ISyE 4111

Advanced Supply Chain Logistics

Fall 2010

Homework 3 solution

This homework is worth 50 points.

Problem 1

The following network represents the network of canals in a famous city (Amsterdam? Venice? Fort Lauderdale?). These canals have to be cleaned regularly. Canals are cleaned by a specially equipped boat that cruises along the canals, dredging the bottoms of the canals and scooping up any floating debris in the water. The boat can cruise along the canals in any direction. The cost of operating the cleaning service (including the cost of the boat, fuel and crew) is represented by costs on the edges of the network. Design a cleaning tour for the boat that will traverse each canal at least once, that will return the boat to its origin (boathouse), and that will incur the minimum total cost. Write a complete LP formulation, explicitly with all applicable data values, for the shortest path problem from all odd degree nodes to the largest indexed node with odd degree. Also write a complete LP formulation, explicitly with all applicable data values, of the matching problem. (Hint: Use an LP solver of your choice, such as LINDO, GAMS, CPlex, XPRESS-MP, AIMMS, AMPL, to solve the optimization problems encountered along the way.) (25)

Answer: Let $N = \{1, 2, \ldots, 23\}$ denote the set of nodes. Let $E = \{\{1,2\}, \{1,3\}, \{1,5\}, \{1,7\}, \{2,8\}, \{2,9\}, \{3,4\}, \{3,11\}, \{4,5\}, \{4,6\}, \{4,16\}, \{5,15\}, \{6,8\}, \{6,17\}, \{7,11\}, \{7,13\}, \{8,10\}, \{9,10\}, \{9,12\}, \{10,18\}, \{11,12\}, \{11,23\}, \{12,19\}, \{13,14\}, \{13,22\}, \{14,15\}, \{15,16\}, \{16,17\}, \{18,19\}, \{19,20\}, \{20,21\}, \{20,23\}, \{21,22\}, \{22,23\}\}$ denote the set of edges. Let $G = (N, E)$ denote the given undirected network. Let c_{ij} denote the cost associated with edge $\{i, j\}$.

The set of odd degree nodes is $O = \{2, 3, 5, 6, 7, 8, 9, 10, 12, 13, 15, 16, 19, 20, 22, 23\}$. The first step is to compute the shortest path and shortest path length between each pair of odd degree nodes. One way to do it is to formulate the following multicommodity network flow problem. For each edge $\{i, j\} \in E$, create two arcs (i, j) and (j, i), each with cost equal to c_{ij}. Let A denote the set of arcs. For each odd degree node (except the one with smallest index), create a commodity — the commodity index is the same as the index of the destination node of the shortest path. Thus the set of commodities is $K = \{3, 5, 6, 7, 8, 9, 10, 12, 13, 15, 16, 19, 20, 22, 23\}$. Then the supply for commodity k at node $i \in O$ is $s_i^k = |O| - 1$ if $i = k$, $s_i^k = -1$ (i.e., demand) if $i \neq k$, and the supply at node $i \notin O$ is $s_i^k = 0$.

Decision variables:

Let x_{ij}^k denote the amount of flow of commodity k on arc (i, j). The optimal solution x_{ij}^{k*} will give the shortest path tree from node k to all other odd-degree nodes (which is the same as the shortest path tree into node k because the network is undirected), and the shortest path lengths can easily be calculated based on that.

Then the formulation of the problem is as follows:

$$\begin{array}{llll} \min & \sum_{(i,j)\in A} \sum_{k\in K} c_{ij} x_{ij}^k & & \\ \text{s.t.} & & & \\ & \sum_{\{j\in N\,:\,(i,j)\in A\}} x_{ij}^k - \sum_{\{j\in N\,:\,(j,i)\in A\}} x_{ji}^k & = s_i^k & \forall\ k \in K,\ \ \forall\ i \in N \\ & x_{ij}^k & \geq 0 & \forall\ (i,j) \in A,\ \ \forall\ k \in K \end{array}$$

With the given data, the formulation is

$$\begin{aligned} \min \quad \{ & 45x_{1,2}^3 + 45x_{1,2}^5 + 45x_{1,2}^6 + 45x_{1,2}^7 + 45x_{1,2}^8 + 45x_{1,2}^9 + 45x_{1,2}^{10} + 45x_{1,2}^{12} \\ & +45x_{1,2}^{13} + 45x_{1,2}^{15} + 45x_{1,2}^{16} + 45x_{1,2}^{19} + 45x_{1,2}^{20} + 45x_{1,2}^{22} + 45x_{1,2}^{23} \\ & +45x_{2,1}^3 + 45x_{2,1}^5 + 45x_{2,1}^6 + 45x_{2,1}^7 + 45x_{2,1}^8 + 45x_{2,1}^9 + 45x_{2,1}^{10} + 45x_{2,1}^{12} \end{aligned}$$

$$+45x_{2,1}^{13} + 45x_{2,1}^{15} + 45x_{2,1}^{16} + 45x_{2,1}^{19} + 45x_{2,1}^{20} + 45x_{2,1}^{22} + 45x_{2,1}^{23}$$
$$+29x_{1,3}^{3} + 29x_{1,3}^{5} + 29x_{1,3}^{6} + 29x_{1,3}^{7} + 29x_{1,3}^{8} + 29x_{1,3}^{9} + 29x_{1,3}^{10} + 29x_{1,3}^{12}$$
$$+29x_{1,3}^{13} + 29x_{1,3}^{15} + 29x_{1,3}^{16} + 29x_{1,3}^{19} + 29x_{1,3}^{20} + 29x_{1,3}^{22} + 29x_{1,3}^{23}$$
$$+29x_{3,1}^{3} + 29x_{3,1}^{5} + 29x_{3,1}^{6} + 29x_{3,1}^{7} + 29x_{3,1}^{8} + 29x_{3,1}^{9} + 29x_{3,1}^{10} + 29x_{3,1}^{12}$$
$$+29x_{3,1}^{13} + 29x_{3,1}^{15} + 29x_{3,1}^{16} + 29x_{3,1}^{19} + 29x_{3,1}^{20} + 29x_{3,1}^{22} + 29x_{3,1}^{23}$$
$$+20x_{1,5}^{3} + 20x_{1,5}^{5} + 20x_{1,5}^{6} + 20x_{1,5}^{7} + 20x_{1,5}^{8} + 20x_{1,5}^{9} + 20x_{1,5}^{10} + 20x_{1,5}^{12}$$
$$+20x_{1,5}^{13} + 20x_{1,5}^{15} + 20x_{1,5}^{16} + 20x_{1,5}^{19} + 20x_{1,5}^{20} + 20x_{1,5}^{22} + 20x_{1,5}^{23}$$
$$+20x_{5,1}^{3} + 20x_{5,1}^{5} + 20x_{5,1}^{6} + 20x_{5,1}^{7} + 20x_{5,1}^{8} + 20x_{5,1}^{9} + 20x_{5,1}^{10} + 20x_{5,1}^{12}$$
$$+20x_{5,1}^{13} + 20x_{5,1}^{15} + 20x_{5,1}^{16} + 20x_{5,1}^{19} + 20x_{5,1}^{20} + 20x_{5,1}^{22} + 20x_{5,1}^{23}$$
$$+30x_{1,7}^{3} + 30x_{1,7}^{5} + 30x_{1,7}^{6} + 30x_{1,7}^{7} + 30x_{1,7}^{8} + 30x_{1,7}^{9} + 30x_{1,7}^{10} + 30x_{1,7}^{12}$$
$$+30x_{1,7}^{13} + 30x_{1,7}^{15} + 30x_{1,7}^{16} + 30x_{1,7}^{19} + 30x_{1,7}^{20} + 30x_{1,7}^{22} + 30x_{1,7}^{23}$$
$$+30x_{7,1}^{3} + 30x_{7,1}^{5} + 30x_{7,1}^{6} + 30x_{7,1}^{7} + 30x_{7,1}^{8} + 30x_{7,1}^{9} + 30x_{7,1}^{10} + 30x_{7,1}^{12}$$
$$+30x_{7,1}^{13} + 30x_{7,1}^{15} + 30x_{7,1}^{16} + 30x_{7,1}^{19} + 30x_{7,1}^{20} + 30x_{7,1}^{22} + 30x_{7,1}^{23}$$
$$+33x_{2,8}^{3} + 33x_{2,8}^{5} + 33x_{2,8}^{6} + 33x_{2,8}^{7} + 33x_{2,8}^{8} + 33x_{2,8}^{9} + 33x_{2,8}^{10} + 33x_{2,8}^{12}$$
$$+33x_{2,8}^{13} + 33x_{2,8}^{15} + 33x_{2,8}^{16} + 33x_{2,8}^{19} + 33x_{2,8}^{20} + 33x_{2,8}^{22} + 33x_{2,8}^{23}$$
$$+33x_{8,2}^{3} + 33x_{8,2}^{5} + 33x_{8,2}^{6} + 33x_{8,2}^{7} + 33x_{8,2}^{8} + 33x_{8,2}^{9} + 33x_{8,2}^{10} + 33x_{8,2}^{12}$$
$$+33x_{8,2}^{13} + 33x_{8,2}^{15} + 33x_{8,2}^{16} + 33x_{8,2}^{19} + 33x_{8,2}^{20} + 33x_{8,2}^{22} + 33x_{8,2}^{23}$$
$$+37x_{2,9}^{3} + 37x_{2,9}^{5} + 37x_{2,9}^{6} + 37x_{2,9}^{7} + 37x_{2,9}^{8} + 37x_{2,9}^{9} + 37x_{2,9}^{10} + 37x_{2,9}^{12}$$
$$+37x_{2,9}^{13} + 37x_{2,9}^{15} + 37x_{2,9}^{16} + 37x_{2,9}^{19} + 37x_{2,9}^{20} + 37x_{2,9}^{22} + 37x_{2,9}^{23}$$
$$+37x_{9,2}^{3} + 37x_{9,2}^{5} + 37x_{9,2}^{6} + 37x_{9,2}^{7} + 37x_{9,2}^{8} + 37x_{9,2}^{9} + 37x_{9,2}^{10} + 37x_{9,2}^{12}$$
$$+37x_{9,2}^{13} + 37x_{9,2}^{15} + 37x_{9,2}^{16} + 37x_{9,2}^{19} + 37x_{9,2}^{20} + 37x_{9,2}^{22} + 37x_{9,2}^{23}$$
$$+44x_{3,4}^{3} + 44x_{3,4}^{5} + 44x_{3,4}^{6} + 44x_{3,4}^{7} + 44x_{3,4}^{8} + 44x_{3,4}^{9} + 44x_{3,4}^{10} + 44x_{3,4}^{12}$$
$$+44x_{3,4}^{13} + 44x_{3,4}^{15} + 44x_{3,4}^{16} + 44x_{3,4}^{19} + 44x_{3,4}^{20} + 44x_{3,4}^{22} + 44x_{3,4}^{23}$$
$$+44x_{4,3}^{3} + 44x_{4,3}^{5} + 44x_{4,3}^{6} + 44x_{4,3}^{7} + 44x_{4,3}^{8} + 44x_{4,3}^{9} + 44x_{4,3}^{10} + 44x_{4,3}^{12}$$
$$+44x_{4,3}^{13} + 44x_{4,3}^{15} + 44x_{4,3}^{16} + 44x_{4,3}^{19} + 44x_{4,3}^{20} + 44x_{4,3}^{22} + 44x_{4,3}^{23}$$
$$+19x_{3,11}^{3} + 19x_{3,11}^{5} + 19x_{3,11}^{6} + 19x_{3,11}^{7} + 19x_{3,11}^{8} + 19x_{3,11}^{9} + 19x_{3,11}^{10} + 19x_{3,11}^{12}$$
$$+19x_{3,11}^{13} + 19x_{3,11}^{15} + 19x_{3,11}^{16} + 19x_{3,11}^{19} + 19x_{3,11}^{20} + 19x_{3,11}^{22} + 19x_{3,11}^{23}$$
$$+19x_{11,3}^{3} + 19x_{11,3}^{5} + 19x_{11,3}^{6} + 19x_{11,3}^{7} + 19x_{11,3}^{8} + 19x_{11,3}^{9} + 19x_{11,3}^{10} + 19x_{11,3}^{12}$$
$$+19x_{11,3}^{13} + 19x_{11,3}^{15} + 19x_{11,3}^{16} + 19x_{11,3}^{19} + 19x_{11,3}^{20} + 19x_{11,3}^{22} + 19x_{11,3}^{23}$$
$$+31x_{4,5}^{3} + 31x_{4,5}^{5} + 31x_{4,5}^{6} + 31x_{4,5}^{7} + 31x_{4,5}^{8} + 31x_{4,5}^{9} + 31x_{4,5}^{10} + 31x_{4,5}^{12}$$
$$+31x_{4,5}^{13} + 31x_{4,5}^{15} + 31x_{4,5}^{16} + 31x_{4,5}^{19} + 31x_{4,5}^{20} + 31x_{4,5}^{22} + 31x_{4,5}^{23}$$
$$+31x_{5,4}^{3} + 31x_{5,4}^{5} + 31x_{5,4}^{6} + 31x_{5,4}^{7} + 31x_{5,4}^{8} + 31x_{5,4}^{9} + 31x_{5,4}^{10} + 31x_{5,4}^{12}$$
$$+31x_{5,4}^{13} + 31x_{5,4}^{15} + 31x_{5,4}^{16} + 31x_{5,4}^{19} + 31x_{5,4}^{20} + 31x_{5,4}^{22} + 31x_{5,4}^{23}$$
$$+34x_{4,6}^{3} + 34x_{4,6}^{5} + 34x_{4,6}^{6} + 34x_{4,6}^{7} + 34x_{4,6}^{8} + 34x_{4,6}^{9} + 34x_{4,6}^{10} + 34x_{4,6}^{12}$$
$$+34x_{4,6}^{13} + 34x_{4,6}^{15} + 34x_{4,6}^{16} + 34x_{4,6}^{19} + 34x_{4,6}^{20} + 34x_{4,6}^{22} + 34x_{4,6}^{23}$$
$$+34x_{6,4}^{3} + 34x_{6,4}^{5} + 34x_{6,4}^{6} + 34x_{6,4}^{7} + 34x_{6,4}^{8} + 34x_{6,4}^{9} + 34x_{6,4}^{10} + 34x_{6,4}^{12}$$
$$+34x_{6,4}^{13} + 34x_{6,4}^{15} + 34x_{6,4}^{16} + 34x_{6,4}^{19} + 34x_{6,4}^{20} + 34x_{6,4}^{22} + 34x_{6,4}^{23}$$
$$+32x_{4,16}^{3} + 32x_{4,16}^{5} + 32x_{4,16}^{6} + 32x_{4,16}^{7} + 32x_{4,16}^{8} + 32x_{4,16}^{9} + 32x_{4,16}^{10} + 32x_{4,16}^{12}$$
$$+32x_{4,16}^{13} + 32x_{4,16}^{15} + 32x_{4,16}^{16} + 32x_{4,16}^{19} + 32x_{4,16}^{20} + 32x_{4,16}^{22} + 32x_{4,16}^{23}$$
$$+32x_{16,4}^{3} + 32x_{16,4}^{5} + 32x_{16,4}^{6} + 32x_{16,4}^{7} + 32x_{16,4}^{8} + 32x_{16,4}^{9} + 32x_{16,4}^{10} + 32x_{16,4}^{12}$$
$$+32x_{16,4}^{13} + 32x_{16,4}^{15} + 32x_{16,4}^{16} + 32x_{16,4}^{19} + 32x_{16,4}^{20} + 32x_{16,4}^{22} + 32x_{16,4}^{23}$$
$$+21x_{5,15}^{3} + 21x_{5,15}^{5} + 21x_{5,15}^{6} + 21x_{5,15}^{7} + 21x_{5,15}^{8} + 21x_{5,15}^{9} + 21x_{5,15}^{10} + 21x_{5,15}^{12}$$
$$+21x_{5,15}^{13} + 21x_{5,15}^{15} + 21x_{5,15}^{16} + 21x_{5,15}^{19} + 21x_{5,15}^{20} + 21x_{5,15}^{22} + 21x_{5,15}^{23}$$
$$+21x_{15,5}^{3} + 21x_{15,5}^{5} + 21x_{15,5}^{6} + 21x_{15,5}^{7} + 21x_{15,5}^{8} + 21x_{15,5}^{9} + 21x_{15,5}^{10} + 21x_{15,5}^{12}$$
$$+21x_{15,5}^{13} + 21x_{15,5}^{15} + 21x_{15,5}^{16} + 21x_{15,5}^{19} + 21x_{15,5}^{20} + 21x_{15,5}^{22} + 21x_{15,5}^{23}$$
$$+35x_{6,8}^{3} + 35x_{6,8}^{5} + 35x_{6,8}^{6} + 35x_{6,8}^{7} + 35x_{6,8}^{8} + 35x_{6,8}^{9} + 35x_{6,8}^{10} + 35x_{6,8}^{12}$$
$$+35x_{6,8}^{13} + 35x_{6,8}^{15} + 35x_{6,8}^{16} + 35x_{6,8}^{19} + 35x_{6,8}^{20} + 35x_{6,8}^{22} + 35x_{6,8}^{23}$$
$$+35x_{8,6}^{3} + 35x_{8,6}^{5} + 35x_{8,6}^{6} + 35x_{8,6}^{7} + 35x_{8,6}^{8} + 35x_{8,6}^{9} + 35x_{8,6}^{10} + 35x_{8,6}^{12}$$
$$+35x_{8,6}^{13} + 35x_{8,6}^{15} + 35x_{8,6}^{16} + 35x_{8,6}^{19} + 35x_{8,6}^{20} + 35x_{8,6}^{22} + 35x_{8,6}^{23}$$

