ISyE 4803

Advanced Supply Chain Logistics

Fall 2009

Homework 3 solution

This homework is worth 50 points.

Problem 1

The following network represents the network of canals in a famous city (Amsterdam? Venice? Fort Lauderdale?). These canals have to be cleaned regularly. Canals are cleaned by a specially equipped boat that cruises along the canals, dredging the bottoms of the canals and scooping up any floating debris in the water. The boat can cruise along the canals in any direction. The cost of operating the cleaning service (including the cost of the boat, fuel and crew) is represented by costs on the edges of the network. Design a cleaning tour for the boat that will traverse each canal at least once, that will return the boat to its origin (boathouse), and that will incur the minimum total cost. Write a complete LP formulation, explicitly with all applicable data values, for the shortest path problem from all odd degree nodes to the largest indexed node with odd degree. Also write a complete LP formulation, explicitly with all applicable data values, of the matching problem. (Hint: Use an LP solver of your choice, such as LINDO, GAMS, CPlex, XPRESS-MP, AIMMS, AMPL, to solve the optimization problems encountered along the way.) (25)

Answer: Let $N = \{1, 2, \ldots, 16\}$ denote the set of nodes. Let $E = \{\{1,2\}, \{2,3\}, \{3,4\}, \{1,5\}, \{2,6\}, \{3,7\}, \{4,8\}, \{5,6\}, \{6,7\}, \{7,8\}, \{5,9\}, \{6,10\}, \{7,11\}, \{8,12\}, \{9,10\}, \{10,11\}, \{11,12\}, \{9,13\}, \{10,14\}, \{11,15\}, \{12,16\}, \{13,14\}, \{14,15\}, \{15,16\}\}$ denote the set of edges. Let $G = (N, E)$ denote the given undirected network.
Let c_{ij} denote the cost associated with edge $\{i, j\}$.
The set of odd degree nodes is $O = \{2, 3, 5, 8, 9, 12, 14, 15\}$. The first step is to compute the shortest path and shortest path length between each pair of odd degree nodes. Since the network is undirected, the shortest path and shortest path length from i to j are the same as the shortest path and shortest path length from j to i. Thus it is only necessary to compute the shortest path and shortest path length from i to j with $i < j$ and $i, j \in O$. One way to do that is to formulate the following multicommodity network flow problem. For each edge $\{i, j\} \in E$, create two arcs (i, j) and (j, i), each with cost equal to c_{ij}. Let A denote the set of arcs. For each odd degree node (except the one with smallest index), create a commodity — the commodity index is the same as the index of the destination node of the shortest path. Thus the set of commodities is $K = \{3, 5, 8, 9, 12, 14, 15\}$. Then the demand for commodity k at node $j \in O$ is $d_{jk} = -1$ (a supply of 1) if $j < k$, $d_{jk} = |\{i \in O : i < j\}|$ if $j = k$, and $d_{jk} = 0$ if $j \notin O$ or $j > k$.
Decision variables:
Let x_{ijk} denote the amount of flow of commodity k on arc (i, j). The optimal solution x^*_{ijk} will give the shortest path tree from all nodes to node k, and the shortest path lengths can easily be calculated based on that.
Then the formulation of the problem is as follows:

$$\begin{array}{lrcll} \min & \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} x_{ijk} & & & \\ \text{s.t.} & & & & \\ & \sum_{\{i\in N : (i,j)\in A\}} x_{ijk} - \sum_{\{i\in N : (j,i)\in A\}} x_{jik} & = & d_{jk} & \forall\, k \in K, \ \ \forall\, j \in N \\ & x_{ijk} & \geq & 0 & \forall\, (i,j) \in A, \ \ \forall\, k \in K \end{array}$$

With the given data, the formulation is

$$\min \quad \{46x_{1,2,3} + 46x_{1,2,5} + 46x_{1,2,8} + 46x_{1,2,9} + 46x_{1,2,12} + 46x_{1,2,14} + 46x_{1,2,15}$$

