Read the materials below in preparation for discussion in class. The questions are intended to guide your reading and thoughts, but are not the only aspects that will be discussed in class. **You should be prepared to explain your answer for any question on the board in class.**


1. How does a base-stock policy work? How does a time-varying base-stock policy work? How does an echelon base-stock policy work? (If you do not know, look these up.)

2. The following setup is the same as in the article, but in slightly different words; I hope it will help to make the setup clear to you. We consider a discrete time model of inventory control for a single product. Let $D(t)$ denote the random demand in period $t$. Choose a distribution for $D(t)$ (the article assumes that $D(t)$ is normally distributed, but we can choose any other distribution — it does not affect our calculations) with mean $\mu(t)$, variance $\sigma^2(t)$, and covariance $\text{Cov}(D(i), D(j))$ between $D(i)$ and $D(j)$. Note that the marginal distribution of $D(t)$ may be different for different time periods $t$, which is why the demand is called nonstationary. For any $t_1 < t_2$, let $d(t_1, t_2) = \sum_{i=t_1+1}^{t_2} D(i)$ denote the total demand in periods $t_1 + 1, \ldots, t_2$. Let $B(t)$ denote the base-stock level chosen for the end of period $t$. For the purpose of this exercise, assume that all the base-stock levels $B(0), B(1), B(2), \ldots,$ are chosen in advance, that is, $B(t)$ does not depend on the observed demands $D(0), D(1), \ldots, D(t)$, and thus $B(t)$ is not random. (This is a restrictive assumption in general.) Let $O(t)$ denote the quantity of product ordered at the end of time period $t$. Let $SI$ denote the number of time periods from the time an order is placed at the supplier until the time that the supplier dispatches the order (sends it with a carrier), and let $T$ denote the transportation lead time from the supplier until the shipment is received. The product that is ordered at the end of time period $t$ will be received at the beginning of time period $t + SI + T$. Let $S$ denote the number of time periods from the time an order is received from a customer until the time that the order is dispatched to the customer. Assume that $S \leq SI + T$. Let $I(t)$ denote the inventory on hand at the end of period $t$, after the replenishment due in time period $t$ has been received from the supplier, and after the customer order dispatched in time period $t$ has been deducted from inventory. Let $SS(t) := E[I(t)]$ denote the expected inventory on hand at the end of period $t$, that is, the safety stock in period $t$. Let $IP(t)$ denote the inventory position in period $t$ after the customer order in time period $t$ has been received, but before the order with the supplier in time
period $t$ has been placed; recall that inventory position is the inventory on hand, plus
all the product ordered from the supplier but not received yet, minus all the product
ordered by customers but not dispatched yet.

In each time period $t$, you may think of events happening in the following sequence:
First, the quantity $O(t - SI - T)$ of product ordered from the supplier in period $t - SI - T$
is received. Then the quantity $D(t - S)$ of product ordered by customers in period $t - S$
is dispatched. Then the orders for quantity $D(t)$ (that was random, and thus unknown,
until now) of product is received from customers. Then the inventory position $IP(t)$
is updated, and then the quantity $O(t)$ of product is ordered from the supplier, where
$O(t)$ is determined by the chosen base-stock level $B(t)$ by $O(t) = B(t) - IP(t)$.

(a) Give an expression for $\mathbb{E}[d(t_1, t_2)]$ and $\text{Var}[d(t_1, t_2)]$.
(b) Assume that $D(0), D(1), D(2), \ldots$, are independent. Give an expression for $\text{Var}[d(t_1, t_2)]$.
(c) The assumption that $D(0), D(1), D(2), \ldots$, are independent is a restrictive assumption. Give an example of a setting when you expect $D(t)$ and $D(t + 1)$ to be positively correlated. Give an example of a setting when you expect $D(t)$ and $D(t + 1)$ to be negatively correlated.
(d) The assumption that all the base-stock levels $B(0), B(1), B(2), \ldots$, are chosen in advance, that is, $B(t)$ does not depend on the observed demands $D(0), D(1), \ldots, D(t)$, is a restrictive assumption in general. Give conditions under which this assumption is not a good assumption, and conditions under which this assumption is alright.
(e) In terms of the base-stock levels, what is the inventory position just after the order for $O(t)$ units of product has been placed?
(f) Write the inventory position $IP(t)$ at time period $t$ in terms of the initial inventory position $IP(0)$, the orders placed with the supplier, and the orders received from the customers.
(g) Write the inventory on hand $I(t)$ at time period $t$ in terms of the initial inventory on hand $I(0)$, the orders placed with the supplier, and the orders received from the customers.
(h) Write the inventory on hand $I(t)$ at time period $t$ in terms of the inventory position $IP(t)$ at time period $t$, the orders placed with the supplier, and the orders received from the customers.
(i) Now write the inventory on hand $I(t)$ at time period $t$ in terms of the inventory position $IP(t - SI - T)$ at time period $t - SI - T$, the orders placed with the supplier, and the orders received from the customers.
(j) Now write the inventory on hand $I(t)$ at time period $t$ in terms of the base-stock $B(t - SI - T)$ at time period $t - SI - T$, and the orders received from the customers.
(k) Explain the derivation of the expression for the base-stock level at time $t$ in
equation (2):

\[ B(t) = \sum_{i=1}^{\text{NRLT}} \mu(t + i) + z(t + \text{NRLT}) \sqrt{\sum_{i=1}^{\text{NRLT}} \sigma^2(t + i)} \]

(l) Explain the derivation of the expression for the safety stock level at time \( t \) in equation (3):

\[ SS(t) = z(t - S) \sqrt{\sum_{i=1}^{\text{NRLT}} \sigma^2(t - S + 1 - i)} \]

(m) Explain the derivation of the expression for the expected quantity ordered from the supplier at time \( t \) in equation (8)

\[ \mathbb{E}[O(t)] = \mu(t + \text{NRLT}) + \Delta SS(t) \]

and the expression for the variance of the quantity ordered from the supplier at time \( t \) in equation (7).

3. What is the landslide effect? (Hint: It will help to refer to equation (3).)

4. Suppose that the inventory manager in the model expects well in advance that a big increase in demand will occur from time period \( t \) to time period \( t + 1 \) (for example, due to a holiday). When will the supplier of our inventory manager experience an increase in demand? (Hint: Refer to equation (8).)

5. Describe two extensions of the basic model in the article, and the effect of these extensions on the supply chain.

6. Describe the effect of limited production capacity and nonstationary demand on the inventory control.

7. Describe the effect of positive correlation between nonstationary random demand and random lead times on the inventory control.

8. Describe the application of the nonstationary inventory control model at Microsoft.

9. Describe the application of the nonstationary inventory control model at Case New Holland.