$$+14x^{3}_{6,17} + 14x^{5}_{6,17} + 14x^{6}_{6,17} + 14x^{7}_{6,17} + 14x^{8}_{6,17} + 14x^{9}_{6,17} + 14x^{10}_{6,17} + 14x^{12}_{6,17}$$
$$+14x^{13}_{6,17} + 14x^{15}_{6,17} + 14x^{16}_{6,17} + 14x^{19}_{6,17} + 14x^{20}_{6,17} + 14x^{22}_{6,17} + 14x^{23}_{6,17}$$
$$+14x^{3}_{17,6} + 14x^{5}_{17,6} + 14x^{6}_{17,6} + 14x^{7}_{17,6} + 14x^{8}_{17,6} + 14x^{9}_{17,6} + 14x^{10}_{17,6} + 14x^{12}_{17,6}$$
$$+14x^{13}_{17,6} + 14x^{15}_{17,6} + 14x^{16}_{17,6} + 14x^{19}_{17,6} + 14x^{20}_{17,6} + 14x^{22}_{17,6} + 14x^{23}_{17,6}$$
$$+43x^{3}_{7,11} + 43x^{5}_{7,11} + 43x^{6}_{7,11} + 43x^{7}_{7,11} + 43x^{8}_{7,11} + 43x^{9}_{7,11} + 43x^{10}_{7,11} + 43x^{12}_{7,11}$$
$$+43x^{13}_{7,11} + 43x^{15}_{7,11} + 43x^{16}_{7,11} + 43x^{19}_{7,11} + 43x^{20}_{7,11} + 43x^{22}_{7,11} + 43x^{23}_{7,11}$$
$$+43x^{3}_{11,7} + 43x^{5}_{11,7} + 43x^{6}_{11,7} + 43x^{7}_{11,7} + 43x^{8}_{11,7} + 43x^{9}_{11,7} + 43x^{10}_{11,7} + 43x^{12}_{11,7}$$
$$+43x^{13}_{11,7} + 43x^{15}_{11,7} + 43x^{16}_{11,7} + 43x^{19}_{11,7} + 43x^{20}_{11,7} + 43x^{22}_{11,7} + 43x^{23}_{11,7}$$
$$+23x^{3}_{7,13} + 23x^{5}_{7,13} + 23x^{6}_{7,13} + 23x^{7}_{7,13} + 23x^{8}_{7,13} + 23x^{9}_{7,13} + 23x^{10}_{7,13} + 23x^{12}_{7,13}$$
$$+23x^{13}_{7,13} + 23x^{15}_{7,13} + 23x^{16}_{7,13} + 23x^{19}_{7,13} + 23x^{20}_{7,13} + 23x^{22}_{7,13} + 23x^{23}_{7,13}$$
$$+23x^{3}_{13,7} + 23x^{5}_{13,7} + 23x^{6}_{13,7} + 23x^{7}_{13,7} + 23x^{8}_{13,7} + 23x^{9}_{13,7} + 23x^{10}_{13,7} + 23x^{12}_{13,7}$$
$$+23x^{13}_{13,7} + 23x^{15}_{13,7} + 23x^{16}_{13,7} + 23x^{19}_{13,7} + 23x^{20}_{13,7} + 23x^{22}_{13,7} + 23x^{23}_{13,7}$$
$$+36x^{3}_{8,10} + 36x^{5}_{8,10} + 36x^{6}_{8,10} + 36x^{7}_{8,10} + 36x^{8}_{8,10} + 36x^{9}_{8,10} + 36x^{10}_{8,10} + 36x^{12}_{8,10}$$
$$+36x^{13}_{8,10} + 36x^{15}_{8,10} + 36x^{16}_{8,10} + 36x^{19}_{8,10} + 36x^{20}_{8,10} + 36x^{22}_{8,10} + 36x^{23}_{8,10}$$
$$+36x^{3}_{10,8} + 36x^{5}_{10,8} + 36x^{6}_{10,8} + 36x^{7}_{10,8} + 36x^{8}_{10,8} + 36x^{9}_{10,8} + 36x^{10}_{10,8} + 36x^{12}_{10,8}$$
$$+36x^{13}_{10,8} + 36x^{15}_{10,8} + 36x^{16}_{10,8} + 36x^{19}_{10,8} + 36x^{20}_{10,8} + 36x^{22}_{10,8} + 36x^{23}_{10,8}$$
$$+38x^{3}_{9,10} + 38x^{5}_{9,10} + 38x^{6}_{9,10} + 38x^{7}_{9,10} + 38x^{8}_{9,10} + 38x^{9}_{9,10} + 38x^{10}_{9,10} + 38x^{12}_{9,10}$$
$$+38x^{13}_{9,10} + 38x^{15}_{9,10} + 38x^{16}_{9,10} + 38x^{19}_{9,10} + 38x^{20}_{9,10} + 38x^{22}_{9,10} + 38x^{23}_{9,10}$$
$$+38x^{3}_{10,9} + 38x^{5}_{10,9} + 38x^{6}_{10,9} + 38x^{7}_{10,9} + 38x^{8}_{10,9} + 38x^{9}_{10,9} + 38x^{10}_{10,9} + 38x^{12}_{10,9}$$
$$+38x^{13}_{10,9} + 38x^{15}_{10,9} + 38x^{16}_{10,9} + 38x^{19}_{10,9} + 38x^{20}_{10,9} + 38x^{22}_{10,9} + 38x^{23}_{10,9}$$
$$+15x^{3}_{9,12} + 15x^{5}_{9,12} + 15x^{6}_{9,12} + 15x^{7}_{9,12} + 15x^{8}_{9,12} + 15x^{9}_{9,12} + 15x^{10}_{9,12} + 15x^{12}_{9,12}$$
$$+15x^{13}_{9,12} + 15x^{15}_{9,12} + 15x^{16}_{9,12} + 15x^{19}_{9,12} + 15x^{20}_{9,12} + 15x^{22}_{9,12} + 15x^{23}_{9,12}$$
$$+15x^{3}_{12,9} + 15x^{5}_{12,9} + 15x^{6}_{12,9} + 15x^{7}_{12,9} + 15x^{8}_{12,9} + 15x^{9}_{12,9} + 15x^{10}_{12,9} + 15x^{12}_{12,9}$$
$$+15x^{13}_{12,9} + 15x^{15}_{12,9} + 15x^{16}_{12,9} + 15x^{19}_{12,9} + 15x^{20}_{12,9} + 15x^{22}_{12,9} + 15x^{23}_{12,9}$$
$$+16x^{3}_{10,18} + 16x^{5}_{10,18} + 16x^{6}_{10,18} + 16x^{7}_{10,18} + 16x^{8}_{10,18} + 16x^{9}_{10,18} + 16x^{10}_{10,18} + 16x^{12}_{10,18}$$
$$+16x^{13}_{10,18} + 16x^{15}_{10,18} + 16x^{16}_{10,18} + 16x^{19}_{10,18} + 16x^{20}_{10,18} + 16x^{22}_{10,18} + 16x^{23}_{10,18}$$
$$+16x^{3}_{18,10} + 16x^{5}_{18,10} + 16x^{6}_{18,10} + 16x^{7}_{18,10} + 16x^{8}_{18,10} + 16x^{9}_{18,10} + 16x^{10}_{18,10} + 16x^{12}_{18,10}$$
$$+16x^{13}_{18,10} + 16x^{15}_{18,10} + 16x^{16}_{18,10} + 16x^{19}_{18,10} + 16x^{20}_{18,10} + 16x^{22}_{18,10} + 16x^{23}_{18,10}$$
$$+41x^{3}_{11,12} + 41x^{5}_{11,12} + 41x^{6}_{11,12} + 41x^{7}_{11,12} + 41x^{8}_{11,12} + 41x^{9}_{11,12} + 41x^{10}_{11,12} + 41x^{12}_{11,12}$$
$$+41x^{13}_{11,12} + 41x^{15}_{11,12} + 41x^{16}_{11,12} + 41x^{19}_{11,12} + 41x^{20}_{11,12} + 41x^{22}_{11,12} + 41x^{23}_{11,12}$$
$$+41x^{3}_{12,11} + 41x^{5}_{12,11} + 41x^{6}_{12,11} + 41x^{7}_{12,11} + 41x^{8}_{12,11} + 41x^{9}_{12,11} + 41x^{10}_{12,11} + 41x^{12}_{12,11}$$
$$+41x^{13}_{12,11} + 41x^{15}_{12,11} + 41x^{16}_{12,11} + 41x^{19}_{12,11} + 41x^{20}_{12,11} + 41x^{22}_{12,11} + 41x^{23}_{12,11}$$
$$+28x^{3}_{11,23} + 28x^{5}_{11,23} + 28x^{6}_{11,23} + 28x^{7}_{11,23} + 28x^{8}_{11,23} + 28x^{9}_{11,23} + 28x^{10}_{11,23} + 28x^{12}_{11,23}$$
$$+28x^{13}_{11,23} + 28x^{15}_{11,23} + 28x^{16}_{11,23} + 28x^{19}_{11,23} + 28x^{20}_{11,23} + 28x^{22}_{11,23} + 28x^{23}_{11,23}$$
$$+28x^{3}_{23,11} + 28x^{5}_{23,11} + 28x^{6}_{23,11} + 28x^{7}_{23,11} + 28x^{8}_{23,11} + 28x^{9}_{23,11} + 28x^{10}_{23,11} + 28x^{12}_{23,11}$$
$$+28x^{13}_{23,11} + 28x^{15}_{23,11} + 28x^{16}_{23,11} + 28x^{19}_{23,11} + 28x^{20}_{23,11} + 28x^{22}_{23,11} + 28x^{23}_{23,11}$$
$$+39x^{3}_{12,19} + 39x^{5}_{12,19} + 39x^{6}_{12,19} + 39x^{7}_{12,19} + 39x^{8}_{12,19} + 39x^{9}_{12,19} + 39x^{10}_{12,19} + 39x^{12}_{12,19}$$
$$+39x^{13}_{12,19} + 39x^{15}_{12,19} + 39x^{16}_{12,19} + 39x^{19}_{12,19} + 39x^{20}_{12,19} + 39x^{22}_{12,19} + 39x^{23}_{12,19}$$
$$+39x^{3}_{19,12} + 39x^{5}_{19,12} + 39x^{6}_{19,12} + 39x^{7}_{19,12} + 39x^{8}_{19,12} + 39x^{9}_{19,12} + 39x^{10}_{19,12} + 39x^{12}_{19,12}$$
$$+39x^{13}_{19,12} + 39x^{15}_{19,12} + 39x^{16}_{19,12} + 39x^{19}_{19,12} + 39x^{20}_{19,12} + 39x^{22}_{19,12} + 39x^{23}_{19,12}$$
$$+22x^{3}_{13,14} + 22x^{5}_{13,14} + 22x^{6}_{13,14} + 22x^{7}_{13,14} + 22x^{8}_{13,14} + 22x^{9}_{13,14} + 22x^{10}_{13,14} + 22x^{12}_{13,14}$$
$$+22x^{13}_{13,14} + 22x^{15}_{13,14} + 22x^{16}_{13,14} + 22x^{19}_{13,14} + 22x^{20}_{13,14} + 22x^{22}_{13,14} + 22x^{23}_{13,14}$$
$$+22x^{3}_{14,13} + 22x^{5}_{14,13} + 22x^{6}_{14,13} + 22x^{7}_{14,13} + 22x^{8}_{14,13} + 22x^{9}_{14,13} + 22x^{10}_{14,13} + 22x^{12}_{14,13}$$
$$+22x^{13}_{14,13} + 22x^{15}_{14,13} + 22x^{16}_{14,13} + 22x^{19}_{14,13} + 22x^{20}_{14,13} + 22x^{22}_{14,13} + 22x^{23}_{14,13}$$
$$+18x^{3}_{13,22} + 18x^{5}_{13,22} + 18x^{6}_{13,22} + 18x^{7}_{13,22} + 18x^{8}_{13,22} + 18x^{9}_{13,22} + 18x^{10}_{13,22} + 18x^{12}_{13,22}$$
$$+18x^{13}_{13,22} + 18x^{15}_{13,22} + 18x^{16}_{13,22} + 18x^{19}_{13,22} + 18x^{20}_{13,22} + 18x^{22}_{13,22} + 18x^{23}_{13,22}$$
$$18x^{3}_{22,13} + 18x^{5}_{22,13} + 18x^{6}_{22,13} + 18x^{7}_{22,13} + 18x^{8}_{22,13} + 18x^{9}_{22,13} + 18x^{10}_{22,13} + 18x^{12}_{22,13}$$
$$+18x^{13}_{22,13} + 18x^{15}_{22,13} + 18x^{16}_{22,13} + 18x^{19}_{22,13} + 18x^{20}_{22,13} + 18x^{22}_{22,13} + 18x^{23}_{22,13}$$
$$+13x^{3}_{14,15} + 13x^{5}_{14,15} + 13x^{6}_{14,15} + 13x^{7}_{14,15} + 13x^{8}_{14,15} + 13x^{9}_{14,15} + 13x^{10}_{14,15} + 13x^{12}_{14,15}$$