$$
\begin{aligned}
&+46x_{2,1,3} + 46x_{2,1,5} + 46x_{2,1,8} + 46x_{2,1,9} + 46x_{2,1,12} + 46x_{2,1,14} + 46x_{2,1,15} \\
&+42x_{2,3,3} + 42x_{2,3,5} + 42x_{2,3,8} + 42x_{2,3,9} + 42x_{2,3,12} + 42x_{2,3,14} + 42x_{2,3,15} \\
&+42x_{3,2,3} + 42x_{3,2,5} + 42x_{3,2,8} + 42x_{3,2,9} + 42x_{3,2,12} + 42x_{3,2,14} + 42x_{3,2,15} \\
&+22x_{3,4,3} + 22x_{3,4,5} + 22x_{3,4,8} + 22x_{3,4,9} + 22x_{3,4,12} + 22x_{3,4,14} + 22x_{3,4,15} \\
&+22x_{4,3,3} + 22x_{4,3,5} + 22x_{4,3,8} + 22x_{4,3,9} + 22x_{4,3,12} + 22x_{4,3,14} + 22x_{4,3,15} \\
&+24x_{1,5,3} + 24x_{1,5,5} + 24x_{1,5,8} + 24x_{1,5,9} + 24x_{1,5,12} + 24x_{1,5,14} + 24x_{1,5,15} \\
&+24x_{5,1,3} + 24x_{5,1,5} + 24x_{5,1,8} + 24x_{5,1,9} + 24x_{5,1,12} + 24x_{5,1,14} + 24x_{5,1,15} \\
&+17x_{2,6,3} + 17x_{2,6,5} + 17x_{2,6,8} + 17x_{2,6,9} + 17x_{2,6,12} + 17x_{2,6,14} + 17x_{2,6,15} \\
&+17x_{6,2,3} + 17x_{6,2,5} + 17x_{6,2,8} + 17x_{6,2,9} + 17x_{6,2,12} + 17x_{6,2,14} + 17x_{6,2,15} \\
&+18x_{3,7,3} + 18x_{3,7,5} + 18x_{3,7,8} + 18x_{3,7,9} + 18x_{3,7,12} + 18x_{3,7,14} + 18x_{3,7,15} \\
&+18x_{7,3,3} + 18x_{7,3,5} + 18x_{7,3,8} + 18x_{7,3,9} + 18x_{7,3,12} + 18x_{7,3,14} + 18x_{7,3,15} \\
&+32x_{4,8,3} + 32x_{4,8,5} + 32x_{4,8,8} + 32x_{4,8,9} + 32x_{4,8,12} + 32x_{4,8,14} + 32x_{4,8,15} \\
&+32x_{8,4,3} + 32x_{8,4,5} + 32x_{8,4,8} + 32x_{8,4,9} + 32x_{8,4,12} + 32x_{8,4,14} + 32x_{8,4,15} \\
&+36x_{5,6,3} + 36x_{5,6,5} + 36x_{5,6,8} + 36x_{5,6,9} + 36x_{5,6,12} + 36x_{5,6,14} + 36x_{5,6,15} \\
&+36x_{6,5,3} + 36x_{6,5,5} + 36x_{6,5,8} + 36x_{6,5,9} + 36x_{6,5,12} + 36x_{6,5,14} + 36x_{6,5,15} \\
&+12x_{6,7,3} + 12x_{6,7,5} + 12x_{6,7,8} + 12x_{6,7,9} + 12x_{6,7,12} + 12x_{6,7,14} + 12x_{6,7,15} \\
&+12x_{7,6,3} + 12x_{7,6,5} + 12x_{7,6,8} + 12x_{7,6,9} + 12x_{7,6,12} + 12x_{7,6,14} + 12x_{7,6,15} \\
&+13x_{7,8,3} + 13x_{7,8,5} + 13x_{7,8,8} + 13x_{7,8,9} + 13x_{7,8,12} + 13x_{7,8,14} + 13x_{7,8,15} \\
&+13x_{8,7,3} + 13x_{8,7,5} + 13x_{8,7,8} + 13x_{8,7,9} + 13x_{8,7,12} + 13x_{8,7,14} + 13x_{8,7,15} \\
&+21x_{5,9,3} + 21x_{5,9,5} + 21x_{5,9,8} + 21x_{5,9,9} + 21x_{5,9,12} + 21x_{5,9,14} + 21x_{5,9,15} \\
&+21x_{9,5,3} + 21x_{9,5,5} + 21x_{9,5,8} + 21x_{9,5,9} + 21x_{9,5,12} + 21x_{9,5,14} + 21x_{9,5,15} \\
&+19x_{6,10,3} + 19x_{6,10,5} + 19x_{6,10,8} + 19x_{6,10,9} + 19x_{6,10,12} + 19x_{6,10,14} + 19x_{6,10,15} \\
&+19x_{10,6,3} + 19x_{10,6,5} + 19x_{10,6,8} + 19x_{10,6,9} + 19x_{10,6,12} + 19x_{10,6,14} + 19x_{10,6,15} \\
&+28x_{7,11,3} + 28x_{7,11,5} + 28x_{7,11,8} + 28x_{7,11,9} + 28x_{7,11,12} + 28x_{7,11,14} + 28x_{7,11,15} \\
&+28x_{11,7,3} + 28x_{11,7,5} + 28x_{11,7,8} + 28x_{11,7,9} + 28x_{11,7,12} + 28x_{11,7,14} + 28x_{11,7,15} \\
&+40x_{8,12,3} + 40x_{8,12,5} + 40x_{8,12,8} + 40x_{8,12,9} + 40x_{8,12,12} + 40x_{8,12,14} + 40x_{8,12,15} \\
&+40x_{12,8,3} + 40x_{12,8,5} + 40x_{12,8,8} + 40x_{12,8,9} + 40x_{12,8,12} + 40x_{12,8,14} + 40x_{12,8,15} \\
&+38x_{9,10,3} + 38x_{9,10,5} + 38x_{9,10,8} + 38x_{9,10,9} + 38x_{9,10,12} + 38x_{9,10,14} + 38x_{9,10,15} \\
&+38x_{10,9,3} + 38x_{10,9,5} + 38x_{10,9,8} + 38x_{10,9,9} + 38x_{10,9,12} + 38x_{10,9,14} + 38x_{10,9,15} \\
&+25x_{10,11,3} + 25x_{10,11,5} + 25x_{10,11,8} + 25x_{10,11,9} + 25x_{10,11,12} + 25x_{10,11,14} + 25x_{10,11,15} \\
&+25x_{11,10,3} + 25x_{11,10,5} + 25x_{11,10,8} + 25x_{11,10,9} + 25x_{11,10,12} + 25x_{11,10,14} + 25x_{11,10,15} \\
&+20x_{11,12,3} + 20x_{11,12,5} + 20x_{11,12,8} + 20x_{11,12,9} + 20x_{11,12,12} + 20x_{11,12,14} + 20x_{11,12,15} \\
&+20x_{12,11,3} + 20x_{12,11,5} + 20x_{12,11,8} + 20x_{12,11,9} + 20x_{12,11,12} + 20x_{12,11,14} + 20x_{12,11,15} \\
&+34x_{9,13,3} + 34x_{9,13,5} + 34x_{9,13,8} + 34x_{9,13,9} + 34x_{9,13,12} + 34x_{9,13,14} + 34x_{9,13,15} \\
&+34x_{13,9,3} + 34x_{13,9,5} + 34x_{13,9,8} + 34x_{13,9,9} + 34x_{13,9,12} + 34x_{13,9,14} + 34x_{13,9,15} \\
&+14x_{10,14,3} + 14x_{10,14,5} + 14x_{10,14,8} + 14x_{10,14,9} + 14x_{10,14,12} + 14x_{10,14,14} + 14x_{10,14,15} \\
&+14x_{14,10,3} + 14x_{14,10,5} + 14x_{14,10,8} + 14x_{14,10,9} + 14x_{14,10,12} + 14x_{14,10,14} + 14x_{14,10,15} \\
&+15x_{11,15,3} + 15x_{11,15,5} + 15x_{11,15,8} + 15x_{11,15,9} + 15x_{11,15,12} + 15x_{11,15,14} + 15x_{11,15,15} \\
&+15x_{15,11,3} + 15x_{15,11,5} + 15x_{15,11,8} + 15x_{15,11,9} + 15x_{15,11,12} + 15x_{15,11,14} + 15x_{15,11,15} \\
&+30x_{12,16,3} + 30x_{12,16,5} + 30x_{12,16,8} + 30x_{12,16,9} + 30x_{12,16,12} + 30x_{12,16,14} + 30x_{12,16,15} \\
&+30x_{16,12,3} + 30x_{16,12,5} + 30x_{16,12,8} + 30x_{16,12,9} + 30x_{16,12,12} + 30x_{16,12,14} + 30x_{16,12,15}
\end{aligned}
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\begin{aligned}
&+26x_{13,14,3} + 26x_{13,14,5} + 26x_{13,14,8} + 26x_{13,14,9} + 26x_{13,14,12} + 26x_{13,14,14} + 26x_{13,14,15} \\
&+26x_{14,13,3} + 26x_{14,13,5} + 26x_{14,13,8} + 26x_{14,13,9} + 26x_{14,13,12} + 26x_{14,13,14} + 26x_{14,13,15} \\
&+37x_{14,15,3} + 37x_{14,15,5} + 37x_{14,15,8} + 37x_{14,15,9} + 37x_{14,15,12} + 37x_{14,15,14} + 37x_{14,15,15} \\
&+37x_{15,14,3} + 37x_{15,14,5} + 37x_{15,14,8} + 37x_{15,14,9} + 37x_{15,14,12} + 37x_{15,14,14} + 37x_{15,14,15} \\
&+23x_{15,16,3} + 23x_{15,16,5} + 23x_{15,16,8} + 23x_{15,16,9} + 23x_{15,16,12} + 23x_{15,16,14} + 23x_{15,16,15} \\
&+23x_{16,15,3} + 23x_{16,15,5} + 23x_{16,15,8} + 23x_{16,15,9} + 23x_{16,15,12} + 23x_{16,15,14} + 23x_{16,15,15}\}
\end{aligned}
$$

s.t.

