Problem 1
Consider closed sets $F, G \subset \mathbb{R}^m$, and suppose that $F \cap G = \emptyset$. Show that there exist open sets $A, B \subset \mathbb{R}^m$ such that $F \subset A$, $G \subset B$, and $A \cap B = \emptyset$.

Problem 2
Let $X \subset \mathbb{R}^m$, $Y \subset \mathbb{R}^k$, and $F : X \mapsto 2^Y$.

1. Show that if $F$ is open, then $F$ is lower hemi-continuous.

2. The following statement comes from Proposition 11.9(d) in Border (1985): “If $F$ is upper hemi-continuous at $x$ and $F(x)$ is a singleton, then $F$ is lower hemi-continuous at $x$.”
   (a) Give a counterexample to the statement above.
   (b) Correct the statement, and prove the correctness of your statement.

3. Show that if $F^{-1}(\{y\})$ is open for all $y \in Y$, then $F$ is lower hemi-continuous.

Problem 3
Let $X \subset \mathbb{R}^m$, $Y \subset \mathbb{R}^k$, and $F : X \mapsto 2^Y$. Show that $F$ is lower hemi-continuous at $x$ if and only if for all sequences $\{x_k\} \subset X$ such that $x_k \to x$ as $k \to \infty$, and all $y \in F(x)$, there is a sequence $\{y_k\} \subset Y$ such that $y_k \in F(x_k)$ for all $k$ and $y_k \to y$ as $k \to \infty$. 