$$+13x^{13}_{14,15} + 13x^{15}_{14,15} + 13x^{16}_{14,15} + 13x^{19}_{14,15} + 13x^{20}_{14,15} + 13x^{22}_{14,15} + 13x^{23}_{14,15}$$
$$+13x^{3}_{15,14} + 13x^{5}_{15,14} + 13x^{6}_{15,14} + 13x^{7}_{15,14} + 13x^{8}_{15,14} + 13x^{9}_{15,14} + 13x^{10}_{15,14} + 13x^{12}_{15,14}$$
$$+13x^{13}_{15,14} + 13x^{15}_{15,14} + 13x^{16}_{15,14} + 13x^{19}_{15,14} + 13x^{20}_{15,14} + 13x^{22}_{15,14} + 13x^{23}_{15,14}$$
$$+24x^{3}_{15,16} + 24x^{5}_{15,16} + 24x^{6}_{15,16} + 24x^{7}_{15,16} + 24x^{8}_{15,16} + 24x^{9}_{15,16} + 24x^{10}_{15,16} + 24x^{12}_{15,16}$$
$$+24x^{13}_{15,16} + 24x^{15}_{15,16} + 24x^{16}_{15,16} + 24x^{19}_{15,16} + 24x^{20}_{15,16} + 24x^{22}_{15,16} + 24x^{23}_{15,16}$$
$$+24x^{3}_{16,15} + 24x^{5}_{16,15} + 24x^{6}_{16,15} + 24x^{7}_{16,15} + 24x^{8}_{16,15} + 24x^{9}_{16,15} + 24x^{10}_{16,15} + 24x^{12}_{16,15}$$
$$+24x^{13}_{16,15} + 24x^{15}_{16,15} + 24x^{16}_{16,15} + 24x^{19}_{16,15} + 24x^{20}_{16,15} + 24x^{22}_{16,15} + 24x^{23}_{16,15}$$
$$+26x^{3}_{16,17} + 26x^{5}_{16,17} + 26x^{6}_{16,17} + 26x^{7}_{16,17} + 26x^{8}_{16,17} + 26x^{9}_{16,17} + 26x^{10}_{16,17} + 26x^{12}_{16,17}$$
$$+26x^{13}_{16,17} + 26x^{15}_{16,17} + 26x^{16}_{16,17} + 26x^{19}_{16,17} + 26x^{20}_{16,17} + 26x^{22}_{16,17} + 26x^{23}_{16,17}$$
$$+26x^{3}_{17,16} + 26x^{5}_{17,16} + 26x^{6}_{17,16} + 26x^{7}_{17,16} + 26x^{8}_{17,16} + 26x^{9}_{17,16} + 26x^{10}_{17,16} + 26x^{12}_{17,16}$$
$$+26x^{13}_{17,16} + 26x^{15}_{17,16} + 26x^{16}_{17,16} + 26x^{19}_{17,16} + 26x^{20}_{17,16} + 26x^{22}_{17,16} + 26x^{23}_{17,16}$$
$$+17x^{3}_{18,19} + 17x^{5}_{18,19} + 17x^{6}_{18,19} + 17x^{7}_{18,19} + 17x^{8}_{18,19} + 17x^{9}_{18,19} + 17x^{10}_{18,19} + 17x^{12}_{18,19}$$
$$+17x^{13}_{18,19} + 17x^{15}_{18,19} + 17x^{16}_{18,19} + 17x^{19}_{18,19} + 17x^{20}_{18,19} + 17x^{22}_{18,19} + 17x^{23}_{18,19}$$
$$+17x^{3}_{19,18} + 17x^{5}_{19,18} + 17x^{6}_{19,18} + 17x^{7}_{19,18} + 17x^{8}_{19,18} + 17x^{9}_{19,18} + 17x^{10}_{19,18} + 17x^{12}_{19,18}$$
$$+17x^{13}_{19,18} + 17x^{15}_{19,18} + 17x^{16}_{19,18} + 17x^{19}_{19,18} + 17x^{20}_{19,18} + 17x^{22}_{19,18} + 17x^{23}_{19,18}$$
$$+25x^{3}_{19,20} + 25x^{5}_{19,20} + 25x^{6}_{19,20} + 25x^{7}_{19,20} + 25x^{8}_{19,20} + 25x^{9}_{19,20} + 25x^{10}_{19,20} + 25x^{12}_{19,20}$$
$$+25x^{13}_{19,20} + 25x^{15}_{19,20} + 25x^{16}_{19,20} + 25x^{19}_{19,20} + 25x^{20}_{19,20} + 25x^{22}_{19,20} + 25x^{23}_{19,20}$$
$$+25x^{3}_{20,19} + 25x^{5}_{20,19} + 25x^{6}_{20,19} + 25x^{7}_{20,19} + 25x^{8}_{20,19} + 25x^{9}_{20,19} + 25x^{10}_{20,19} + 25x^{12}_{20,19}$$
$$+25x^{13}_{20,19} + 25x^{15}_{20,19} + 25x^{16}_{20,19} + 25x^{19}_{20,19} + 25x^{20}_{20,19} + 25x^{22}_{20,19} + 25x^{23}_{20,19}$$
$$+27x^{3}_{20,21} + 27x^{5}_{20,21} + 27x^{6}_{20,21} + 27x^{7}_{20,21} + 27x^{8}_{20,21} + 27x^{9}_{20,21} + 27x^{10}_{20,21} + 27x^{12}_{20,21}$$
$$+27x^{13}_{20,21} + 27x^{15}_{20,21} + 27x^{16}_{20,21} + 27x^{19}_{20,21} + 27x^{20}_{20,21} + 27x^{22}_{20,21} + 27x^{23}_{20,21}$$
$$+27x^{3}_{21,20} + 27x^{5}_{21,20} + 27x^{6}_{21,20} + 27x^{7}_{21,20} + 27x^{8}_{21,20} + 27x^{9}_{21,20} + 27x^{10}_{21,20} + 27x^{12}_{21,20}$$
$$+27x^{13}_{21,20} + 27x^{15}_{21,20} + 27x^{16}_{21,20} + 27x^{19}_{21,20} + 27x^{20}_{21,20} + 27x^{22}_{21,20} + 27x^{23}_{21,20}$$
$$+40x^{3}_{20,23} + 40x^{5}_{20,23} + 40x^{6}_{20,23} + 40x^{7}_{20,23} + 40x^{8}_{20,23} + 40x^{9}_{20,23} + 40x^{10}_{20,23} + 40x^{12}_{20,23}$$
$$+40x^{13}_{20,23} + 40x^{15}_{20,23} + 40x^{16}_{20,23} + 40x^{19}_{20,23} + 40x^{20}_{20,23} + 40x^{22}_{20,23} + 40x^{23}_{20,23}$$
$$+40x^{3}_{23,20} + 40x^{5}_{23,20} + 40x^{6}_{23,20} + 40x^{7}_{23,20} + 40x^{8}_{23,20} + 40x^{9}_{23,20} + 40x^{10}_{23,20} + 40x^{12}_{23,20}$$
$$+40x^{13}_{23,20} + 40x^{15}_{23,20} + 40x^{16}_{23,20} + 40x^{19}_{23,20} + 40x^{20}_{23,20} + 40x^{22}_{23,20} + 40x^{23}_{23,20}$$
$$+12x^{3}_{21,22} + 12x^{5}_{21,22} + 12x^{6}_{21,22} + 12x^{7}_{21,22} + 12x^{8}_{21,22} + 12x^{9}_{21,22} + 12x^{10}_{21,22} + 12x^{12}_{21,22}$$
$$+12x^{13}_{21,22} + 12x^{15}_{21,22} + 12x^{16}_{21,22} + 12x^{19}_{21,22} + 12x^{20}_{21,22} + 12x^{22}_{21,22} + 12x^{23}_{21,22}$$
$$+12x^{3}_{22,21} + 12x^{5}_{22,21} + 12x^{6}_{22,21} + 12x^{7}_{22,21} + 12x^{8}_{22,21} + 12x^{9}_{22,21} + 12x^{10}_{22,21} + 12x^{12}_{22,21}$$
$$+12x^{13}_{22,21} + 12x^{15}_{22,21} + 12x^{16}_{22,21} + 12x^{19}_{22,21} + 12x^{20}_{22,21} + 12x^{22}_{22,21} + 12x^{23}_{22,21}$$
$$+42x^{3}_{22,23} + 42x^{5}_{22,23} + 42x^{6}_{22,23} + 42x^{7}_{22,23} + 42x^{8}_{22,23} + 42x^{9}_{22,23} + 42x^{10}_{22,23} + 42x^{12}_{22,23}$$
$$+42x^{13}_{22,23} + 42x^{15}_{22,23} + 42x^{16}_{22,23} + 42x^{19}_{22,23} + 42x^{20}_{22,23} + 42x^{22}_{22,23} + 42x^{23}_{22,23}$$
$$+42x^{3}_{23,22} + 42x^{5}_{23,22} + 42x^{6}_{23,22} + 42x^{7}_{23,22} + 42x^{8}_{23,22} + 42x^{9}_{23,22} + 42x^{10}_{23,22} + 42x^{12}_{23,22}$$
$$+42x^{13}_{23,22} + 42x^{15}_{23,22} + 42x^{16}_{23,22} + 42x^{19}_{23,22} + 42x^{20}_{23,22} + 42x^{22}_{23,22} + 42x^{23}_{23,22}\}$$

s.t.

$$x^{3}_{1,2} + x^{3}_{1,3} + x^{3}_{1,5} + x^{3}_{1,7} - x^{3}_{2,1} - x^{3}_{3,1} - x^{3}_{5,1} - x^{3}_{7,1} = 0$$
$$x^{3}_{2,1} + x^{3}_{2,8} + x^{3}_{2,9} - x^{3}_{1,2} - x^{3}_{8,2} - x^{3}_{9,2} = -1$$
$$x^{3}_{3,1} + x^{3}_{3,4} + x^{3}_{3,11} - x^{3}_{1,3} - x^{3}_{4,3} - x^{3}_{11,3} = 15$$
$$x^{3}_{4,3} + x^{3}_{4,5} + x^{3}_{4,6} + x^{3}_{4,16} - x^{3}_{3,4} - x^{3}_{5,4} - x^{3}_{6,4} - x^{3}_{16,4} = 0$$
$$x^{3}_{5,1} + x^{3}_{5,4} + x^{3}_{5,15} - x^{3}_{1,5} - x^{3}_{4,5} - x^{3}_{15,5} = -1$$
$$x^{3}_{6,4} + x^{3}_{6,8} + x^{3}_{6,17} - x^{3}_{4,6} - x^{3}_{8,6} - x^{3}_{17,6} = -1$$
$$x^{3}_{7,1} + x^{3}_{7,11} + x^{3}_{7,13} - x^{3}_{1,7} - x^{3}_{11,7} - x^{3}_{13,7} = -1$$
$$x^{3}_{8,2} + x^{3}_{8,6} + x^{3}_{8,10} - x^{3}_{2,8} - x^{3}_{6,8} - x^{3}_{10,8} = -1$$
$$x^{3}_{9,2} + x^{3}_{9,10} + x^{3}_{9,12} - x^{3}_{2,9} - x^{3}_{10,9} - x^{3}_{12,9} = -1$$
$$x^{3}_{10,8} + x^{3}_{10,9} + x^{3}_{10,18} - x^{3}_{8,10} - x^{3}_{9,10} - x^{3}_{18,10} = -1$$
$$x^{3}_{11,3} + x^{3}_{11,7} + x^{3}_{11,12} + x^{3}_{11,23} - x^{3}_{3,11} - x^{3}_{7,11} - x^{3}_{12,11} - x^{3}_{23,11} = 0$$
$$x^{3}_{12,9} + x^{3}_{12,11} + x^{3}_{12,19} - x^{3}_{9,12} - x^{3}_{11,12} - x^{3}_{19,12} = -1$$
$$x^{3}_{13,7} + x^{3}_{13,14} + x^{3}_{13,22} - x^{3}_{7,13} - x^{3}_{14,13} - x^{3}_{22,13} = -1$$
$$x^{3}_{14,13} + x^{3}_{14,15} - x^{3}_{13,14} - x^{3}_{15,14} = 0$$