$$
\begin{aligned}
x_{2,1,3} + x_{5,1,3} - x_{1,2,3} - x_{1,5,3} &= 0 \\
x_{2,1,5} + x_{5,1,5} - x_{1,2,5} - x_{1,5,5} &= 0 \\
x_{2,1,8} + x_{5,1,8} - x_{1,2,8} - x_{1,5,8} &= 0 \\
x_{2,1,9} + x_{5,1,9} - x_{1,2,9} - x_{1,5,9} &= 0 \\
x_{2,1,12} + x_{5,1,12} - x_{1,2,12} - x_{1,5,12} &= 0 \\
x_{2,1,14} + x_{5,1,14} - x_{1,2,14} - x_{1,5,14} &= 0 \\
x_{2,1,15} + x_{5,1,15} - x_{1,2,15} - x_{1,5,15} &= 0 \\
x_{1,2,3} + x_{3,2,3} + x_{6,2,3} - x_{2,1,3} - x_{2,3,3} - x_{2,6,3} &= -1 \\
x_{1,2,5} + x_{3,2,5} + x_{6,2,5} - x_{2,1,5} - x_{2,3,5} - x_{2,6,5} &= -1 \\
x_{1,2,8} + x_{3,2,8} + x_{6,2,8} - x_{2,1,8} - x_{2,3,8} - x_{2,6,8} &= -1 \\
x_{1,2,9} + x_{3,2,9} + x_{6,2,9} - x_{2,1,9} - x_{2,3,9} - x_{2,6,9} &= -1 \\
x_{1,2,12} + x_{3,2,12} + x_{6,2,12} - x_{2,1,12} - x_{2,3,12} - x_{2,6,12} &= -1 \\
x_{1,2,14} + x_{3,2,14} + x_{6,2,14} - x_{2,1,14} - x_{2,3,14} - x_{2,6,14} &= -1 \\
x_{1,2,15} + x_{3,2,15} + x_{6,2,15} - x_{2,1,15} - x_{2,3,15} - x_{2,6,15} &= -1 \\
x_{2,3,3} + x_{4,3,3} + x_{7,3,3} - x_{3,2,3} - x_{3,4,3} - x_{3,7,3} &= 1 \\
x_{2,3,5} + x_{4,3,5} + x_{7,3,5} - x_{3,2,5} - x_{3,4,5} - x_{3,7,5} &= -1 \\
x_{2,3,8} + x_{4,3,8} + x_{7,3,8} - x_{3,2,8} - x_{3,4,8} - x_{3,7,8} &= -1 \\
x_{2,3,9} + x_{4,3,9} + x_{7,3,9} - x_{3,2,9} - x_{3,4,9} - x_{3,7,9} &= -1 \\
x_{2,3,12} + x_{4,3,12} + x_{7,3,12} - x_{3,2,12} - x_{3,4,12} - x_{3,7,12} &= -1 \\
x_{2,3,14} + x_{4,3,14} + x_{7,3,14} - x_{3,2,14} - x_{3,4,14} - x_{3,7,14} &= -1 \\
x_{2,3,15} + x_{4,3,15} + x_{7,3,15} - x_{3,2,15} - x_{3,4,15} - x_{3,7,15} &= -1 \\
x_{3,4,3} + x_{8,4,3} - x_{4,3,3} - x_{4,8,3} &= 0 \\
x_{3,4,5} + x_{8,4,5} - x_{4,3,5} - x_{4,8,5} &= 0 \\
x_{3,4,8} + x_{8,4,8} - x_{4,3,8} - x_{4,8,8} &= 0 \\
x_{3,4,9} + x_{8,4,9} - x_{4,3,9} - x_{4,8,9} &= 0 \\
x_{3,4,12} + x_{8,4,12} - x_{4,3,12} - x_{4,8,12} &= 0 \\
x_{3,4,14} + x_{8,4,14} - x_{4,3,14} - x_{4,8,14} &= 0 \\
x_{3,4,15} + x_{8,4,15} - x_{4,3,15} - x_{4,8,15} &= 0 \\
x_{1,5,3} + x_{6,5,3} + x_{9,5,3} - x_{5,1,3} - x_{5,6,3} - x_{5,9,3} &= 0 \\
x_{1,5,5} + x_{6,5,5} + x_{9,5,5} - x_{5,1,5} - x_{5,6,5} - x_{5,9,5} &= 2 \\
x_{1,5,8} + x_{6,5,8} + x_{9,5,8} - x_{5,1,8} - x_{5,6,8} - x_{5,9,8} &= -1 \\
x_{1,5,9} + x_{6,5,9} + x_{9,5,9} - x_{5,1,9} - x_{5,6,9} - x_{5,9,9} &= -1 \\
x_{1,5,12} + x_{6,5,12} + x_{9,5,12} - x_{5,1,12} - x_{5,6,12} - x_{5,9,12} &= -1 \\
x_{1,5,14} + x_{6,5,14} + x_{9,5,14} - x_{5,1,14} - x_{5,6,14} - x_{5,9,14} &= -1 \\
x_{1,5,15} + x_{6,5,15} + x_{9,5,15} - x_{5,1,15} - x_{5,6,15} - x_{5,9,15} &= -1
\end{aligned}
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\begin{aligned}
x_{2,6,3} + x_{5,6,3} + x_{7,6,3} + x_{10,6,3} - x_{6,2,3} - x_{6,5,3} - x_{6,7,3} - x_{6,10,3} &= 0 \\
x_{2,6,5} + x_{5,6,5} + x_{7,6,5} + x_{10,6,5} - x_{6,2,5} - x_{6,5,5} - x_{6,7,5} - x_{6,10,5} &= 0 \\
x_{2,6,8} + x_{5,6,8} + x_{7,6,8} + x_{10,6,8} - x_{6,2,8} - x_{6,5,8} - x_{6,7,8} - x_{6,10,8} &= 0 \\
x_{2,6,9} + x_{5,6,9} + x_{7,6,9} + x_{10,6,9} - x_{6,2,9} - x_{6,5,9} - x_{6,7,9} - x_{6,10,9} &= 0 \\
x_{2,6,12} + x_{5,6,12} + x_{7,6,12} + x_{10,6,12} - x_{6,2,12} - x_{6,5,12} - x_{6,7,12} - x_{6,10,12} &= 0 \\
x_{2,6,14} + x_{5,6,14} + x_{7,6,14} + x_{10,6,14} - x_{6,2,14} - x_{6,5,14} - x_{6,7,14} - x_{6,10,14} &= 0 \\
x_{2,6,15} + x_{5,6,15} + x_{7,6,15} + x_{10,6,15} - x_{6,2,15} - x_{6,5,15} - x_{6,7,15} - x_{6,10,15} &= 0 \\
x_{3,7,3} + x_{6,7,3} + x_{8,7,3} + x_{11,7,3} - x_{7,3,3} - x_{7,6,3} - x_{7,8,3} - x_{7,11,3} &= 0 \\
x_{3,7,5} + x_{6,7,5} + x_{8,7,5} + x_{11,7,5} - x_{7,3,5} - x_{7,6,5} - x_{7,8,5} - x_{7,11,5} &= 0 \\
x_{3,7,8} + x_{6,7,8} + x_{8,7,8} + x_{11,7,8} - x_{7,3,8} - x_{7,6,8} - x_{7,8,8} - x_{7,11,8} &= 0 \\
x_{3,7,9} + x_{6,7,9} + x_{8,7,9} + x_{11,7,9} - x_{7,3,9} - x_{7,6,9} - x_{7,8,9} - x_{7,11,9} &= 0 \\
x_{3,7,12} + x_{6,7,12} + x_{8,7,12} + x_{11,7,12} - x_{7,3,12} - x_{7,6,12} - x_{7,8,12} - x_{7,11,12} &= 0 \\
x_{3,7,14} + x_{6,7,14} + x_{8,7,14} + x_{11,7,14} - x_{7,3,14} - x_{7,6,14} - x_{7,8,14} - x_{7,11,14} &= 0 \\
x_{3,7,15} + x_{6,7,15} + x_{8,7,15} + x_{11,7,15} - x_{7,3,15} - x_{7,6,15} - x_{7,8,15} - x_{7,11,15} &= 0 \\
x_{4,8,3} + x_{7,8,3} + x_{12,8,3} - x_{8,4,3} - x_{8,7,3} - x_{8,12,3} &= 0 \\
x_{4,8,5} + x_{7,8,5} + x_{12,8,5} - x_{8,4,5} - x_{8,7,5} - x_{8,12,5} &= 0 \\
x_{4,8,8} + x_{7,8,8} + x_{12,8,8} - x_{8,4,8} - x_{8,7,8} - x_{8,12,8} &= 3 \\
x_{4,8,9} + x_{7,8,9} + x_{12,8,9} - x_{8,4,9} - x_{8,7,9} - x_{8,12,9} &= -1 \\
x_{4,8,12} + x_{7,8,12} + x_{12,8,12} - x_{8,4,12} - x_{8,7,12} - x_{8,12,12} &= -1 \\
x_{4,8,14} + x_{7,8,14} + x_{12,8,14} - x_{8,4,14} - x_{8,7,14} - x_{8,12,14} &= -1 \\
x_{4,8,15} + x_{7,8,15} + x_{12,8,15} - x_{8,4,15} - x_{8,7,15} - x_{8,12,15} &= -1 \\
x_{5,9,3} + x_{10,9,3} + x_{13,9,3} - x_{9,5,3} - x_{9,10,3} - x_{9,13,3} &= 0 \\
x_{5,9,5} + x_{10,9,5} + x_{13,9,5} - x_{9,5,5} - x_{9,10,5} - x_{9,13,5} &= 0 \\
x_{5,9,8} + x_{10,9,8} + x_{13,9,8} - x_{9,5,8} - x_{9,10,8} - x_{9,13,8} &= 0 \\
x_{5,9,9} + x_{10,9,9} + x_{13,9,9} - x_{9,5,9} - x_{9,10,9} - x_{9,13,9} &= 4 \\
x_{5,9,12} + x_{10,9,12} + x_{13,9,12} - x_{9,5,12} - x_{9,10,12} - x_{9,13,12} &= -1 \\
x_{5,9,14} + x_{10,9,14} + x_{13,9,14} - x_{9,5,14} - x_{9,10,14} - x_{9,13,14} &= -1 \\
x_{5,9,15} + x_{10,9,15} + x_{13,9,15} - x_{9,5,15} - x_{9,10,15} - x_{9,13,15} &= -1 \\
x_{6,10,3} + x_{9,10,3} + x_{11,10,3} + x_{14,10,3} - x_{10,6,3} - x_{10,9,3} - x_{10,11,3} - x_{10,14,3} &= 0 \\
x_{6,10,5} + x_{9,10,5} + x_{11,10,5} + x_{14,10,5} - x_{10,6,5} - x_{10,9,5} - x_{10,11,5} - x_{10,14,5} &= 0 \\
x_{6,10,8} + x_{9,10,8} + x_{11,10,8} + x_{14,10,8} - x_{10,6,8} - x_{10,9,8} - x_{10,11,8} - x_{10,14,8} &= 0 \\
x_{6,10,9} + x_{9,10,9} + x_{11,10,9} + x_{14,10,9} - x_{10,6,9} - x_{10,9,9} - x_{10,11,9} - x_{10,14,9} &= 0 \\
x_{6,10,12} + x_{9,10,12} + x_{11,10,12} + x_{14,10,12} - x_{10,6,12} - x_{10,9,12} - x_{10,11,12} - x_{10,14,12} &= 0 \\
x_{6,10,14} + x_{9,10,14} + x_{11,10,14} + x_{14,10,14} - x_{10,6,14} - x_{10,9,14} - x_{10,11,14} - x_{10,14,14} &= 0 \\
x_{6,10,15} + x_{9,10,15} + x_{11,10,15} + x_{14,10,15} - x_{10,6,15} - x_{10,9,15} - x_{10,11,15} - x_{10,14,15} &= 0 \\
x_{7,11,3} + x_{10,11,3} + x_{12,11,3} + x_{15,11,3} - x_{11,7,3} - x_{11,10,3} - x_{11,12,3} - x_{11,15,3} &= 0 \\
x_{7,11,5} + x_{10,11,5} + x_{12,11,5} + x_{15,11,5} - x_{11,7,5} - x_{11,10,5} - x_{11,12,5} - x_{11,15,5} &= 0 \\
x_{7,11,8} + x_{10,11,8} + x_{12,11,8} + x_{15,11,8} - x_{11,7,8} - x_{11,10,8} - x_{11,12,8} - x_{11,15,8} &= 0 \\
x_{7,11,9} + x_{10,11,9} + x_{12,11,9} + x_{15,11,9} - x_{11,7,9} - x_{11,10,9} - x_{11,12,9} - x_{11,15,9} &= 0 \\
x_{7,11,12} + x_{10,11,12} + x_{12,11,12} + x_{15,11,12} - x_{11,7,12} - x_{11,10,12} - x_{11,12,12} - x_{11,15,12} &= 0 \\
x_{7,11,14} + x_{10,11,14} + x_{12,11,14} + x_{15,11,14} - x_{11,7,14} - x_{11,10,14} - x_{11,12,14} - x_{11,15,14} &= 0
\end{aligned}
$$

