_1) + \text{cl} (f_2) \leq \text{cl} (f_1 + f_2)$.

2. Show that if $\text{ri} (\text{dom} (f_1)) \cap \text{ri} (\text{dom} (f_2)) \neq \emptyset$ then $\text{cl} (f_1) + \text{cl} (f_2) = \text{cl} (f_1 + f_2)$.

3. Give an example to show that the equality in (2) fails if $\text{ri} (\text{dom} (f_1)) \cap \text{ri} (\text{dom} (f_2)) = \emptyset$.

4. State without proof analogous results for (1) and (2) in the case where $m$ functions $f_1, \ldots, f_m \in \text{Conv} \mathbb{R}^n$ are given.

Problem 2
Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$. Show that:

1. if $\text{dom} (f)$ is closed and $f$ restricted to $\text{dom} (f)$ is lower semi-continuous then $f$ is lower semi-continuous;

2. if $f \in \text{Conv} \mathbb{R}^n$ and $\text{dom} (f)$ is an affine manifold then $f \in \text{Conv} \mathbb{R}^n$.

Problem 3
Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{\pm \infty\}$. Show that:

1. for all $x \in \mathbb{R}^n$, $\text{cl} (f)(x) = \sup \{g(x) : g \in \mathcal{C}\}$ where $\mathcal{C}$ is the collection of all lower semi-continuous functions $g : \mathbb{R}^n \to \mathbb{R} \cup \{\pm \infty\}$ satisfying $g \leq f$;

2. $\text{cl} (f)$ is the largest function in $\mathcal{C}$, that is, $\text{cl} (f) \in \mathcal{C}$ and $\text{cl} (f) \geq g$ for all $g \in \mathcal{C}$. 