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\begin{aligned}
x^3_{15,5} + x^3_{15,14} + x^3_{15,16} - x^3_{5,15} - x^3_{14,15} - x^3_{16,15} &= -1\\
x^3_{16,15} + x^3_{16,17} - x^3_{15,16} - x^3_{17,16} &= -1\\
x^3_{17,6} + x^3_{17,16} - x^3_{6,17} - x^3_{16,17} &= 0\\
x^3_{18,10} + x^3_{18,19} - x^3_{10,18} - x^3_{19,18} &= 0\\
x^3_{19,12} + x^3_{19,18} + x^3_{19,20} - x^3_{12,19} - x^3_{18,19} - x^3_{20,19} &= -1\\
x^3_{20,19} + x^3_{20,21} + x^3_{20,23} - x^3_{19,20} - x^3_{21,20} - x^3_{23,20} &= -1\\
x^3_{21,20} + x^3_{21,22} - x^3_{20,21} - x^3_{22,21} &= 0\\
x^3_{22,21} + x^3_{22,23} - x^3_{21,22} - x^3_{23,22} &= -1\\
x^3_{23,11} + x^3_{23,20} + x^3_{23,22} - x^3_{11,23} - x^3_{20,23} - x^3_{22,23} &= -1\\
x^5_{1,2} + x^5_{1,3} + x^5_{1,5} + x^5_{1,7} - x^5_{2,1} - x^5_{3,1} - x^5_{5,1} - x^5_{7,1} &= 0\\
x^5_{2,1} + x^5_{2,8} + x^5_{2,9} - x^5_{1,2} - x^5_{8,2} - x^5_{9,2} &= -1\\
x^5_{3,1} + x^5_{3,4} + x^5_{3,11} - x^5_{1,3} - x^5_{4,3} - x^5_{11,3} &= -1\\
x^5_{4,3} + x^5_{4,5} + x^5_{4,6} + x^5_{4,16} - x^5_{3,4} - x^5_{5,4} - x^5_{6,4} - x^5_{16,4} &= 0\\
x^5_{5,1} + x^5_{5,4} + x^5_{5,15} - x^5_{1,5} - x^5_{4,5} - x^5_{15,5} &= 15\\
x^5_{6,4} + x^5_{6,8} + x^5_{6,17} - x^5_{4,6} - x^5_{8,6} - x^5_{17,6} &= -1\\
x^5_{7,1} + x^5_{7,11} + x^5_{7,13} - x^5_{1,7} - x^5_{11,7} - x^5_{13,7} &= -1\\
x^5_{8,2} + x^5_{8,6} + x^5_{8,10} - x^5_{2,8} - x^5_{6,8} - x^5_{10,8} &= -1\\
x^5_{9,2} + x^5_{9,10} + x^5_{9,12} - x^5_{2,9} - x^5_{10,9} - x^5_{12,9} &= -1\\
x^5_{10,8} + x^5_{10,9} + x^5_{10,18} - x^5_{8,10} - x^5_{9,10} - x^5_{18,10} &= -1\\
x^5_{11,3} + x^5_{11,7} + x^5_{11,12} + x^5_{11,23} - x^5_{3,11} - x^5_{7,11} - x^5_{12,11} - x^5_{23,11} &= 0\\
x^5_{12,9} + x^5_{12,11} + x^5_{12,19} - x^5_{9,12} - x^5_{11,12} - x^5_{19,12} &= -1\\
x^5_{13,7} + x^5_{13,14} + x^5_{13,22} - x^5_{7,13} - x^5_{14,13} - x^5_{22,13} &= -1\\
x^5_{14,13} + x^5_{14,15} - x^5_{13,14} - x^5_{15,14} &= 0\\
x^5_{15,5} + x^5_{15,14} + x^5_{15,16} - x^5_{5,15} - x^5_{14,15} - x^5_{16,15} &= -1\\
x^5_{16,15} + x^5_{16,17} - x^5_{15,16} - x^5_{17,16} &= -1\\
x^5_{17,6} + x^5_{17,16} - x^5_{6,17} - x^5_{16,17} &= 0\\
x^5_{18,10} + x^5_{18,19} - x^5_{10,18} - x^5_{19,18} &= 0\\
x^5_{19,12} + x^5_{19,18} + x^5_{19,20} - x^5_{12,19} - x^5_{18,19} - x^5_{20,19} &= -1\\
x^5_{20,19} + x^5_{20,21} + x^5_{20,23} - x^5_{19,20} - x^5_{21,20} - x^5_{23,20} &= -1\\
x^5_{21,20} + x^5_{21,22} - x^5_{20,21} - x^5_{22,21} &= 0\\
x^5_{22,21} + x^5_{22,23} - x^5_{21,22} - x^5_{23,22} &= -1\\
x^5_{23,11} + x^5_{23,20} + x^5_{23,22} - x^5_{11,23} - x^5_{20,23} - x^5_{22,23} &= -1\\
x^6_{1,2} + x^6_{1,3} + x^6_{1,5} + x^6_{1,7} - x^6_{2,1} - x^6_{3,1} - x^6_{5,1} - x^6_{7,1} &= 0\\
x^6_{2,1} + x^6_{2,8} + x^6_{2,9} - x^6_{1,2} - x^6_{8,2} - x^6_{9,2} &= -1\\
x^6_{3,1} + x^6_{3,4} + x^6_{3,11} - x^6_{1,3} - x^6_{4,3} - x^6_{11,3} &= -1\\
x^6_{4,3} + x^6_{4,5} + x^6_{4,6} + x^6_{4,16} - x^6_{3,4} - x^6_{5,4} - x^6_{6,4} - x^6_{16,4} &= 0\\
x^6_{5,1} + x^6_{5,4} + x^6_{5,15} - x^6_{1,5} - x^6_{4,5} - x^6_{15,5} &= -1\\
x^6_{6,4} + x^6_{6,8} + x^6_{6,17} - x^6_{4,6} - x^6_{8,6} - x^6_{17,6} &= 15\\
x^6_{7,1} + x^6_{7,11} + x^6_{7,13} - x^6_{1,7} - x^6_{11,7} - x^6_{13,7} &= -1\\
x^6_{8,2} + x^6_{8,6} + x^6_{8,10} - x^6_{2,8} - x^6_{6,8} - x^6_{10,8} &= -1\\
x^6_{9,2} + x^6_{9,10} + x^6_{9,12} - x^6_{2,9} - x^6_{10,9} - x^6_{12,9} &= -1\\
x^6_{10,8} + x^6_{10,9} + x^6_{10,18} - x^6_{8,10} - x^6_{9,10} - x^6_{18,10} &= -1\\
x^6_{11,3} + x^6_{11,7} + x^6_{11,12} + x^6_{11,23} - x^6_{3,11} - x^6_{7,11} - x^6_{12,11} - x^6_{23,11} &= 0\\
x^6_{12,9} + x^6_{12,11} + x^6_{12,19} - x^6_{9,12} - x^6_{11,12} - x^6_{19,12} &= -1\\
x^6_{13,7} + x^6_{13,14} + x^6_{13,22} - x^6_{7,13} - x^6_{14,13} - x^6_{22,13} &= -1\\
x^6_{14,13} + x^6_{14,15} - x^6_{13,14} - x^6_{15,14} &= 0\\
x^6_{15,5} + x^6_{15,14} + x^6_{15,16} - x^6_{5,15} - x^6_{14,15} - x^6_{16,15} &= -1\\
x^6_{16,15} + x^6_{16,17} - x^6_{15,16} - x^6_{17,16} &= -1\\
x^6_{17,6} + x^6_{17,16} - x^6_{6,17} - x^6_{16,17} &= 0
\end{aligned}
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\begin{aligned}
x^6_{18,10} + x^6_{18,19} - x^6_{10,18} - x^6_{19,18} &= 0\\
x^6_{19,12} + x^6_{19,18} + x^6_{19,20} - x^6_{12,19} - x^6_{18,19} - x^6_{20,19} &= -1\\
x^6_{20,19} + x^6_{20,21} + x^6_{20,23} - x^6_{19,20} - x^6_{21,20} - x^6_{23,20} &= -1\\
x^6_{21,20} + x^6_{21,22} - x^6_{20,21} - x^6_{22,21} &= 0\\
x^6_{22,21} + x^6_{22,23} - x^6_{21,22} - x^6_{23,22} &= -1\\
x^6_{23,11} + x^6_{23,20} + x^6_{23,22} - x^6_{11,23} - x^6_{20,23} - x^6_{22,23} &= -1\\
x^7_{1,2} + x^7_{1,3} + x^7_{1,5} + x^7_{1,7} - x^7_{2,1} - x^7_{3,1} - x^7_{5,1} - x^7_{7,1} &= 0\\
x^7_{2,1} + x^7_{2,8} + x^7_{2,9} - x^7_{1,2} - x^7_{8,2} - x^7_{9,2} &= -1\\
x^7_{3,1} + x^7_{3,4} + x^7_{3,11} - x^7_{1,3} - x^7_{4,3} - x^7_{11,3} &= -1\\
x^7_{4,3} + x^7_{4,5} + x^7_{4,6} + x^7_{4,16} - x^7_{3,4} - x^7_{5,4} - x^7_{6,4} - x^7_{16,4} &= 0\\
x^7_{5,1} + x^7_{5,4} + x^7_{5,15} - x^7_{1,5} - x^7_{4,5} - x^7_{15,5} &= -1\\
x^7_{6,4} + x^7_{6,8} + x^7_{6,17} - x^7_{4,6} - x^7_{8,6} - x^7_{17,6} &= -1\\
x^7_{7,1} + x^7_{7,11} + x^7_{7,13} - x^7_{1,7} - x^7_{11,7} - x^7_{13,7} &= 15\\
x^7_{8,2} + x^7_{8,6} + x^7_{8,10} - x^7_{2,8} - x^7_{6,8} - x^7_{10,8} &= -1\\
x^7_{9,2} + x^7_{9,10} + x^7_{9,12} - x^7_{2,9} - x^7_{10,9} - x^7_{12,9} &= -1\\
x^7_{10,8} + x^7_{10,9} + x^7_{10,18} - x^7_{8,10} - x^7_{9,10} - x^7_{18,10} &= -1\\
x^7_{11,3} + x^7_{11,7} + x^7_{11,12} + x^7_{11,23} - x^7_{3,11} - x^7_{7,11} - x^7_{12,11} - x^7_{23,11} &= 0\\
x^7_{12,9} + x^7_{12,11} + x^7_{12,19} - x^7_{9,12} - x^7_{11,12} - x^7_{19,12} &= -1\\
x^7_{13,7} + x^7_{13,14} + x^7_{13,22} - x^7_{7,13} - x^7_{14,13} - x^7_{22,13} &= -1\\
x^7_{14,13} + x^7_{14,15} - x^7_{13,14} - x^7_{15,14} &= 0\\
x^7_{15,5} + x^7_{15,14} + x^7_{15,16} - x^7_{5,15} - x^7_{14,15} - x^7_{16,15} &= -1\\
x^7_{16,15} + x^7_{16,17} - x^7_{15,16} - x^7_{17,16} &= -1\\
x^7_{17,6} + x^7_{17,16} - x^7_{6,17} - x^7_{16,17} &= 0\\
x^7_{18,10} + x^7_{18,19} - x^7_{10,18} - x^7_{19,18} &= 0\\
x^7_{19,12} + x^7_{19,18} + x^7_{19,20} - x^7_{12,19} - x^7_{18,19} - x^7_{20,19} &= -1\\
x^7_{20,19} + x^7_{20,21} + x^7_{20,23} - x^7_{19,20} - x^7_{21,20} - x^7_{23,20} &= -1\\
x^7_{21,20} + x^7_{21,22} - x^7_{20,21} - x^7_{22,21} &= 0\\
x^7_{22,21} + x^7_{22,23} - x^7_{21,22} - x^7_{23,22} &= -1\\
x^7_{23,11} + x^7_{23,20} + x^7_{23,22} - x^7_{11,23} - x^7_{20,23} - x^7_{22,23} &= -1\\
x^8_{1,2} + x^8_{1,3} + x^8_{1,5} + x^8_{1,7} - x^8_{2,1} - x^8_{3,1} - x^8_{5,1} - x^8_{7,1} &= 0\\
x^8_{2,1} + x^8_{2,8} + x^8_{2,9} - x^8_{1,2} - x^8_{8,2} - x^8_{9,2} &= -1\\
x^8_{3,1} + x^8_{3,4} + x^8_{3,11} - x^8_{1,3} - x^8_{4,3} - x^8_{11,3} &= -1\\
x^8_{4,3} + x^8_{4,5} + x^8_{4,6} + x^8_{4,16} - x^8_{3,4} - x^8_{5,4} - x^8_{6,4} - x^8_{16,4} &= 0\\
x^8_{5,1} + x^8_{5,4} + x^8_{5,15} - x^8_{1,5} - x^8_{4,5} - x^8_{15,5} &= -1\\
x^8_{6,4} + x^8_{6,8} + x^8_{6,17} - x^8_{4,6} - x^8_{8,6} - x^8_{17,6} &= -1\\
x^8_{7,1} + x^8_{7,11} + x^8_{7,13} - x^8_{1,7} - x^8_{11,7} - x^8_{13,7} &= -1\\
x^8_{8,2} + x^8_{8,6} + x^8_{8,10} - x^8_{2,8} - x^8_{6,8} - x^8_{10,8} &= 15\\
x^8_{9,2} + x^8_{9,10} + x^8_{9,12} - x^8_{2,9} - x^8_{10,9} - x^8_{12,9} &= -1\\
x^8_{10,8} + x^8_{10,9} + x^8_{10,18} - x^8_{8,10} - x^8_{9,10} - x^8_{18,10} &= -1\\
x^8_{11,3} + x^8_{11,7} + x^8_{11,12} + x^8_{11,23} - x^8_{3,11} - x^8_{7,11} - x^8_{12,11} - x^8_{23,11} &= 0\\
x^8_{12,9} + x^8_{12,11} + x^8_{12,19} - x^8_{9,12} - x^8_{11,12} - x^8_{19,12} &= -1\\
x^8_{13,7} + x^8_{13,14} + x^8_{13,22} - x^8_{7,13} - x^8_{14,13} - x^8_{22,13} &= -1\\
x^8_{14,13} + x^8_{14,15} - x^8_{13,14} - x^8_{15,14} &= 0\\
x^8_{15,5} + x^8_{15,14} + x^8_{15,16} - x^8_{5,15} - x^8_{14,15} - x^8_{16,15} &= -1\\
x^8_{16,15} + x^8_{16,17} - x^8_{15,16} - x^8_{17,16} &= -1\\
x^8_{17,6} + x^8_{17,16} - x^8_{6,17} - x^8_{16,17} &= 0\\
x^8_{18,10} + x^8_{18,19} - x^8_{10,18} - x^8_{19,18} &= 0\\
x^8_{19,12} + x^8_{19,18} + x^8_{19,20} - x^8_{12,19} - x^8_{18,19} - x^8_{20,19} &= -1\\
x^8_{20,19} + x^8_{20,21} + x^8_{20,23} - x^8_{19,20} - x^8_{21,20} - x^8_{23,20} &= -1
\end{aligned}
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\begin{aligned}
x^{8}_{21,20} + x^{8}_{21,22} - x^{8}_{20,21} - x^{8}_{22,21} &= 0\\
x^{8}_{22,21} + x^{8}_{22,23} - x^{8}_{21,22} - x^{8}_{23,22} &= -1\\
x^{8}_{23,11} + x^{8}_{23,20} + x^{8}_{23,22} - x^{8}_{11,23} - x^{8}_{20,23} - x^{8}_{22,23} &= -1\\
x^{9}_{1,2} + x^{9}_{1,3} + x^{9}_{1,5} + x^{9}_{1,7} - x^{9}_{2,1} - x^{9}_{3,1} - x^{9}_{5,1} - x^{9}_{7,1} &= 0\\
x^{9}_{2,1} + x^{9}_{2,8} + x^{9}_{2,9} - x^{9}_{1,2} - x^{9}_{8,2} - x^{9}_{9,2} &= -1\\
x^{9}_{3,1} + x^{9}_{3,4} + x^{9}_{3,11} - x^{9}_{1,3} - x^{9}_{4,3} - x^{9}_{11,3} &= -1\\
x^{9}_{4,3} + x^{9}_{4,5} + x^{9}_{4,6} + x^{9}_{4,16} - x^{9}_{3,4} - x^{9}_{5,4} - x^{9}_{6,4} - x^{9}_{16,4} &= 0\\
x^{9}_{5,1} + x^{9}_{5,4} + x^{9}_{5,15} - x^{9}_{1,5} - x^{9}_{4,5} - x^{9}_{15,5} &= -1\\
x^{9}_{6,4} + x^{9}_{6,8} + x^{9}_{6,17} - x^{9}_{4,6} - x^{9}_{8,6} - x^{9}_{17,6} &= -1\\
x^{9}_{7,1} + x^{9}_{7,11} + x^{9}_{7,13} - x^{9}_{1,7} - x^{9}_{11,7} - x^{9}_{13,7} &= -1\\
x^{9}_{8,2} + x^{9}_{8,6} + x^{9}_{8,10} - x^{9}_{2,8} - x^{9}_{6,8} - x^{9}_{10,8} &= -1\\
x^{9}_{9,2} + x^{9}_{9,10} + x^{9}_{9,12} - x^{9}_{2,9} - x^{9}_{10,9} - x^{9}_{12,9} &= 15\\
x^{9}_{10,8} + x^{9}_{10,9} + x^{9}_{10,18} - x^{9}_{8,10} - x^{9}_{9,10} - x^{9}_{18,10} &= -1\\
x^{9}_{11,3} + x^{9}_{11,7} + x^{9}_{11,12} + x^{9}_{11,23} - x^{9}_{3,11} - x^{9}_{7,11} - x^{9}_{12,11} - x^{9}_{23,11} &= 0\\
x^{9}_{12,9} + x^{9}_{12,11} + x^{9}_{12,19} - x^{9}_{9,12} - x^{9}_{11,12} - x^{9}_{19,12} &= -1\\
x^{9}_{13,7} + x^{9}_{13,14} + x^{9}_{13,22} - x^{9}_{7,13} - x^{9}_{14,13} - x^{9}_{22,13} &= -1\\
x^{9}_{14,13} + x^{9}_{14,15} - x^{9}_{13,14} - x^{9}_{15,14} &= 0\\
x^{9}_{15,5} + x^{9}_{15,14} + x^{9}_{15,16} - x^{9}_{5,15} - x^{9}_{14,15} - x^{9}_{16,15} &= -1\\
x^{9}_{16,15} + x^{9}_{16,17} - x^{9}_{15,16} - x^{9}_{17,16} &= -1\\
x^{9}_{17,6} + x^{9}_{17,16} - x^{9}_{6,17} - x^{9}_{16,17} &= 0\\
x^{9}_{18,10} + x^{9}_{18,19} - x^{9}_{10,18} - x^{9}_{19,18} &= 0\\
x^{9}_{19,12} + x^{9}_{19,18} + x^{9}_{19,20} - x^{9}_{12,19} - x^{9}_{18,19} - x^{9}_{20,19} &= -1\\
x^{9}_{20,19} + x^{9}_{20,21} + x^{9}_{20,23} - x^{9}_{19,20} - x^{9}_{21,20} - x^{9}_{23,20} &= -1\\
x^{9}_{21,20} + x^{9}_{21,22} - x^{9}_{20,21} - x^{9}_{22,21} &= 0\\
x^{9}_{22,21} + x^{9}_{22,23} - x^{9}_{21,22} - x^{9}_{23,22} &= -1\\
x^{9}_{23,11} + x^{9}_{23,20} + x^{9}_{23,22} - x^{9}_{11,23} - x^{9}_{20,23} - x^{9}_{22,23} &= -1\\
x^{10}_{1,2} + x^{10}_{1,3} + x^{10}_{1,5} + x^{10}_{1,7} - x^{10}_{2,1} - x^{10}_{3,1} - x^{10}_{5,1} - x^{10}_{7,1} &= 0\\
x^{10}_{2,1} + x^{10}_{2,8} + x^{10}_{2,9} - x^{10}_{1,2} - x^{10}_{8,2} - x^{10}_{9,2} &= -1\\
x^{10}_{3,1} + x^{10}_{3,4} + x^{10}_{3,11} - x^{10}_{1,3} - x^{10}_{4,3} - x^{10}_{11,3} &= -1\\
x^{10}_{4,3} + x^{10}_{4,5} + x^{10}_{4,6} + x^{10}_{4,16} - x^{10}_{3,4} - x^{10}_{5,4} - x^{10}_{6,4} - x^{10}_{16,4} &= 0\\
x^{10}_{5,1} + x^{10}_{5,4} + x^{10}_{5,15} - x^{10}_{1,5} - x^{10}_{4,5} - x^{10}_{15,5} &= -1\\
x^{10}_{6,4} + x^{10}_{6,8} + x^{10}_{6,17} - x^{10}_{4,6} - x^{10}_{8,6} - x^{10}_{17,6} &= -1\\
x^{10}_{7,1} + x^{10}_{7,11} + x^{10}_{7,13} - x^{10}_{1,7} - x^{10}_{11,7} - x^{10}_{13,7} &= -1\\
x^{10}_{8,2} + x^{10}_{8,6} + x^{10}_{8,10} - x^{10}_{2,8} - x^{10}_{6,8} - x^{10}_{10,8} &= -1\\