$$
\begin{aligned}
x_{7,11,15} + x_{10,11,15} + x_{12,11,15} + x_{15,11,15} - x_{11,7,15} - x_{11,10,15} - x_{11,12,15} - x_{11,15,15} &= 0 \\
x_{8,12,3} + x_{11,12,3} + x_{16,12,3} - x_{12,8,3} - x_{12,11,3} - x_{12,16,3} &= 0 \\
x_{8,12,5} + x_{11,12,5} + x_{16,12,5} - x_{12,8,5} - x_{12,11,5} - x_{12,16,5} &= 0 \\
x_{8,12,8} + x_{11,12,8} + x_{16,12,8} - x_{12,8,8} - x_{12,11,8} - x_{12,16,8} &= 0 \\
x_{8,12,9} + x_{11,12,9} + x_{16,12,9} - x_{12,8,9} - x_{12,11,9} - x_{12,16,9} &= 0 \\
x_{8,12,12} + x_{11,12,12} + x_{16,12,12} - x_{12,8,12} - x_{12,11,12} - x_{12,16,12} &= 5 \\
x_{8,12,14} + x_{11,12,14} + x_{16,12,14} - x_{12,8,14} - x_{12,11,14} - x_{12,16,14} &= -1 \\
x_{8,12,15} + x_{11,12,15} + x_{16,12,15} - x_{12,8,15} - x_{12,11,15} - x_{12,16,15} &= -1 \\
x_{9,13,3} + x_{14,13,3} - x_{13,9,3} - x_{13,14,3} &= 0 \\
x_{9,13,5} + x_{14,13,5} - x_{13,9,5} - x_{13,14,5} &= 0 \\
x_{9,13,8} + x_{14,13,8} - x_{13,9,8} - x_{13,14,8} &= 0 \\
x_{9,13,9} + x_{14,13,9} - x_{13,9,9} - x_{13,14,9} &= 0 \\
x_{9,13,12} + x_{14,13,12} - x_{13,9,12} - x_{13,14,12} &= 0 \\
x_{9,13,14} + x_{14,13,14} - x_{13,9,14} - x_{13,14,14} &= 0 \\
x_{9,13,15} + x_{14,13,15} - x_{13,9,15} - x_{13,14,15} &= 0 \\
x_{10,14,3} + x_{13,14,3} + x_{15,14,3} - x_{14,10,3} - x_{14,13,3} - x_{14,15,3} &= 0 \\
x_{10,14,5} + x_{13,14,5} + x_{15,14,5} - x_{14,10,5} - x_{14,13,5} - x_{14,15,5} &= 0 \\
x_{10,14,8} + x_{13,14,8} + x_{15,14,8} - x_{14,10,8} - x_{14,13,8} - x_{14,15,8} &= 0 \\
x_{10,14,9} + x_{13,14,9} + x_{15,14,9} - x_{14,10,9} - x_{14,13,9} - x_{14,15,9} &= 0 \\
x_{10,14,12} + x_{13,14,12} + x_{15,14,12} - x_{14,10,12} - x_{14,13,12} - x_{14,15,12} &= 0 \\
x_{10,14,14} + x_{13,14,14} + x_{15,14,14} - x_{14,10,14} - x_{14,13,14} - x_{14,15,14} &= 6 \\
x_{10,14,15} + x_{13,14,15} + x_{15,14,15} - x_{14,10,15} - x_{14,13,15} - x_{14,15,15} &= -1 \\
x_{11,15,3} + x_{14,15,3} + x_{16,15,3} - x_{15,11,3} - x_{15,14,3} - x_{15,16,3} &= 0 \\
x_{11,15,5} + x_{14,15,5} + x_{16,15,5} - x_{15,11,5} - x_{15,14,5} - x_{15,16,5} &= 0 \\
x_{11,15,8} + x_{14,15,8} + x_{16,15,8} - x_{15,11,8} - x_{15,14,8} - x_{15,16,8} &= 0 \\
x_{11,15,9} + x_{14,15,9} + x_{16,15,9} - x_{15,11,9} - x_{15,14,9} - x_{15,16,9} &= 0 \\
x_{11,15,12} + x_{14,15,12} + x_{16,15,12} - x_{15,11,12} - x_{15,14,12} - x_{15,16,12} &= 0 \\
x_{11,15,14} + x_{14,15,14} + x_{16,15,14} - x_{15,11,14} - x_{15,14,14} - x_{15,16,14} &= 0 \\
x_{11,15,15} + x_{14,15,15} + x_{16,15,15} - x_{15,11,15} - x_{15,14,15} - x_{15,16,15} &= 7 \\
x_{12,16,3} + x_{15,16,3} - x_{16,12,3} - x_{16,15,3} &= 0 \\
x_{12,16,5} + x_{15,16,5} - x_{16,12,5} - x_{16,15,5} &= 0 \\
x_{12,16,8} + x_{15,16,8} - x_{16,12,8} - x_{16,15,8} &= 0 \\
x_{12,16,9} + x_{15,16,9} - x_{16,12,9} - x_{16,15,9} &= 0 \\
x_{12,16,12} + x_{15,16,12} - x_{16,12,12} - x_{16,15,12} &= 0 \\
x_{12,16,14} + x_{15,16,14} - x_{16,12,14} - x_{16,15,14} &= 0 \\
x_{12,16,15} + x_{15,16,15} - x_{16,12,15} - x_{16,15,15} &= 0 \\
x_{1,2,3}, x_{1,2,5}, x_{1,2,8}, x_{1,2,9}, x_{1,2,12}, x_{1,2,14}, x_{1,2,15} &\geq 0 \\
x_{2,1,3}, x_{2,1,5}, x_{2,1,8}, x_{2,1,9}, x_{2,1,12}, x_{2,1,14}, x_{2,1,15} &\geq 0 \\
x_{2,3,3}, x_{2,3,5}, x_{2,3,8}, x_{2,3,9}, x_{2,3,12}, x_{2,3,14}, x_{2,3,15} &\geq 0 \\
x_{3,2,3}, x_{3,2,5}, x_{3,2,8}, x_{3,2,9}, x_{3,2,12}, x_{3,2,14}, x_{3,2,15} &\geq 0 \\
x_{3,4,3}, x_{3,4,5}, x_{3,4,8}, x_{3,4,9}, x_{3,4,12}, x_{3,4,14}, x_{3,4,15} &\geq 0
\end{aligned}
$$

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\begin{aligned}
x_{4,3,3}, x_{4,3,5}, x_{4,3,8}, x_{4,3,9}, x_{4,3,12}, x_{4,3,14}, x_{4,3,15} &\geq 0 \\
x_{1,5,3}, x_{1,5,5}, x_{1,5,8}, x_{1,5,9}, x_{1,5,12}, x_{1,5,14}, x_{1,5,15} &\geq 0 \\
x_{5,1,3}, x_{5,1,5}, x_{5,1,8}, x_{5,1,9}, x_{5,1,12}, x_{5,1,14}, x_{5,1,15} &\geq 0 \\
x_{2,6,3}, x_{2,6,5}, x_{2,6,8}, x_{2,6,9}, x_{2,6,12}, x_{2,6,14}, x_{2,6,15} &\geq 0 \\
x_{6,2,3}, x_{6,2,5}, x_{6,2,8}, x_{6,2,9}, x_{6,2,12}, x_{6,2,14}, x_{6,2,15} &\geq 0 \\
x_{3,7,3}, x_{3,7,5}, x_{3,7,8}, x_{3,7,9}, x_{3,7,12}, x_{3,7,14}, x_{3,7,15} &\geq 0 \\
x_{7,3,3}, x_{7,3,5}, x_{7,3,8}, x_{7,3,9}, x_{7,3,12}, x_{7,3,14}, x_{7,3,15} &\geq 0 \\
x_{4,8,3}, x_{4,8,5}, x_{4,8,8}, x_{4,8,9}, x_{4,8,12}, x_{4,8,14}, x_{4,8,15} &\geq 0 \\
x_{8,4,3}, x_{8,4,5}, x_{8,4,8}, x_{8,4,9}, x_{8,4,12}, x_{8,4,14}, x_{8,4,15} &\geq 0 \\
x_{5,6,3}, x_{5,6,5}, x_{5,6,8}, x_{5,6,9}, x_{5,6,12}, x_{5,6,14}, x_{5,6,15} &\geq 0 \\
x_{6,5,3}, x_{6,5,5}, x_{6,5,8}, x_{6,5,9}, x_{6,5,12}, x_{6,5,14}, x_{6,5,15} &\geq 0 \\
x_{6,7,3}, x_{6,7,5}, x_{6,7,8}, x_{6,7,9}, x_{6,7,12}, x_{6,7,14}, x_{6,7,15} &\geq 0 \\
x_{7,6,3}, x_{7,6,5}, x_{7,6,8}, x_{7,6,9}, x_{7,6,12}, x_{7,6,14}, x_{7,6,15} &\geq 0 \\
x_{7,8,3}, x_{7,8,5}, x_{7,8,8}, x_{7,8,9}, x_{7,8,12}, x_{7,8,14}, x_{7,8,15} &\geq 0 \\
x_{8,7,3}, x_{8,7,5}, x_{8,7,8}, x_{8,7,9}, x_{8,7,12}, x_{8,7,14}, x_{8,7,15} &\geq 0 \\
x_{5,9,3}, x_{5,9,5}, x_{5,9,8}, x_{5,9,9}, x_{5,9,12}, x_{5,9,14}, x_{5,9,15} &\geq 0 \\
x_{9,5,3}, x_{9,5,5}, x_{9,5,8}, x_{9,5,9}, x_{9,5,12}, x_{9,5,14}, x_{9,5,15} &\geq 0 \\
x_{6,10,3}, x_{6,10,5}, x_{6,10,8}, x_{6,10,9}, x_{6,10,12}, x_{6,10,14}, x_{6,10,15} &\geq 0 \\
x_{10,6,3}, x_{10,6,5}, x_{10,6,8}, x_{10,6,9}, x_{10,6,12}, x_{10,6,14}, x_{10,6,15} &\geq 0 \\
x_{7,11,3}, x_{7,11,5}, x_{7,11,8}, x_{7,11,9}, x_{7,11,12}, x_{7,11,14}, x_{7,11,15} &\geq 0 \\
x_{11,7,3}, x_{11,7,5}, x_{11,7,8}, x_{11,7,9}, x_{11,7,12}, x_{11,7,14}, x_{11,7,15} &\geq 0 \\
x_{8,12,3}, x_{8,12,5}, x_{8,12,8}, x_{8,12,9}, x_{8,12,12}, x_{8,12,14}, x_{8,12,15} &\geq 0 \\
x_{12,8,3}, x_{12,8,5}, x_{12,8,8}, x_{12,8,9}, x_{12,8,12}, x_{12,8,14}, x_{12,8,15} &\geq 0 \\
x_{9,10,3}, x_{9,10,5}, x_{9,10,8}, x_{9,10,9}, x_{9,10,12}, x_{9,10,14}, x_{9,10,15} &\geq 0 \\
x_{10,9,3}, x_{10,9,5}, x_{10,9,8}, x_{10,9,9}, x_{10,9,12}, x_{10,9,14}, x_{10,9,15} &\geq 0 \\
x_{10,11,3}, x_{10,11,5}, x_{10,11,8}, x_{10,11,9}, x_{10,11,12}, x_{10,11,14}, x_{10,11,15} &\geq 0 \\
x_{11,10,3}, x_{11,10,5}, x_{11,10,8}, x_{11,10,9}, x_{11,10,12}, x_{11,10,14}, x_{11,10,15} &\geq 0 \\
x_{11,12,3}, x_{11,12,5}, x_{11,12,8}, x_{11,12,9}, x_{11,12,12}, x_{11,12,14}, x_{11,12,15} &\geq 0 \\
x_{12,11,3}, x_{12,11,5}, x_{12,11,8}, x_{12,11,9}, x_{12,11,12}, x_{12,11,14}, x_{12,11,15} &\geq 0 \\
x_{9,13,3}, x_{9,13,5}, x_{9,13,8}, x_{9,13,9}, x_{9,13,12}, x_{9,13,14}, x_{9,13,15} &\geq 0 \\
x_{13,9,3}, x_{13,9,5}, x_{13,9,8}, x_{13,9,9}, x_{13,9,12}, x_{13,9,14}, x_{13,9,15} &\geq 0 \\
x_{10,14,3}, x_{10,14,5}, x_{10,14,8}, x_{10,14,9}, x_{10,14,12}, x_{10,14,14}, x_{10,14,15} &\geq 0 \\
x_{14,10,3}, x_{14,10,5}, x_{14,10,8}, x_{14,10,9}, x_{14,10,12}, x_{14,10,14}, x_{14,10,15} &\geq 0 \\
x_{11,15,3}, x_{11,15,5}, x_{11,15,8}, x_{11,15,9}, x_{11,15,12}, x_{11,15,14}, x_{11,15,15} &\geq 0 \\
x_{15,11,3}, x_{15,11,5}, x_{15,11,8}, x_{15,11,9}, x_{15,11,12}, x_{15,11,14}, x_{15,11,15} &\geq 0 \\
x_{12,16,3}, x_{12,16,5}, x_{12,16,8}, x_{12,16,9}, x_{12,16,12}, x_{12,16,14}, x_{12,16,15} &\geq 0 \\
x_{16,12,3}, x_{16,12,5}, x_{16,12,8}, x_{16,12,9}, x_{16,12,12}, x_{16,12,14}, x_{16,12,15} &\geq 0 \\
x_{13,14,3}, x_{13,14,5}, x_{13,14,8}, x_{13,14,9}, x_{13,14,12}, x_{13,14,14}, x_{13,14,15} &\geq 0 \\
x_{14,13,3}, x_{14,13,5}, x_{14,13,8}, x_{14,13,9}, x_{14,13,12}, x_{14,13,14}, x_{14,13,15} &\geq 0 \\
x_{14,15,3}, x_{14,15,5}, x_{14,15,8}, x_{14,15,9}, x_{14,15,12}, x_{14,15,14}, x_{14,15,15} &\geq 0 \\
x_{15,14,3}, x_{15,14,5}, x_{15,14,8}, x_{15,14,9}, x_{15,14,12}, x_{15,14,14}, x_{15,14,15} &\geq 0
\end{aligned}
$$

$$x_{15,16,3}, x_{15,16,5}, x_{15,16,8}, x_{15,16,9}, x_{15,16,12}, x_{15,16,14}, x_{15,16,15} \geq 0$$
$$x_{16,15,3}, x_{16,15,5}, x_{16,15,8}, x_{16,15,9}, x_{16,15,12}, x_{16,15,14}, x_{16,15,15} \geq 0$$

See Figures 1–8 for the shortest path trees for all odd degree nodes obtained by solving the problem above:

Thus we obtain the shortest path distances between each pair of odd degree nodes, given in Table 1.

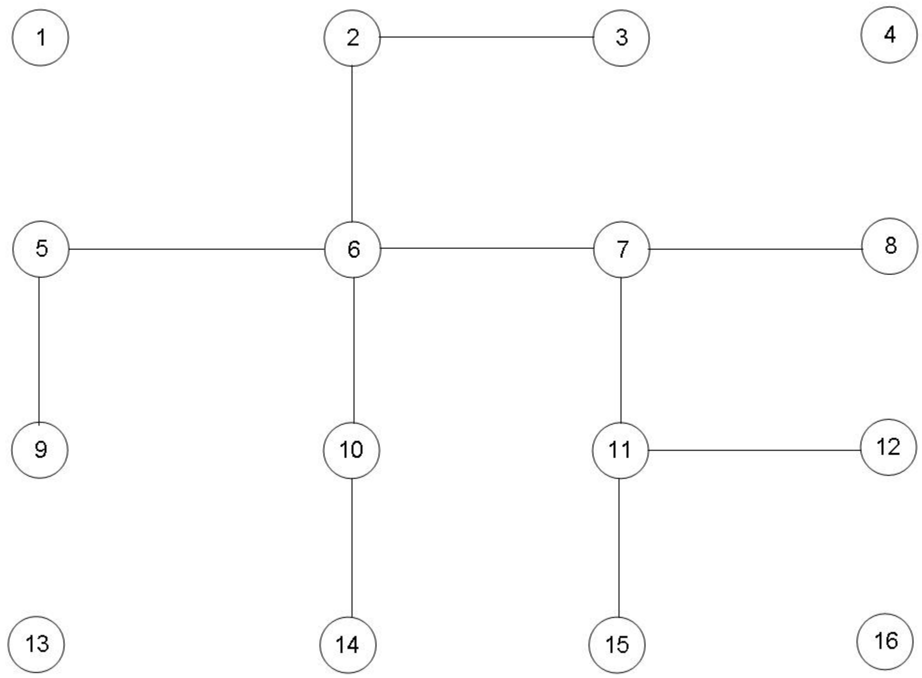

Figure 1: Shortest path tree between node 2 and all odd degree nodes.