x^{10}_{9,2} + x^{10}_{9,10} + x^{10}_{9,12} - x^{10}_{2,9} - x^{10}_{10,9} - x^{10}_{12,9} &= -1\\
x^{10}_{10,8} + x^{10}_{10,9} + x^{10}_{10,18} - x^{10}_{8,10} - x^{10}_{9,10} - x^{10}_{18,10} &= 15\\
x^{10}_{11,3} + x^{10}_{11,7} + x^{10}_{11,12} + x^{10}_{11,23} - x^{10}_{3,11} - x^{10}_{7,11} - x^{10}_{12,11} - x^{10}_{23,11} &= 0\\
x^{10}_{12,9} + x^{10}_{12,11} + x^{10}_{12,19} - x^{10}_{9,12} - x^{10}_{11,12} - x^{10}_{19,12} &= -1\\
x^{10}_{13,7} + x^{10}_{13,14} + x^{10}_{13,22} - x^{10}_{7,13} - x^{10}_{14,13} - x^{10}_{22,13} &= -1\\
x^{10}_{14,13} + x^{10}_{14,15} - x^{10}_{13,14} - x^{10}_{15,14} &= 0\\
x^{10}_{15,5} + x^{10}_{15,14} + x^{10}_{15,16} - x^{10}_{5,15} - x^{10}_{14,15} - x^{10}_{16,15} &= -1\\
x^{10}_{16,15} + x^{10}_{16,17} - x^{10}_{15,16} - x^{10}_{17,16} &= -1\\
x^{10}_{17,6} + x^{10}_{17,16} - x^{10}_{6,17} - x^{10}_{16,17} &= 0\\
x^{10}_{18,10} + x^{10}_{18,19} - x^{10}_{10,18} - x^{10}_{19,18} &= 0\\
x^{10}_{19,12} + x^{10}_{19,18} + x^{10}_{19,20} - x^{10}_{12,19} - x^{10}_{18,19} - x^{10}_{20,19} &= -1\\
x^{10}_{20,19} + x^{10}_{20,21} + x^{10}_{20,23} - x^{10}_{19,20} - x^{10}_{21,20} - x^{10}_{23,20} &= -1\\
x^{10}_{21,20} + x^{10}_{21,22} - x^{10}_{20,21} - x^{10}_{22,21} &= 0\\
x^{10}_{22,21} + x^{10}_{22,23} - x^{10}_{21,22} - x^{10}_{23,22} &= -1\\
x^{10}_{23,11} + x^{10}_{23,20} + x^{10}_{23,22} - x^{10}_{11,23} - x^{10}_{20,23} - x^{10}_{22,23} &= -1
\end{aligned}
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\begin{aligned}
x^{12}_{1,2} + x^{12}_{1,3} + x^{12}_{1,5} + x^{12}_{1,7} - x^{12}_{2,1} - x^{12}_{3,1} - x^{12}_{5,1} - x^{12}_{7,1} &= 0\\
x^{12}_{2,1} + x^{12}_{2,8} + x^{12}_{2,9} - x^{12}_{1,2} - x^{12}_{8,2} - x^{12}_{9,2} &= -1\\
x^{12}_{3,1} + x^{12}_{3,4} + x^{12}_{3,11} - x^{12}_{1,3} - x^{12}_{4,3} - x^{12}_{11,3} &= -1\\
x^{12}_{4,3} + x^{12}_{4,5} + x^{12}_{4,6} + x^{12}_{4,16} - x^{12}_{3,4} - x^{12}_{5,4} - x^{12}_{6,4} - x^{12}_{16,4} &= 0\\
x^{12}_{5,1} + x^{12}_{5,4} + x^{12}_{5,15} - x^{12}_{1,5} - x^{12}_{4,5} - x^{12}_{15,5} &= -1\\
x^{12}_{6,4} + x^{12}_{6,8} + x^{12}_{6,17} - x^{12}_{4,6} - x^{12}_{8,6} - x^{12}_{17,6} &= -1\\
x^{12}_{7,1} + x^{12}_{7,11} + x^{12}_{7,13} - x^{12}_{1,7} - x^{12}_{11,7} - x^{12}_{13,7} &= -1\\
x^{12}_{8,2} + x^{12}_{8,6} + x^{12}_{8,10} - x^{12}_{2,8} - x^{12}_{6,8} - x^{12}_{10,8} &= -1\\
x^{12}_{9,2} + x^{12}_{9,10} + x^{12}_{9,12} - x^{12}_{2,9} - x^{12}_{10,9} - x^{12}_{12,9} &= -1\\
x^{12}_{10,8} + x^{12}_{10,9} + x^{12}_{10,18} - x^{12}_{8,10} - x^{12}_{9,10} - x^{12}_{18,10} &= -1\\
x^{12}_{11,3} + x^{12}_{11,7} + x^{12}_{11,12} + x^{12}_{11,23} - x^{12}_{3,11} - x^{12}_{7,11} - x^{12}_{12,11} - x^{12}_{23,11} &= 0\\
x^{12}_{12,9} + x^{12}_{12,11} + x^{12}_{12,19} - x^{12}_{9,12} - x^{12}_{11,12} - x^{12}_{19,12} &= 15\\
x^{12}_{13,7} + x^{12}_{13,14} + x^{12}_{13,22} - x^{12}_{7,13} - x^{12}_{14,13} - x^{12}_{22,13} &= -1\\
x^{12}_{14,13} + x^{12}_{14,15} - x^{12}_{13,14} - x^{12}_{15,14} &= 0\\
x^{12}_{15,5} + x^{12}_{15,14} + x^{12}_{15,16} - x^{12}_{5,15} - x^{12}_{14,15} - x^{12}_{16,15} &= -1\\
x^{12}_{16,15} + x^{12}_{16,17} - x^{12}_{15,16} - x^{12}_{17,16} &= -1\\
x^{12}_{17,6} + x^{12}_{17,16} - x^{12}_{6,17} - x^{12}_{16,17} &= 0\\
x^{12}_{18,10} + x^{12}_{18,19} - x^{12}_{10,18} - x^{12}_{19,18} &= 0\\
x^{12}_{19,12} + x^{12}_{19,18} + x^{12}_{19,20} - x^{12}_{12,19} - x^{12}_{18,19} - x^{12}_{20,19} &= -1\\
x^{12}_{20,19} + x^{12}_{20,21} + x^{12}_{20,23} - x^{12}_{19,20} - x^{12}_{21,20} - x^{12}_{23,20} &= -1\\
x^{12}_{21,20} + x^{12}_{21,22} - x^{12}_{20,21} - x^{12}_{22,21} &= 0\\
x^{12}_{22,21} + x^{12}_{22,23} - x^{12}_{21,22} - x^{12}_{23,22} &= -1\\
x^{12}_{23,11} + x^{12}_{23,20} + x^{12}_{23,22} - x^{12}_{11,23} - x^{12}_{20,23} - x^{12}_{22,23} &= -1\\
x^{13}_{1,2} + x^{13}_{1,3} + x^{13}_{1,5} + x^{13}_{1,7} - x^{13}_{2,1} - x^{13}_{3,1} - x^{13}_{5,1} - x^{13}_{7,1} &= 0\\
x^{13}_{2,1} + x^{13}_{2,8} + x^{13}_{2,9} - x^{13}_{1,2} - x^{13}_{8,2} - x^{13}_{9,2} &= -1\\
x^{13}_{3,1} + x^{13}_{3,4} + x^{13}_{3,11} - x^{13}_{1,3} - x^{13}_{4,3} - x^{13}_{11,3} &= -1\\
x^{13}_{4,3} + x^{13}_{4,5} + x^{13}_{4,6} + x^{13}_{4,16} - x^{13}_{3,4} - x^{13}_{5,4} - x^{13}_{6,4} - x^{13}_{16,4} &= 0\\
x^{13}_{5,1} + x^{13}_{5,4} + x^{13}_{5,15} - x^{13}_{1,5} - x^{13}_{4,5} - x^{13}_{15,5} &= -1\\
x^{13}_{6,4} + x^{13}_{6,8} + x^{13}_{6,17} - x^{13}_{4,6} - x^{13}_{8,6} - x^{13}_{17,6} &= -1\\
x^{13}_{7,1} + x^{13}_{7,11} + x^{13}_{7,13} - x^{13}_{1,7} - x^{13}_{11,7} - x^{13}_{13,7} &= -1\\
x^{13}_{8,2} + x^{13}_{8,6} + x^{13}_{8,10} - x^{13}_{2,8} - x^{13}_{6,8} - x^{13}_{10,8} &= -1\\
x^{13}_{9,2} + x^{13}_{9,10} + x^{13}_{9,12} - x^{13}_{2,9} - x^{13}_{10,9} - x^{13}_{12,9} &= -1\\
x^{13}_{10,8} + x^{13}_{10,9} + x^{13}_{10,18} - x^{13}_{8,10} - x^{13}_{9,10} - x^{13}_{18,10} &= -1\\
x^{13}_{11,3} + x^{13}_{11,7} + x^{13}_{11,12} + x^{13}_{11,23} - x^{13}_{3,11} - x^{13}_{7,11} - x^{13}_{12,11} - x^{13}_{23,11} &= 0\\
x^{13}_{12,9} + x^{13}_{12,11} + x^{13}_{12,19} - x^{13}_{9,12} - x^{13}_{11,12} - x^{13}_{19,12} &= -1\\
x^{13}_{13,7} + x^{13}_{13,14} + x^{13}_{13,22} - x^{13}_{7,13} - x^{13}_{14,13} - x^{13}_{22,13} &= 15\\
x^{13}_{14,13} + x^{13}_{14,15} - x^{13}_{13,14} - x^{13}_{15,14} &= 0\\
x^{13}_{15,5} + x^{13}_{15,14} + x^{13}_{15,16} - x^{13}_{5,15} - x^{13}_{14,15} - x^{13}_{16,15} &= -1\\
x^{13}_{16,15} + x^{13}_{16,17} - x^{13}_{15,16} - x^{13}_{17,16} &= -1\\
x^{13}_{17,6} + x^{13}_{17,16} - x^{13}_{6,17} - x^{13}_{16,17} &= 0\\
x^{13}_{18,10} + x^{13}_{18,19} - x^{13}_{10,18} - x^{13}_{19,18} &= 0\\
x^{13}_{19,12} + x^{13}_{19,18} + x^{13}_{19,20} - x^{13}_{12,19} - x^{13}_{18,19} - x^{13}_{20,19} &= -1\\
x^{13}_{20,19} + x^{13}_{20,21} + x^{13}_{20,23} - x^{13}_{19,20} - x^{13}_{21,20} - x^{13}_{23,20} &= -1\\
x^{13}_{21,20} + x^{13}_{21,22} - x^{13}_{20,21} - x^{13}_{22,21} &= 0\\
x^{13}_{22,21} + x^{13}_{22,23} - x^{13}_{21,22} - x^{13}_{23,22} &= -1\\
x^{13}_{23,11} + x^{13}_{23,20} + x^{13}_{23,22} - x^{13}_{11,23} - x^{13}_{20,23} - x^{13}_{22,23} &= -1\\
x^{15}_{1,2} + x^{15}_{1,3} + x^{15}_{1,5} + x^{15}_{1,7} - x^{15}_{2,1} - x^{15}_{3,1} - x^{15}_{5,1} - x^{15}_{7,1} &= 0\\
x^{15}_{2,1} + x^{15}_{2,8} + x^{15}_{2,9} - x^{15}_{1,2} - x^{15}_{8,2} - x^{15}_{9,2} &= -1\\
x^{15}_{3,1} + x^{15}_{3,4} + x^{15}_{3,11} - x^{15}_{1,3} - x^{15}_{4,3} - x^{15}_{11,3} &= -1
\end{aligned}
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$$x^{15}_{4,3} + x^{15}_{4,5} + x^{15}_{4,6} + x^{15}_{4,16} - x^{15}_{3,4} - x^{15}_{5,4} - x^{15}_{6,4} - x^{15}_{16,4} = 0$$
$$x^{15}_{5,1} + x^{15}_{5,4} + x^{15}_{5,15} - x^{15}_{1,5} - x^{15}_{4,5} - x^{15}_{15,5} = -1$$
$$x^{15}_{6,4} + x^{15}_{6,8} + x^{15}_{6,17} - x^{15}_{4,6} - x^{15}_{8,6} - x^{15}_{17,6} = -1$$
$$x^{15}_{7,1} + x^{15}_{7,11} + x^{15}_{7,13} - x^{15}_{1,7} - x^{15}_{11,7} - x^{15}_{13,7} = -1$$
$$x^{15}_{8,2} + x^{15}_{8,6} + x^{15}_{8,10} - x^{15}_{2,8} - x^{15}_{6,8} - x^{15}_{10,8} = -1$$
$$x^{15}_{9,2} + x^{15}_{9,10} + x^{15}_{9,12} - x^{15}_{2,9} - x^{15}_{10,9} - x^{15}_{12,9} = -1$$
$$x^{15}_{10,8} + x^{15}_{10,9} + x^{15}_{10,18} - x^{15}_{8,10} - x^{15}_{9,10} - x^{15}_{18,10} = -1$$
$$x^{15}_{11,3} + x^{15}_{11,7} + x^{15}_{11,12} + x^{15}_{11,23} - x^{15}_{3,11} - x^{15}_{7,11} - x^{15}_{12,11} - x^{15}_{23,11} = 0$$
$$x^{15}_{12,9} + x^{15}_{12,11} + x^{15}_{12,19} - x^{15}_{9,12} - x^{15}_{11,12} - x^{15}_{19,12} = -1$$
$$x^{15}_{13,7} + x^{15}_{13,14} + x^{15}_{13,22} - x^{15}_{7,13} - x^{15}_{14,13} - x^{15}_{22,13} = -1$$
$$x^{15}_{14,13} + x^{15}_{14,15} - x^{15}_{13,14} - x^{15}_{15,14} = 0$$
$$x^{15}_{15,5} + x^{15}_{15,14} + x^{15}_{15,16} - x^{15}_{5,15} - x^{15}_{14,15} - x^{15}_{16,15} = 15$$
$$x^{15}_{16,15} + x^{15}_{16,17} - x^{15}_{15,16} - x^{15}_{17,16} = -1$$
$$x^{15}_{17,6} + x^{15}_{17,16} - x^{15}_{6,17} - x^{15}_{16,17} = 0$$
$$x^{15}_{18,10} + x^{15}_{18,19} - x^{15}_{10,18} - x^{15}_{19,18} = 0$$
$$x^{15}_{19,12} + x^{15}_{19,18} + x^{15}_{19,20} - x^{15}_{12,19} - x^{15}_{18,19} - x^{15}_{20,19} = -1$$
$$x^{15}_{20,19} + x^{15}_{20,21} + x^{15}_{20,23} - x^{15}_{19,20} - x^{15}_{21,20} - x^{15}_{23,20} = -1$$
$$x^{15}_{21,20} + x^{15}_{21,22} - x^{15}_{20,21} - x^{15}_{22,21} = 0$$
$$x^{15}_{22,21} + x^{15}_{22,23} - x^{15}_{21,22} - x^{15}_{23,22} = -1$$
$$x^{15}_{23,11} + x^{15}_{23,20} + x^{15}_{23,22} - x^{15}_{11,23} - x^{15}_{20,23} - x^{15}_{22,23} = -1$$
$$x^{16}_{1,2} + x^{16}_{1,3} + x^{16}_{1,5} + x^{16}_{1,7} - x^{16}_{2,1} - x^{16}_{3,1} - x^{16}_{5,1} - x^{16}_{7,1} = 0$$
$$x^{16}_{2,1} + x^{16}_{2,8} + x^{16}_{2,9} - x^{16}_{1,2} - x^{16}_{8,2} - x^{16}_{9,2} = -1$$
$$x^{16}_{3,1} + x^{16}_{3,4} + x^{16}_{3,11} - x^{16}_{1,3} - x^{16}_{4,3} - x^{16}_{11,3} = -1$$
$$x^{16}_{4,3} + x^{16}_{4,5} + x^{16}_{4,6} + x^{16}_{4,16} - x^{16}_{3,4} - x^{16}_{5,4} - x^{16}_{6,4} - x^{16}_{16,4} = 0$$
$$x^{16}_{5,1} + x^{16}_{5,4} + x^{16}_{5,15} - x^{16}_{1,5} - x^{16}_{4,5} - x^{16}_{15,5} = -1$$
$$x^{16}_{6,4} + x^{16}_{6,8} + x^{16}_{6,17} - x^{16}_{4,6} - x^{16}_{8,6} - x^{16}_{17,6} = -1$$
$$x^{16}_{7,1} + x^{16}_{7,11} + x^{16}_{7,13} - x^{16}_{1,7} - x^{16}_{11,7} - x^{16}_{13,7} = -1$$
$$x^{16}_{8,2} + x^{16}_{8,6} + x^{16}_{8,10} - x^{16}_{2,8} - x^{16}_{6,8} - x^{16}_{10,8} = -1$$
$$x^{16}_{9,2} + x^{16}_{9,10} + x^{16}_{9,12} - x^{16}_{2,9} - x^{16}_{10,9} - x^{16}_{12,9} = -1$$
$$x^{16}_{10,8} + x^{16}_{10,9} + x^{16}_{10,18} - x^{16}_{8,10} - x^{16}_{9,10} - x^{16}_{18,10} = -1$$
$$x^{16}_{11,3} + x^{16}_{11,7} + x^{16}_{11,12} + x^{16}_{11,23} - x^{16}_{3,11} - x^{16}_{7,11} - x^{16}_{12,11} - x^{16}_{23,11} = 0$$
$$x^{16}_{12,9} + x^{16}_{12,11} + x^{16}_{12,19} - x^{16}_{9,12} - x^{16}_{11,12} - x^{16}_{19,12} = -1$$
$$x^{16}_{13,7} + x^{16}_{13,14} + x^{16}_{13,22} - x^{16}_{7,13} - x^{16}_{14,13} - x^{16}_{22,13} = -1$$
$$x^{16}_{14,13} + x^{16}_{14,15} - x^{16}_{13,14} - x^{16}_{15,14} = 0$$
$$x^{16}_{15,5} + x^{16}_{15,14} + x^{16}_{15,16} - x^{16}_{5,15} - x^{16}_{14,15} - x^{16}_{16,15} = -1$$
$$x^{16}_{16,15} + x^{16}_{16,17} - x^{16}_{15,16} - x^{16}_{17,16} = 15$$
$$x^{16}_{17,6} + x^{16}_{17,16} - x^{16}_{6,17} - x^{16}_{16,17} = 0$$
$$x^{16}_{18,10} + x^{16}_{18,19} - x^{16}_{10,18} - x^{16}_{19,18} = 0$$
$$x^{16}_{19,12} + x^{16}_{19,18} + x^{16}_{19,20} - x^{16}_{12,19} - x^{16}_{18,19} - x^{16}_{20,19} = -1$$
$$x^{16}_{20,19} + x^{16}_{20,21} + x^{16}_{20,23} - x^{16}_{19,20} - x^{16}_{21,20} - x^{16}_{23,20} = -1$$
$$x^{16}_{21,20} + x^{16}_{21,22} - x^{16}_{20,21} - x^{16}_{22,21} = 0$$
$$x^{16}_{22,21} + x^{16}_{22,23} - x^{16}_{21,22} - x^{16}_{23,22} = -1$$
$$x^{16}_{23,11} + x^{16}_{23,20} + x^{16}_{23,22} - x^{16}_{11,23} - x^{16}_{20,23} - x^{16}_{22,23} = -1$$
$$x^{19}_{1,2} + x^{19}_{1,3} + x^{19}_{1,5} + x^{19}_{1,7} - x^{19}_{2,1} - x^{19}_{3,1} - x^{19}_{5,1} - x^{19}_{7,1} = 0$$
$$x^{19}_{2,1} + x^{19}_{2,8} + x^{19}_{2,9} - x^{19}_{1,2} - x^{19}_{8,2} - x^{19}_{9,2} = -1$$
$$x^{19}_{3,1} + x^{19}_{3,4} + x^{19}_{3,11} - x^{19}_{1,3} - x^{19}_{4,3} - x^{19}_{11,3} = -1$$
$$x^{19}_{4,3} + x^{19}_{4,5} + x^{19}_{4,6} + x^{19}_{4,16} - x^{19}_{3,4} - x^{19}_{5,4} - x^{19}_{6,4} - x^{19}_{16,4} = 0$$
$$x^{19}_{5,1} + x^{19}_{5,4} + x^{19}_{5,15} - x^{19}_{1,5} - x^{19}_{4,5} - x^{19}_{15,5} = -1$$
$$x^{19}_{6,4} + x^{19}_{6,8} + x^{19}_{6,17} - x^{19}_{4,6} - x^{19}_{8,6} - x^{19}_{17,6} = -1$$

$$x^{19}_{7,1} + x^{19}_{7,11} + x^{19}_{7,13} - x^{19}_{1,7} - x^{19}_{11,7} - x^{19}_{13,7} = -1$$
$$x^{19}_{8,2} + x^{19}_{8,6} + x^{19}_{8,10} - x^{19}_{2,8} - x^{19}_{6,8} - x^{19}_{10,8} = -1$$
$$x^{19}_{9,2} + x^{19}_{9,10} + x^{19}_{9,12} - x^{19}_{2,9} - x^{19}_{10,9} - x^{19}_{12,9} = -1$$
$$x^{19}_{10,8} + x^{19}_{10,9} + x^{19}_{10,18} - x^{19}_{8,10} - x^{19}_{9,10} - x^{19}_{18,10} = -1$$
$$x^{19}_{11,3} + x^{19}_{11,7} + x^{19}_{11,12} + x^{19}_{11,23} - x^{19}_{3,11} - x^{19}_{7,11} - x^{19}_{12,11} - x^{19}_{23,11} = 0$$
$$x^{19}_{12,9} + x^{19}_{12,11} + x^{19}_{12,19} - x^{19}_{9,12} - x^{19}_{11,12} - x^{19}_{19,12} = -1$$
$$x^{19}_{13,7} + x^{19}_{13,14} + x^{19}_{13,22} - x^{19}_{7,13} - x^{19}_{14,13} - x^{19}_{22,13} = -1$$
$$x^{19}_{14,13} + x^{19}_{14,15} - x^{19}_{13,14} - x^{19}_{15,14} = 0$$
$$x^{19}_{15,5} + x^{19}_{15,14} + x^{19}_{15,16} - x^{19}_{5,15} - x^{19}_{14,15} - x^{19}_{16,15} = -1$$
$$x^{19}_{16,15} + x^{19}_{16,17} - x^{19}_{15,16} - x^{19}_{17,16} = -1$$
$$x^{19}_{17,6} + x^{19}_{17,16} - x^{19}_{6,17} - x^{19}_{16,17} = 0$$
$$x^{19}_{18,10} + x^{19}_{18,19} - x^{19}_{10,18} - x^{19}_{19,18} = 0$$
$$x^{19}_{19,12} + x^{19}_{19,18} + x^{19}_{19,20} - x^{19}_{12,19} - x^{19}_{18,19} - x^{19}_{20,19} = 15$$
$$x^{19}_{20,19} + x^{19}_{20,21} + x^{19}_{20,23} - x^{19}_{19,20} - x^{19}_{21,20} - x^{19}_{23,20} = -1$$
$$x^{19}_{21,20} + x^{19}_{21,22} - x^{19}_{20,21} - x^{19}_{22,21} = 0$$
$$x^{19}_{22,21} + x^{19}_{22,23} - x^{19}_{21,22} - x^{19}_{23,22} = -1$$
$$x^{19}_{23,11} + x^{19}_{23,20} + x^{19}_{23,22} - x^{19}_{11,23} - x^{19}_{20,23} - x^{19}_{22,23} = -1$$
$$x^{20}_{1,2} + x^{20}_{1,3} + x^{20}_{1,5} + x^{20}_{1,7} - x^{20}_{2,1} - x^{20}_{3,1} - x^{20}_{5,1} - x^{20}_{7,1} = 0$$
$$x^{20}_{2,1} + x^{20}_{2,8} + x^{20}_{2,9} - x^{20}_{1,2} - x^{20}_{8,2} - x^{20}_{9,2} = -1$$
$$x^{20}_{3,1} + x^{20}_{3,4} + x^{20}_{3,11} - x^{20}_{1,3} - x^{20}_{4,3} - x^{20}_{11,3} = -1$$
$$x^{20}_{4,3} + x^{20}_{4,5} + x^{20}_{4,6} + x^{20}_{4,16} - x^{20}_{3,4} - x^{20}_{5,4} - x^{20}_{6,4} - x^{20}_{16,4} = 0$$
$$x^{20}_{5,1} + x^{20}_{5,4} + x^{20}_{5,15} - x^{20}_{1,5} - x^{20}_{4,5} - x^{20}_{15,5} = -1$$
$$x^{20}_{6,4} + x^{20}_{6,8} + x^{20}_{6,17} - x^{20}_{4,6} - x^{20}_{8,6} - x^{20}_{17,6} = -1$$
$$x^{20}_{7,1} + x^{20}_{7,11} + x^{20}_{7,13} - x^{20}_{1,7} - x^{20}_{11,7} - x^{20}_{13,7} = -1$$
$$x^{20}_{8,2} + x^{20}_{8,6} + x^{20}_{8,10} - x^{20}_{2,8} - x^{20}_{6,8} - x^{20}_{10,8} = -1$$
$$x^{20}_{9,2} + x^{20}_{9,10} + x^{20}_{9,12} - x^{20}_{2,9} - x^{20}_{10,9} - x^{20}_{12,9} = -1$$
$$x^{20}_{10,8} + x^{20}_{10,9} + x^{20}_{10,18} - x^{20}_{8,10} - x^{20}_{9,10} - x^{20}_{18,10} = -1$$
$$x^{20}_{11,3} + x^{20}_{11,7} + x^{20}_{11,12} + x^{20}_{11,23} - x^{20}_{3,11} - x^{20}_{7,11} - x^{20}_{12,11} - x^{20}_{23,11} = 0$$
$$x^{20}_{12,9} + x^{20}_{12,11} + x^{20}_{12,19} - x^{20}_{9,12} - x^{20}_{11,12} - x^{20}_{19,12} = -1$$
$$x^{20}_{13,7} + x^{20}_{13,14} + x^{20}_{13,22} - x^{20}_{7,13} - x^{20}_{14,13} - x^{20}_{22,13} = -1$$
$$x^{20}_{14,13} + x^{20}_{14,15} - x^{20}_{13,14} - x^{20}_{15,14} = 0$$
$$x^{20}_{15,5} + x^{20}_{15,14} + x^{20}_{15,16} - x^{20}_{5,15} - x^{20}_{14,15} - x^{20}_{16,15} = -1$$
$$x^{20}_{16,15} + x^{20}_{16,17} - x^{20}_{15,16} - x^{20}_{17,16} = -1$$
$$x^{20}_{17,6} + x^{20}_{17,16} - x^{20}_{6,17} - x^{20}_{16,17} = 0$$
$$x^{20}_{18,10} + x^{20}_{18,19} - x^{20}_{10,18} - x^{20}_{19,18} = 0$$
$$x^{20}_{19,12} + x^{20}_{19,18} + x^{20}_{19,20} - x^{20}_{12,19} - x^{20}_{18,19} - x^{20}_{20,19} = -1$$
$$x^{20}_{20,19} + x^{20}_{20,21} + x^{20}_{20,23} - x^{20}_{19,20} - x^{20}_{21,20} - x^{20}_{23,20} = 15$$
$$x^{20}_{21,20} + x^{20}_{21,22} - x^{20}_{20,21} - x^{20}_{22,21} = 0$$
$$x^{20}_{22,21} + x^{20}_{22,23} - x^{20}_{21,22} - x^{20}_{23,22} = -1$$
$$x^{20}_{23,11} + x^{20}_{23,20} + x^{20}_{23,22} - x^{20}_{11,23} - x^{20}_{20,23} - x^{20}_{22,23} = -1$$
$$x^{22}_{1,2} + x^{22}_{1,3} + x^{22}_{1,5} + x^{22}_{1,7} - x^{22}_{2,1} - x^{22}_{3,1} - x^{22}_{5,1} - x^{22}_{7,1} = 0$$
$$x^{22}_{2,1} + x^{22}_{2,8} + x^{22}_{2,9} - x^{22}_{1,2} - x^{22}_{8,2} - x^{22}_{9,2} = -1$$
$$x^{22}_{3,1} + x^{22}_{3,4} + x^{22}_{3,11} - x^{22}_{1,3} - x^{22}_{4,3} - x^{22}_{11,3} = -1$$
$$x^{22}_{4,3} + x^{22}_{4,5} + x^{22}_{4,6} + x^{22}_{4,16} - x^{22}_{3,4} - x^{22}_{5,4} - x^{22}_{6,4} - x^{22}_{16,4} = 0$$
$$x^{22}_{5,1} + x^{22}_{5,4} + x^{22}_{5,15} - x^{22}_{1,5} - x^{22}_{4,5} - x^{22}_{15,5} = -1$$
$$x^{22}_{6,4} + x^{22}_{6,8} + x^{22}_{6,17} - x^{22}_{4,6} - x^{22}_{8,6} - x^{22}_{17,6} = -1$$
$$x^{22}_{7,1} + x^{22}_{7,11} + x^{22}_{7,13} - x^{22}_{1,7} - x^{22}_{11,7} - x^{22}_{13,7} = -1$$
$$x^{22}_{8,2} + x^{22}_{8,6} + x^{22}_{8,10} - x^{22}_{2,8} - x^{22}_{6,8} - x^{22}_{10,8} = -1$$
$$x^{22}_{9,2} + x^{22}_{9,10} + x^{22}_{9,12} - x^{22}_{2,9} - x^{22}_{10,9} - x^{22}_{12,9} = -1$$