For odd degree nodes $i, j \in O$, let ℓ_{ij} denote the shortest path distance between i and j, as given in Table 1.

Node	2	3	5	8	9	12	14	15
2	0	42	53	42	73	77	50	72
3	42	0	66	31	87	66	63	61
5	53	66	0	61	21	96	69	91
8	42	31	61	0	82	40	58	56
9	73	87	21	82	0	83	52	78
12	77	66	96	40	83	0	59	35
14	50	63	69	58	52	59	0	37
15	72	61	91	56	78	35	37	0

Table 1: Shortest path distances between each pair of odd degree nodes.

Next we formulate a matching problem to find the least cost matching of odd degree nodes.

Decision variables:

Let $z_{ij} = \begin{cases} 1 & \text{if we match } i \text{ and } j \text{ (duplicate the edges on the shortest path between } i \text{ and } j) \\ 0 & \text{o/w} \end{cases}$

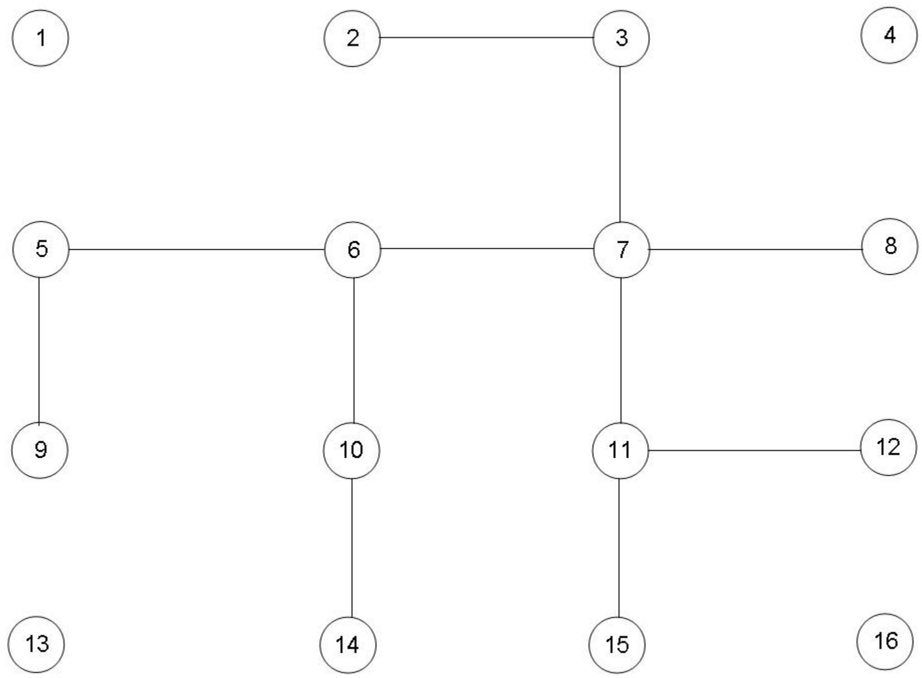

Figure 2: Shortest path tree between node 3 and all odd degree nodes.

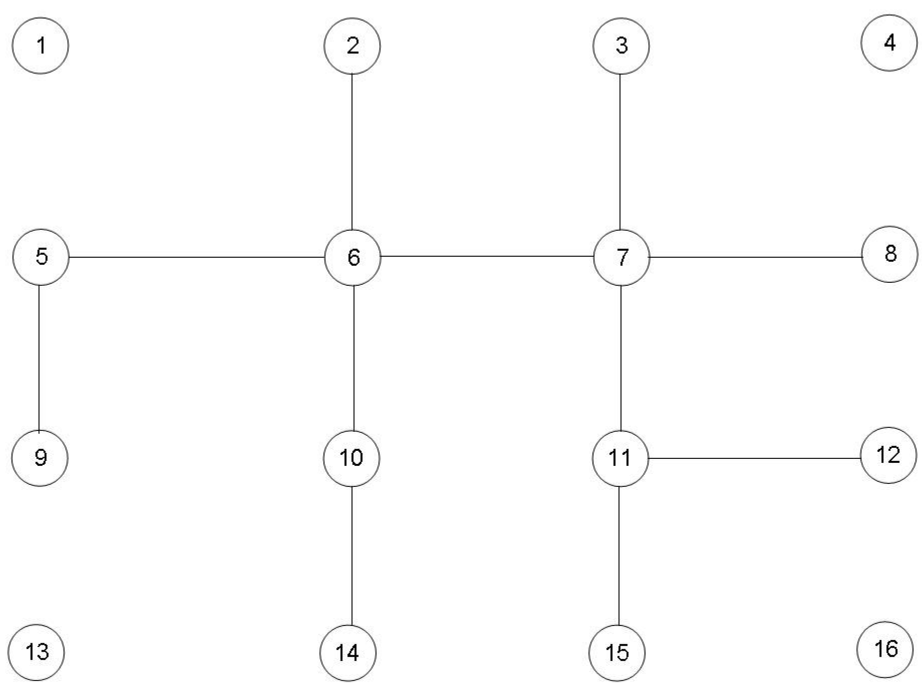

Figure 3: Shortest path tree between node 5 and all odd degree nodes.

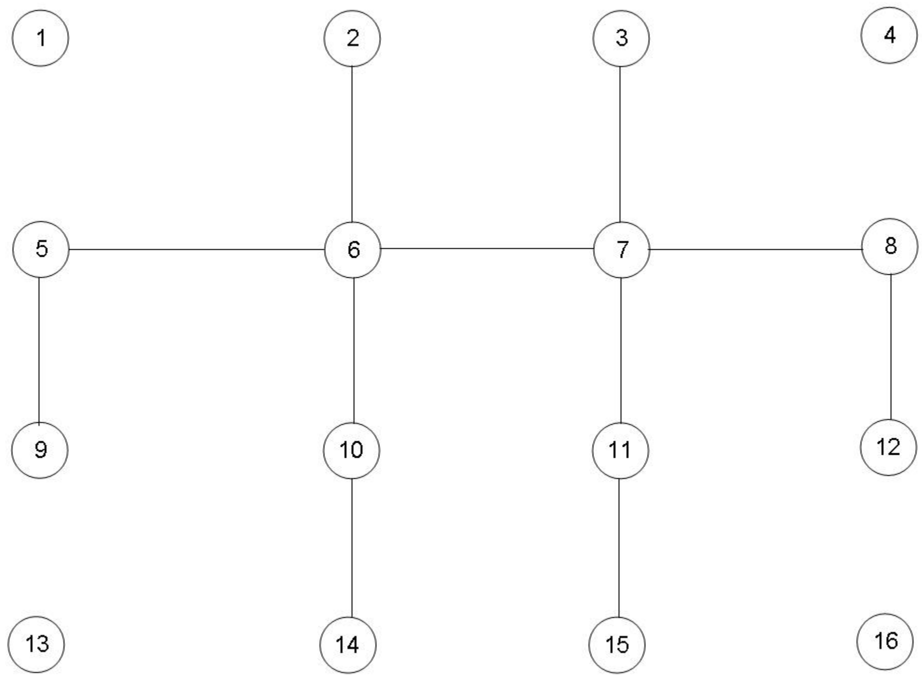

Figure 4: Shortest path tree between node 8 and all odd degree nodes.

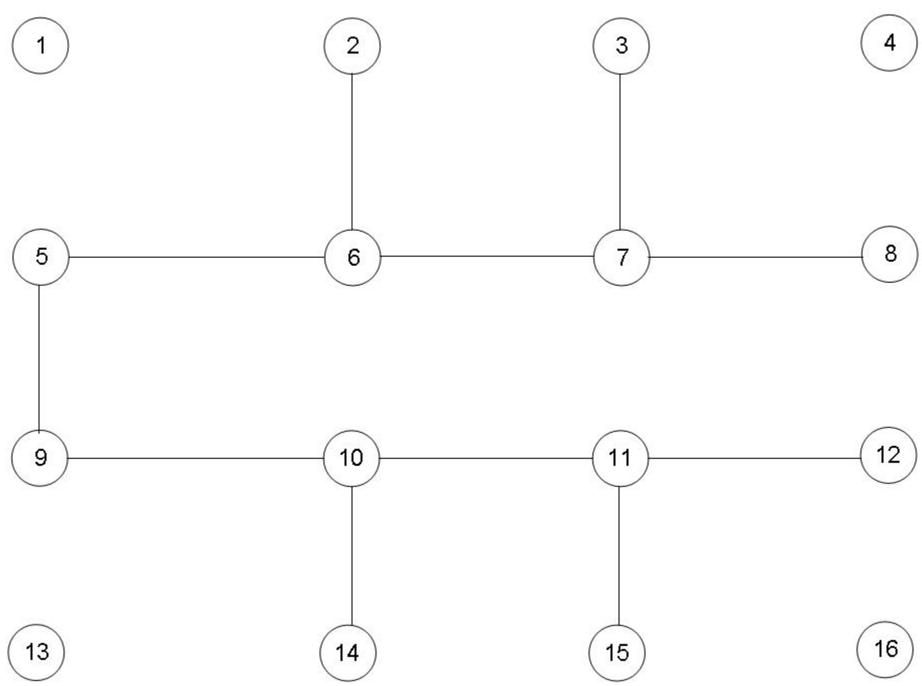

Figure 5: Shortest path tree between node 9 and all odd degree nodes.

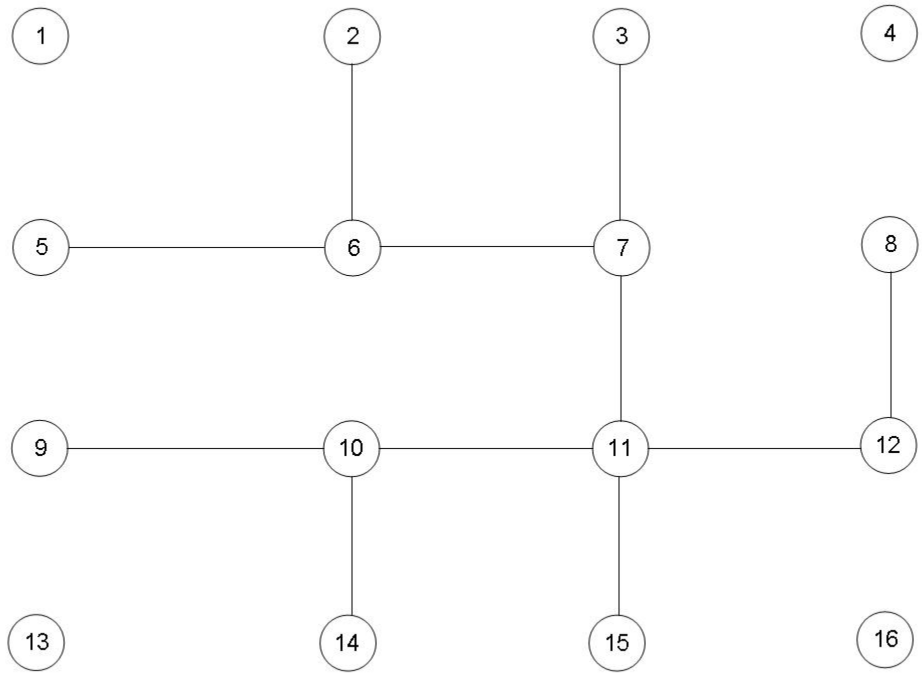

Figure 6: Shortest path tree between node 12 and all odd degree nodes.

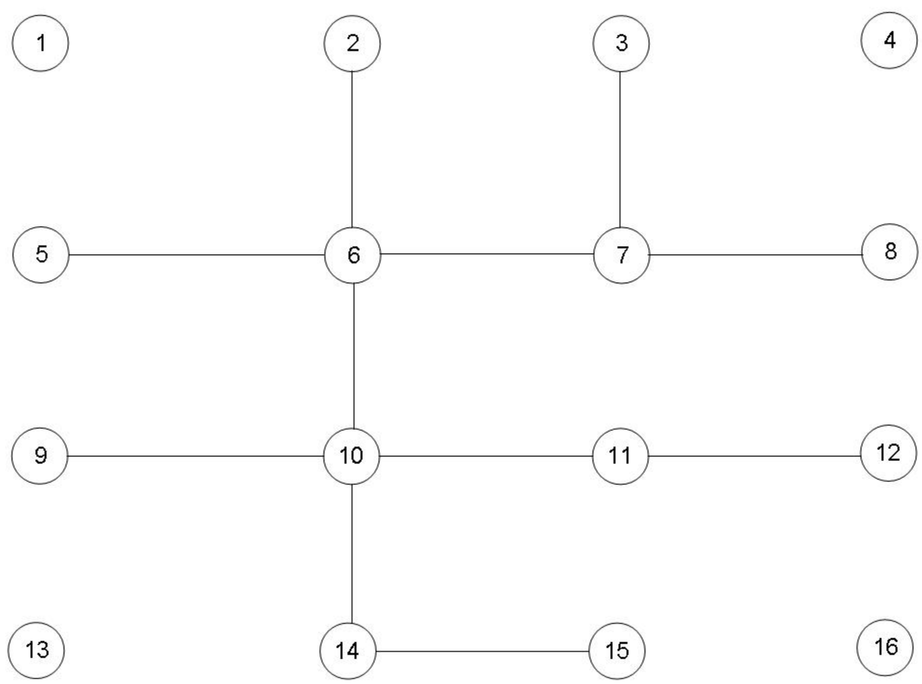

Figure 7: Shortest path tree between node 14 and all odd degree nodes.

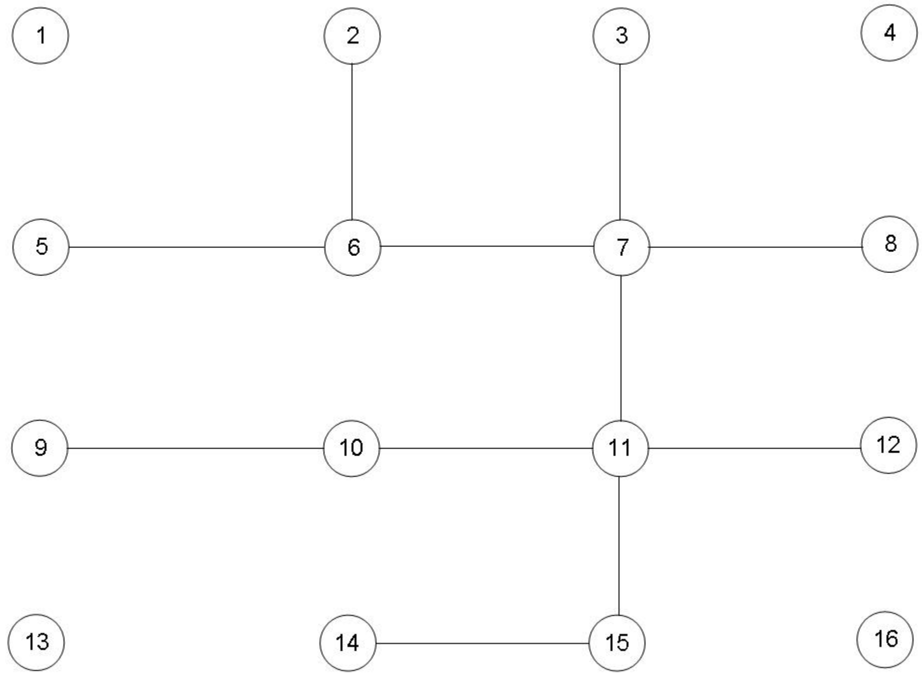

Figure 8: Shortest path tree between node 15 and all odd degree nodes.

Then the formulation of the problem is as follows:

$$\begin{array}{lrcll} \min & \sum_{i\in O}\sum_{j\in O, j>i} \ell_{ij} z_{ij} & & & \\ \text{s.t.} & & & & \\ & \sum_{\{j\in O\,:\,j<i\}} z_{ji} + \sum_{\{j\in O\,:\,j>i\}} z_{ij} & = & 1 & \forall\, i\in O \\ & z_{ij} & \in & \{0,1\} & \forall\, i,j\in O,\ i<j \end{array}$$

With the given data, the formulation is

$$\begin{array}{lrcl} \min & \{42z_{2,3}+53z_{2,5}+42z_{2,8}+73z_{2,9}+77z_{2,12}+50z_{2,14}+72z_{2,15} & & \\ & +66z_{3,5}+31z_{3,8}+87z_{3,9}+66z_{3,12}+63z_{3,14}+61z_{3,15}+61z_{5,8} & & \\ & +21z_{5,9}+96z_{5,12}+69z_{5,14}+91z_{5,15}+82z_{8,9}+40z_{8,12}+58z_{8,14} & & \\ & +56z_{8,15}+83z_{9,12}+52z_{9,14}+78z_{9,15}+59z_{12,14}+35z_{12,15}+37z_{14,15}\} & & \\ \text{s.t.} & z_{2,3}+z_{2,5}+z_{2,8}+z_{2,9}+z_{2,12}+z_{2,14}+z_{2,15} & = & 1 \\ & z_{2,3}+z_{3,5}+z_{3,8}+z_{3,9}+z_{3,12}+z_{3,14}+z_{3,15} & = & 1 \\ & z_{2,5}+z_{3,5}+z_{5,8}+z_{5,9}+z_{5,12}+z_{5,14}+z_{5,15} & = & 1 \\ & z_{2,8}+z_{3,8}+z_{5,8}+z_{8,9}+z_{8,12}+z_{8,14}+z_{8,15} & = & 1 \\ & z_{2,9}+z_{3,9}+z_{5,9}+z_{8,9}+z_{9,12}+z_{9,14}+z_{9,15} & = & 1 \\ & z_{2,12}+z_{3,12}+z_{5,12}+z_{8,12}+z_{9,12}+z_{12,14}+z_{12,15} & = & 1 \\ & z_{2,14}+z_{3,14}+z_{5,14}+z_{8,14}+z_{9,14}+z_{12,14}+z_{14,15} & = & 1 \\ & z_{2,15}+z_{3,15}+z_{5,15}+z_{8,15}+z_{9,15}+z_{12,15}+z_{14,15} & = & 1 \\ & z_{2,3}, z_{2,5}, z_{2,8}, z_{2,9}, z_{2,12}, z_{2,14}, z_{2,15} & \in & \{0,1\} \\ & z_{3,5}, z_{3,8}, z_{3,9}, z_{3,12}, z_{3,14}, z_{3,15}, z_{5,8} & \in & \{0,1\} \\ & z_{8,15}, z_{9,12}, z_{9,14}, z_{9,15}, z_{12,14}, z_{12,15}, z_{14,15} & \in & \{0,1\} \end{array}$$