$$
\begin{aligned}
x^{22}_{10,8} + x^{22}_{10,9} + x^{22}_{10,18} - x^{22}_{8,10} - x^{22}_{9,10} - x^{22}_{18,10} &= -1 \\
x^{22}_{11,3} + x^{22}_{11,7} + x^{22}_{11,12} + x^{22}_{11,23} - x^{22}_{3,11} - x^{22}_{7,11} - x^{22}_{12,11} - x^{22}_{23,11} &= 0 \\
x^{22}_{12,9} + x^{22}_{12,11} + x^{22}_{12,19} - x^{22}_{9,12} - x^{22}_{11,12} - x^{22}_{19,12} &= -1 \\
x^{22}_{13,7} + x^{22}_{13,14} + x^{22}_{13,22} - x^{22}_{7,13} - x^{22}_{14,13} - x^{22}_{22,13} &= -1 \\
x^{22}_{14,13} + x^{22}_{14,15} - x^{22}_{13,14} - x^{22}_{15,14} &= 0 \\
x^{22}_{15,5} + x^{22}_{15,14} + x^{22}_{15,16} - x^{22}_{5,15} - x^{22}_{14,15} - x^{22}_{16,15} &= -1 \\
x^{22}_{16,15} + x^{22}_{16,17} - x^{22}_{15,16} - x^{22}_{17,16} &= -1 \\
x^{22}_{17,6} + x^{22}_{17,16} - x^{22}_{6,17} - x^{22}_{16,17} &= 0 \\
x^{22}_{18,10} + x^{22}_{18,19} - x^{22}_{10,18} - x^{22}_{19,18} &= 0 \\
x^{22}_{19,12} + x^{22}_{19,18} + x^{22}_{19,20} - x^{22}_{12,19} - x^{22}_{18,19} - x^{22}_{20,19} &= -1 \\
x^{22}_{20,19} + x^{22}_{20,21} + x^{22}_{20,23} - x^{22}_{19,20} - x^{22}_{21,20} - x^{22}_{23,20} &= -1 \\
x^{22}_{21,20} + x^{22}_{21,22} - x^{22}_{20,21} - x^{22}_{22,21} &= 0 \\
x^{22}_{22,21} + x^{22}_{22,23} - x^{22}_{21,22} - x^{22}_{23,22} &= 15 \\
x^{22}_{23,11} + x^{22}_{23,20} + x^{22}_{23,22} - x^{22}_{11,23} - x^{22}_{20,23} - x^{22}_{22,23} &= -1 \\
x^{23}_{1,2} + x^{23}_{1,3} + x^{23}_{1,5} + x^{23}_{1,7} - x^{23}_{2,1} - x^{23}_{3,1} - x^{23}_{5,1} - x^{23}_{7,1} &= 0 \\
x^{23}_{2,1} + x^{23}_{2,8} + x^{23}_{2,9} - x^{23}_{1,2} - x^{23}_{8,2} - x^{23}_{9,2} &= -1 \\
x^{23}_{3,1} + x^{23}_{3,4} + x^{23}_{3,11} - x^{23}_{1,3} - x^{23}_{4,3} - x^{23}_{11,3} &= -1 \\
x^{23}_{4,3} + x^{23}_{4,5} + x^{23}_{4,6} + x^{23}_{4,16} - x^{23}_{3,4} - x^{23}_{5,4} - x^{23}_{6,4} - x^{23}_{16,4} &= 0 \\
x^{23}_{5,1} + x^{23}_{5,4} + x^{23}_{5,15} - x^{23}_{1,5} - x^{23}_{4,5} - x^{23}_{15,5} &= -1 \\
x^{23}_{6,4} + x^{23}_{6,8} + x^{23}_{6,17} - x^{23}_{4,6} - x^{23}_{8,6} - x^{23}_{17,6} &= -1 \\
x^{23}_{7,1} + x^{23}_{7,11} + x^{23}_{7,13} - x^{23}_{1,7} - x^{23}_{11,7} - x^{23}_{13,7} &= -1 \\
x^{23}_{8,2} + x^{23}_{8,6} + x^{23}_{8,10} - x^{23}_{2,8} - x^{23}_{6,8} - x^{23}_{10,8} &= -1 \\
x^{23}_{9,2} + x^{23}_{9,10} + x^{23}_{9,12} - x^{23}_{2,9} - x^{23}_{10,9} - x^{23}_{12,9} &= -1 \\
x^{23}_{10,8} + x^{23}_{10,9} + x^{23}_{10,18} - x^{23}_{8,10} - x^{23}_{9,10} - x^{23}_{18,10} &= -1 \\
x^{23}_{11,3} + x^{23}_{11,7} + x^{23}_{11,12} + x^{23}_{11,23} - x^{23}_{3,11} - x^{23}_{7,11} - x^{23}_{12,11} - x^{23}_{23,11} &= 0 \\
x^{23}_{12,9} + x^{23}_{12,11} + x^{23}_{12,19} - x^{23}_{9,12} - x^{23}_{11,12} - x^{23}_{19,12} &= -1 \\
x^{23}_{13,7} + x^{23}_{13,14} + x^{23}_{13,22} - x^{23}_{7,13} - x^{23}_{14,13} - x^{23}_{22,13} &= -1 \\
x^{23}_{14,13} + x^{23}_{14,15} - x^{23}_{13,14} - x^{23}_{15,14} &= 0 \\
x^{23}_{15,5} + x^{23}_{15,14} + x^{23}_{15,16} - x^{23}_{5,15} - x^{23}_{14,15} - x^{23}_{16,15} &= -1 \\
x^{23}_{16,15} + x^{23}_{16,17} - x^{23}_{15,16} - x^{23}_{17,16} &= -1 \\
x^{23}_{17,6} + x^{23}_{17,16} - x^{23}_{6,17} - x^{23}_{16,17} &= 0 \\
x^{23}_{18,10} + x^{23}_{18,19} - x^{23}_{10,18} - x^{23}_{19,18} &= 0 \\
x^{23}_{19,12} + x^{23}_{19,18} + x^{23}_{19,20} - x^{23}_{12,19} - x^{23}_{18,19} - x^{23}_{20,19} &= -1 \\
x^{23}_{20,19} + x^{23}_{20,21} + x^{23}_{20,23} - x^{23}_{19,20} - x^{23}_{21,20} - x^{23}_{23,20} &= -1 \\
x^{23}_{21,20} + x^{23}_{21,22} - x^{23}_{20,21} - x^{23}_{22,21} &= 0 \\
x^{23}_{22,21} + x^{23}_{22,23} - x^{23}_{21,22} - x^{23}_{23,22} &= -1 \\
x^{23}_{23,11} + x^{23}_{23,20} + x^{23}_{23,22} - x^{23}_{11,23} - x^{23}_{20,23} - x^{23}_{22,23} &= 15 \\
x^{3}_{1,2}, x^{5}_{1,2}, x^{6}_{1,2}, x^{7}_{1,2}, x^{8}_{1,2}, x^{9}_{1,2}, x^{10}_{1,2}, x^{12}_{1,2}, x^{13}_{1,2}, x^{15}_{1,2}, x^{16}_{1,2}, x^{19}_{1,2}, x^{20}_{1,2}, x^{22}_{1,2}, x^{23}_{1,2} &\geq 0 \\
x^{5}_{2,1}, x^{6}_{2,1}, x^{7}_{2,1}, x^{8}_{2,1}, x^{9}_{2,1}, x^{10}_{2,1}, x^{12}_{2,1}, x^{13}_{2,1}, x^{15}_{2,1}, x^{16}_{2,1}, x^{19}_{2,1}, x^{20}_{2,1}, x^{22}_{2,1}, x^{23}_{2,1}, x_{2,1} &\geq 0 \\
x^{3}_{1,3}, x^{5}_{1,3}, x^{6}_{1,3}, x^{7}_{1,3}, x^{8}_{1,3}, x^{9}_{1,3}, x^{10}_{1,3}, x^{12}_{1,3}, x^{13}_{1,3}, x^{15}_{1,3}, x^{16}_{1,3}, x^{19}_{1,3}, x^{20}_{1,3}, x^{22}_{1,3}, x^{23}_{1,3} &\geq 0 \\
x^{5}_{3,1}, x^{6}_{3,1}, x^{7}_{3,1}, x^{8}_{3,1}, x^{9}_{3,1}, x^{10}_{3,1}, x^{12}_{3,1}, x^{13}_{3,1}, x^{15}_{3,1}, x^{16}_{3,1}, x^{19}_{3,1}, x^{20}_{3,1}, x^{22}_{3,1}, x^{23}_{3,1}, x_{3,1} &\geq 0 \\
x^{3}_{1,5}, x^{5}_{1,5}, x^{6}_{1,5}, x^{7}_{1,5}, x^{8}_{1,5}, x^{9}_{1,5}, x^{10}_{1,5}, x^{12}_{1,5}, x^{13}_{1,5}, x^{15}_{1,5}, x^{16}_{1,5}, x^{19}_{1,5}, x^{20}_{1,5}, x^{22}_{1,5}, x^{23}_{1,5} &\geq 0 \\
x^{5}_{5,1}, x^{6}_{5,1}, x^{7}_{5,1}, x^{8}_{5,1}, x^{9}_{5,1}, x^{10}_{5,1}, x^{12}_{5,1}, x^{13}_{5,1}, x^{15}_{5,1}, x^{16}_{5,1}, x^{19}_{5,1}, x^{20}_{5,1}, x^{22}_{5,1}, x^{23}_{5,1}, x_{5,1} &\geq 0 \\
x^{3}_{1,7}, x^{5}_{1,7}, x^{6}_{1,7}, x^{7}_{1,7}, x^{8}_{1,7}, x^{9}_{1,7}, x^{10}_{1,7}, x^{12}_{1,7}, x^{13}_{1,7}, x^{15}_{1,7}, x^{16}_{1,7}, x^{19}_{1,7}, x^{20}_{1,7}, x^{22}_{1,7}, x^{23}_{1,7} &\geq 0 \\
x^{5}_{7,1}, x^{6}_{7,1}, x^{7}_{7,1}, x^{8}_{7,1}, x^{9}_{7,1}, x^{10}_{7,1}, x^{12}_{7,1}, x^{13}_{7,1}, x^{15}_{7,1}, x^{16}_{7,1}, x^{19}_{7,1}, x^{20}_{7,1}, x^{22}_{7,1}, x^{23}_{7,1}, x_{7,1} &\geq 0 \\
x^{3}_{2,8}, x^{5}_{2,8}, x^{6}_{2,8}, x^{7}_{2,8}, x^{8}_{2,8}, x^{9}_{2,8}, x^{10}_{2,8}, x^{12}_{2,8}, x^{13}_{2,8}, x^{15}_{2,8}, x^{16}_{2,8}, x^{19}_{2,8}, x^{20}_{2,8}, x^{22}_{2,8}, x^{23}_{2,8} &\geq 0 \\
x^{5}_{8,2}, x^{6}_{8,2}, x^{7}_{8,2}, x^{8}_{8,2}, x^{9}_{8,2}, x^{10}_{8,2}, x^{12}_{8,2}, x^{13}_{8,2}, x^{15}_{8,2}, x^{16}_{8,2}, x^{19}_{8,2}, x^{20}_{8,2}, x^{22}_{8,2}, x^{23}_{8,2}, x_{8,2} &\geq 0 \\
x^{3}_{2,9}, x^{5}_{2,9}, x^{6}_{2,9}, x^{7}_{2,9}, x^{8}_{2,9}, x^{9}_{2,9}, x^{10}_{2,9}, x^{12}_{2,9}, x^{13}_{2,9}, x^{15}_{2,9}, x^{16}_{2,9}, x^{19}_{2,9}, x^{20}_{2,9}, x^{22}_{2,9}, x^{23}_{2,9} &\geq 0 \\
x^{5}_{9,2}, x^{6}_{9,2}, x^{7}_{9,2}, x^{8}_{9,2}, x^{9}_{9,2}, x^{10}_{9,2}, x^{12}_{9,2}, x^{13}_{9,2}, x^{15}_{9,2}, x^{16}_{9,2}, x^{19}_{9,2}, x^{20}_{9,2}, x^{22}_{9,2}, x^{23}_{9,2}, x_{9,2} &\geq 0
\end{aligned}
$$