The optimal solution is $z^*_{2,14} = 1$, $z^*_{3,8} = 1$, $z^*_{5,9} = 1$, $z^*_{12,15} = 1$, all other $z^*_{i,j} = 0$, that is, it is optimal to duplicate the edges on the shortest paths between $2, 14$, between $3, 8$, between $5, 9$, and between $12, 15$.
Thus we duplicate the edges on 2–6–10–14, 3–7–8, 5–9, 12–11–15
An Euler tour on the new augmented network can be obtained with the depth first procedure covered in class.
An Optimal Euler tour = 1–2–3–4–8–12–16–15–14–13–9–5–6–2–6–7–3–7–8–7–11–12–11–15–11–10–14–10–6–10–9–5–1
Optimal Cost = 769

Problem 2

The following network represents the network of one way streets in some city. These streets have to be cleared each time it snows. Streets are cleared by a specially equipped snowplow that cruises along the streets, scraping the snow to the sides. The snowplow can cruise along the streets only in the designated directions. The cost of operating the snowplowing service (including the cost of the plow, fuel and crew) is represented by costs on the edges of the network. Design a snowplowing tour for the plow that will traverse each street at least once, that will return the plow to its origin (plowhouse), and that will incur the minimum total cost. As a first step to designing plow routes, consider the case with a single snowplow, and ignore other constraints such as turn constraints at intersections, and route duration constraints. Write a complete LP formulation, explicitly with all applicable data values, of all problems you solve. (Hint: Use an LP solver of your choice, such as LINDO, GAMS, CPlex, XPRESS-MP, AIMMS, AMPL, to solve the optimization problems encountered along the way.) (25)

Answer: Let $N = \{1, 2, \dots, 16\}$ denote the set of nodes. Let $A = \{(1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (1,5), (5,1), (2,6), (7,3), (4,8), (8,4), (5,6), (6,7), (7,8), (5,9), (9,5), (6,10), (11,7), (8,12), (12,8), (10,9), (11,10), (12,11), (9,13), (13,9), (10,14), (15,11), (12,16), (16,12), (13,14), (14,13), (14,15), (15,14), (15,16), (16,15)\}$ denote the set of arcs. Let $G = (N, A)$ denote the given directed network.
Let c_{ij} denote the cost associated with arc (i, j).
For each node $i \in N$, let d_i denote the degree of i = outdegree - indegree of node i.
The first problem is to determine how many additional times (in addition to the requirement of at least one traversal) to traverse each arc to obtain a balanced network with minimum cost. Decision variables: Let x_{ij} denote the number of additional times to traverse arc (i, j).
Then the formulation of the problem is as follows:

$$\begin{array}{lrcll} \min & \sum_{(i,j)\in A} c_{ij}x_{ij} & & & \\ \text{s.t.} & & & & \\ & \sum_{\{j\in N\,:\,(j,i)\in A\}} x_{ji} - \sum_{\{j\in N\,:\,(i,j)\in A\}} x_{ij} & = & d_i & \forall\ i \in N \\ & x_{ij} & \geq & 0 & \forall\ (i,j) \in A \end{array}$$

With the given data, the formulation is

$$\begin{array}{lrcl} \min & \{16x_{1,2} + 16x_{2,1} + 62x_{2,3} + 62x_{3,2} + 19x_{3,4} + 19x_{4,3} & & \\ & +14x_{1,5} + 14x_{5,1} + 74x_{2,6} + 63x_{7,3} + 15x_{4,8} + 15x_{8,4} & & \\ & +47x_{5,6} + 60x_{6,7} + 36x_{7,8} + 11x_{5,9} + 11x_{9,5} + 56x_{6,10} & & \\ & +52x_{11,7} + 10x_{8,12} + 10x_{12,8} + 26x_{10,9} + 25x_{11,10} + 4x_{12,11} & & \\ & +13x_{9,13} + 13x_{13,9} + 65x_{10,14} + 61x_{15,11} + 8x_{12,16} + 8x_{16,12} & & \\ & +12x_{13,14} + 12x_{14,13} + 67x_{14,15} + 67x_{15,14} + 9x_{15,16} + 9x_{16,15}\} & & \\ \text{s.t.} & x_{2,1} + x_{5,1} - x_{1,2} - x_{1,5} & = & 0 \\ & x_{1,2} + x_{3,2} - x_{2,1} - x_{2,3} - x_{2,6} & = & 1 \\ & x_{2,3} + x_{4,3} + x_{7,3} - x_{3,2} - x_{3,4} & = & -1 \\ & x_{3,4} + x_{8,4} - x_{4,3} - x_{4,8} & = & 0 \\ & x_{1,5} + x_{9,5} - x_{5,1} - x_{5,6} - x_{5,9} & = & 1 \\ & x_{2,6} + x_{5,6} - x_{6,7} - x_{6,10} & = & 0 \\ & x_{6,7} + x_{11,7} - x_{7,3} - x_{7,8} & = & 0 \\ & x_{4,8} + x_{7,8} + x_{12,8} - x_{8,4} - x_{8,12} & = & -1 \\ & x_{5,9} + x_{10,9} + x_{13,9} - x_{9,5} - x_{9,13} & = & -1 \\ & x_{6,10} + x_{11,10} - x_{10,9} - x_{10,14} & = & 0 \\ & x_{12,11} + x_{15,11} - x_{11,7} - x_{11,10} & = & 0 \\ & x_{8,12} + x_{16,12} - x_{12,8} - x_{12,11} - x_{12,16} & = & 1 \end{array}$$

$$
\begin{aligned}
x_{9,13} + x_{14,13} - x_{13,9} - x_{13,14} &= 0 \\
x_{10,14} + x_{13,14} + x_{15,14} - x_{14,13} - x_{14,15} &= -1 \\
x_{14,15} + x_{16,15} - x_{15,11} - x_{15,14} - x_{15,16} &= 1 \\
x_{12,16} + x_{15,16} - x_{16,12} - x_{16,15} &= 0 \\
x_{1,2}, x_{2,1}, x_{2,3}, x_{3,2}, x_{3,4}, x_{4,3}, x_{1,5}, x_{5,1}, x_{2,6} &\geq 0 \\
x_{7,3}, x_{4,8}, x_{8,4}, x_{5,6}, x_{6,7}, x_{7,8}, x_{5,9}, x_{9,5}, x_{6,10} &\geq 0 \\
x_{11,7}, x_{8,12}, x_{12,8}, x_{10,9}, x_{11,10}, x_{12,11}, x_{9,13}, x_{13,9}, x_{10,14} &\geq 0 \\
x_{15,11}, x_{12,16}, x_{16,12}, x_{13,14}, x_{14,13}, x_{14,15}, x_{15,14}, x_{15,16}, x_{16,15} &\geq 0
\end{aligned}
$$

The optimal solution is $x^*_{1,2} = 1$, $x^*_{3,4} = 1$, $x^*_{5,1} = 1$, $x^*_{4,8} = 1$, $x^*_{9,5} = 2$, $x^*_{8,12} = 2$, $x^*_{13,9} = 1$, $x^*_{12,16} = 1$, $x^*_{14,13} = 1$, $x^*_{16,15} = 1$, all other $x^*_{i,j} = 0$. This gives the number of additional times (in addition to the requirement of at least one traversal) to traverse each arc to obtain a balanced network with minimum cost. An Euler tour on the new augmented network can be obtained with the depth first procedure covered in class.

An Optimal Euler tour = 1–2–3–4–8–12–16–15–14–13–9–5–1–2–6–10–14–15–16–12–8–4–3–2–1–5–9–13–14–13–9–5–6–7–8–12–16–15–11–7–3–4–8–12–11–10–9–5–1

Optimal Cost = 1229