$$x^{3}_{3,4}, x^{5}_{3,4}, x^{6}_{3,4}, x^{7}_{3,4}, x^{8}_{3,4}, x^{9}_{3,4}, x^{10}_{3,4}, x^{12}_{3,4}, x^{13}_{3,4}, x^{15}_{3,4}, x^{16}_{3,4}, x^{19}_{3,4}, x^{20}_{3,4}, x^{22}_{3,4}, x^{23}_{3,4} \geq 0$$
$$x^{5}_{4,3}, x^{6}_{4,3}, x^{7}_{4,3}, x^{8}_{4,3}, x^{9}_{4,3}, x^{10}_{4,3}, x^{12}_{4,3}, x^{13}_{4,3}, x^{15}_{4,3}, x^{16}_{4,3}, x^{19}_{4,3}, x^{20}_{4,3}, x^{22}_{4,3}, x^{23}_{4,3}, x_{4,3} \geq 0$$
$$x^{3}_{3,11}, x^{5}_{3,11}, x^{6}_{3,11}, x^{7}_{3,11}, x^{8}_{3,11}, x^{9}_{3,11}, x^{10}_{3,11}, x^{12}_{3,11}, x^{13}_{3,11}, x^{15}_{3,11}, x^{16}_{3,11}, x^{19}_{3,11}, x^{20}_{3,11}, x^{22}_{3,11}, x^{23}_{3,11} \geq 0$$
$$x^{5}_{11,3}, x^{6}_{11,3}, x^{7}_{11,3}, x^{8}_{11,3}, x^{9}_{11,3}, x^{10}_{11,3}, x^{12}_{11,3}, x^{13}_{11,3}, x^{15}_{11,3}, x^{16}_{11,3}, x^{19}_{11,3}, x^{20}_{11,3}, x^{22}_{11,3}, x^{23}_{11,3}, x_{11,3} \geq 0$$
$$x^{3}_{4,5}, x^{5}_{4,5}, x^{6}_{4,5}, x^{7}_{4,5}, x^{8}_{4,5}, x^{9}_{4,5}, x^{10}_{4,5}, x^{12}_{4,5}, x^{13}_{4,5}, x^{15}_{4,5}, x^{16}_{4,5}, x^{19}_{4,5}, x^{20}_{4,5}, x^{22}_{4,5}, x^{23}_{4,5} \geq 0$$
$$x^{5}_{5,4}, x^{6}_{5,4}, x^{7}_{5,4}, x^{8}_{5,4}, x^{9}_{5,4}, x^{10}_{5,4}, x^{12}_{5,4}, x^{13}_{5,4}, x^{15}_{5,4}, x^{16}_{5,4}, x^{19}_{5,4}, x^{20}_{5,4}, x^{22}_{5,4}, x^{23}_{5,4}, x_{5,4} \geq 0$$
$$x^{3}_{4,6}, x^{5}_{4,6}, x^{6}_{4,6}, x^{7}_{4,6}, x^{8}_{4,6}, x^{9}_{4,6}, x^{10}_{4,6}, x^{12}_{4,6}, x^{13}_{4,6}, x^{15}_{4,6}, x^{16}_{4,6}, x^{19}_{4,6}, x^{20}_{4,6}, x^{22}_{4,6}, x^{23}_{4,6} \geq 0$$
$$x^{5}_{6,4}, x^{6}_{6,4}, x^{7}_{6,4}, x^{8}_{6,4}, x^{9}_{6,4}, x^{10}_{6,4}, x^{12}_{6,4}, x^{13}_{6,4}, x^{15}_{6,4}, x^{16}_{6,4}, x^{19}_{6,4}, x^{20}_{6,4}, x^{22}_{6,4}, x^{23}_{6,4}, x_{6,4} \geq 0$$
$$x^{3}_{4,16}, x^{5}_{4,16}, x^{6}_{4,16}, x^{7}_{4,16}, x^{8}_{4,16}, x^{9}_{4,16}, x^{10}_{4,16}, x^{12}_{4,16}, x^{13}_{4,16}, x^{15}_{4,16}, x^{16}_{4,16}, x^{19}_{4,16}, x^{20}_{4,16}, x^{22}_{4,16}, x^{23}_{4,16} \geq 0$$
$$x^{5}_{16,4}, x^{6}_{16,4}, x^{7}_{16,4}, x^{8}_{16,4}, x^{9}_{16,4}, x^{10}_{16,4}, x^{12}_{16,4}, x^{13}_{16,4}, x^{15}_{16,4}, x^{16}_{16,4}, x^{19}_{16,4}, x^{20}_{16,4}, x^{22}_{16,4}, x^{23}_{16,4}, x_{16,4} \geq 0$$
$$x^{3}_{5,15}, x^{5}_{5,15}, x^{6}_{5,15}, x^{7}_{5,15}, x^{8}_{5,15}, x^{9}_{5,15}, x^{10}_{5,15}, x^{12}_{5,15}, x^{13}_{5,15}, x^{15}_{5,15}, x^{16}_{5,15}, x^{19}_{5,15}, x^{20}_{5,15}, x^{22}_{5,15}, x^{23}_{5,15} \geq 0$$
$$x^{5}_{15,5}, x^{6}_{15,5}, x^{7}_{15,5}, x^{8}_{15,5}, x^{9}_{15,5}, x^{10}_{15,5}, x^{12}_{15,5}, x^{13}_{15,5}, x^{15}_{15,5}, x^{16}_{15,5}, x^{19}_{15,5}, x^{20}_{15,5}, x^{22}_{15,5}, x^{23}_{15,5}, x_{15,5} \geq 0$$
$$x^{3}_{6,8}, x^{5}_{6,8}, x^{6}_{6,8}, x^{7}_{6,8}, x^{8}_{6,8}, x^{9}_{6,8}, x^{10}_{6,8}, x^{12}_{6,8}, x^{13}_{6,8}, x^{15}_{6,8}, x^{16}_{6,8}, x^{19}_{6,8}, x^{20}_{6,8}, x^{22}_{6,8}, x^{23}_{6,8} \geq 0$$
$$x^{5}_{8,6}, x^{6}_{8,6}, x^{7}_{8,6}, x^{8}_{8,6}, x^{9}_{8,6}, x^{10}_{8,6}, x^{12}_{8,6}, x^{13}_{8,6}, x^{15}_{8,6}, x^{16}_{8,6}, x^{19}_{8,6}, x^{20}_{8,6}, x^{22}_{8,6}, x^{23}_{8,6}, x_{8,6} \geq 0$$
$$x^{3}_{6,17}, x^{5}_{6,17}, x^{6}_{6,17}, x^{7}_{6,17}, x^{8}_{6,17}, x^{9}_{6,17}, x^{10}_{6,17}, x^{12}_{6,17}, x^{13}_{6,17}, x^{15}_{6,17}, x^{16}_{6,17}, x^{19}_{6,17}, x^{20}_{6,17}, x^{22}_{6,17}, x^{23}_{6,17} \geq 0$$
$$x^{5}_{17,6}, x^{6}_{17,6}, x^{7}_{17,6}, x^{8}_{17,6}, x^{9}_{17,6}, x^{10}_{17,6}, x^{12}_{17,6}, x^{13}_{17,6}, x^{15}_{17,6}, x^{16}_{17,6}, x^{19}_{17,6}, x^{20}_{17,6}, x^{22}_{17,6}, x^{23}_{17,6}, x_{17,6} \geq 0$$
$$x^{3}_{7,11}, x^{5}_{7,11}, x^{6}_{7,11}, x^{7}_{7,11}, x^{8}_{7,11}, x^{9}_{7,11}, x^{10}_{7,11}, x^{12}_{7,11}, x^{13}_{7,11}, x^{15}_{7,11}, x^{16}_{7,11}, x^{19}_{7,11}, x^{20}_{7,11}, x^{22}_{7,11}, x^{23}_{7,11} \geq 0$$
$$x^{5}_{11,7}, x^{6}_{11,7}, x^{7}_{11,7}, x^{8}_{11,7}, x^{9}_{11,7}, x^{10}_{11,7}, x^{12}_{11,7}, x^{13}_{11,7}, x^{15}_{11,7}, x^{16}_{11,7}, x^{19}_{11,7}, x^{20}_{11,7}, x^{22}_{11,7}, x^{23}_{11,7}, x_{11,7} \geq 0$$
$$x^{3}_{7,13}, x^{5}_{7,13}, x^{6}_{7,13}, x^{7}_{7,13}, x^{8}_{7,13}, x^{9}_{7,13}, x^{10}_{7,13}, x^{12}_{7,13}, x^{13}_{7,13}, x^{15}_{7,13}, x^{16}_{7,13}, x^{19}_{7,13}, x^{20}_{7,13}, x^{22}_{7,13}, x^{23}_{7,13} \geq 0$$
$$x^{5}_{13,7}, x^{6}_{13,7}, x^{7}_{13,7}, x^{8}_{13,7}, x^{9}_{13,7}, x^{10}_{13,7}, x^{12}_{13,7}, x^{13}_{13,7}, x^{15}_{13,7}, x^{16}_{13,7}, x^{19}_{13,7}, x^{20}_{13,7}, x^{22}_{13,7}, x^{23}_{13,7}, x_{13,7} \geq 0$$
$$x^{3}_{8,10}, x^{5}_{8,10}, x^{6}_{8,10}, x^{7}_{8,10}, x^{8}_{8,10}, x^{9}_{8,10}, x^{10}_{8,10}, x^{12}_{8,10}, x^{13}_{8,10}, x^{15}_{8,10}, x^{16}_{8,10}, x^{19}_{8,10}, x^{20}_{8,10}, x^{22}_{8,10}, x^{23}_{8,10} \geq 0$$
$$x^{5}_{10,8}, x^{6}_{10,8}, x^{7}_{10,8}, x^{8}_{10,8}, x^{9}_{10,8}, x^{10}_{10,8}, x^{12}_{10,8}, x^{13}_{10,8}, x^{15}_{10,8}, x^{16}_{10,8}, x^{19}_{10,8}, x^{20}_{10,8}, x^{22}_{10,8}, x^{23}_{10,8}, x_{10,8} \geq 0$$
$$x^{3}_{9,10}, x^{5}_{9,10}, x^{6}_{9,10}, x^{7}_{9,10}, x^{8}_{9,10}, x^{9}_{9,10}, x^{10}_{9,10}, x^{12}_{9,10}, x^{13}_{9,10}, x^{15}_{9,10}, x^{16}_{9,10}, x^{19}_{9,10}, x^{20}_{9,10}, x^{22}_{9,10}, x^{23}_{9,10} \geq 0$$
$$x^{5}_{10,9}, x^{6}_{10,9}, x^{7}_{10,9}, x^{8}_{10,9}, x^{9}_{10,9}, x^{10}_{10,9}, x^{12}_{10,9}, x^{13}_{10,9}, x^{15}_{10,9}, x^{16}_{10,9}, x^{19}_{10,9}, x^{20}_{10,9}, x^{22}_{10,9}, x^{23}_{10,9}, x_{10,9} \geq 0$$
$$x^{3}_{9,12}, x^{5}_{9,12}, x^{6}_{9,12}, x^{7}_{9,12}, x^{8}_{9,12}, x^{9}_{9,12}, x^{10}_{9,12}, x^{12}_{9,12}, x^{13}_{9,12}, x^{15}_{9,12}, x^{16}_{9,12}, x^{19}_{9,12}, x^{20}_{9,12}, x^{22}_{9,12}, x^{23}_{9,12} \geq 0$$
$$x^{5}_{12,9}, x^{6}_{12,9}, x^{7}_{12,9}, x^{8}_{12,9}, x^{9}_{12,9}, x^{10}_{12,9}, x^{12}_{12,9}, x^{13}_{12,9}, x^{15}_{12,9}, x^{16}_{12,9}, x^{19}_{12,9}, x^{20}_{12,9}, x^{22}_{12,9}, x^{23}_{12,9}, x_{12,9} \geq 0$$
$$x^{3}_{10,18}, x^{5}_{10,18}, x^{6}_{10,18}, x^{7}_{10,18}, x^{8}_{10,18}, x^{9}_{10,18}, x^{10}_{10,18}, x^{12}_{10,18}, x^{13}_{10,18}, x^{15}_{10,18}, x^{16}_{10,18}, x^{19}_{10,18}, x^{20}_{10,18}, x^{22}_{10,18}, x^{23}_{10,18} \geq 0$$
$$x^{5}_{18,10}, x^{6}_{18,10}, x^{7}_{18,10}, x^{8}_{18,10}, x^{9}_{18,10}, x^{10}_{18,10}, x^{12}_{18,10}, x^{13}_{18,10}, x^{15}_{18,10}, x^{16}_{18,10}, x^{19}_{18,10}, x^{20}_{18,10}, x^{22}_{18,10}, x^{23}_{18,10}, x_{18,10} \geq 0$$
$$x^{3}_{11,12}, x^{5}_{11,12}, x^{6}_{11,12}, x^{7}_{11,12}, x^{8}_{11,12}, x^{9}_{11,12}, x^{10}_{11,12}, x^{12}_{11,12}, x^{13}_{11,12}, x^{15}_{11,12}, x^{16}_{11,12}, x^{19}_{11,12}, x^{20}_{11,12}, x^{22}_{11,12}, x^{23}_{11,12} \geq 0$$
$$x^{5}_{12,11}, x^{6}_{12,11}, x^{7}_{12,11}, x^{8}_{12,11}, x^{9}_{12,11}, x^{10}_{12,11}, x^{12}_{12,11}, x^{13}_{12,11}, x^{15}_{12,11}, x^{16}_{12,11}, x^{19}_{12,11}, x^{20}_{12,11}, x^{22}_{12,11}, x^{23}_{12,11}, x_{12,11} \geq 0$$
$$x^{3}_{11,23}, x^{5}_{11,23}, x^{6}_{11,23}, x^{7}_{11,23}, x^{8}_{11,23}, x^{9}_{11,23}, x^{10}_{11,23}, x^{12}_{11,23}, x^{13}_{11,23}, x^{15}_{11,23}, x^{16}_{11,23}, x^{19}_{11,23}, x^{20}_{11,23}, x^{22}_{11,23}, x^{23}_{11,23} \geq 0$$
$$x^{5}_{23,11}, x^{6}_{23,11}, x^{7}_{23,11}, x^{8}_{23,11}, x^{9}_{23,11}, x^{10}_{23,11}, x^{12}_{23,11}, x^{13}_{23,11}, x^{15}_{23,11}, x^{16}_{23,11}, x^{19}_{23,11}, x^{20}_{23,11}, x^{22}_{23,11}, x^{23}_{23,11}, x_{23,11} \geq 0$$
$$x^{3}_{12,19}, x^{5}_{12,19}, x^{6}_{12,19}, x^{7}_{12,19}, x^{8}_{12,19}, x^{9}_{12,19}, x^{10}_{12,19}, x^{12}_{12,19}, x^{13}_{12,19}, x^{15}_{12,19}, x^{16}_{12,19}, x^{19}_{12,19}, x^{20}_{12,19}, x^{22}_{12,19}, x^{23}_{12,19} \geq 0$$
$$x^{5}_{19,12}, x^{6}_{19,12}, x^{7}_{19,12}, x^{8}_{19,12}, x^{9}_{19,12}, x^{10}_{19,12}, x^{12}_{19,12}, x^{13}_{19,12}, x^{15}_{19,12}, x^{16}_{19,12}, x^{19}_{19,12}, x^{20}_{19,12}, x^{22}_{19,12}, x^{23}_{19,12}, x_{19,12} \geq 0$$
$$x^{3}_{13,14}, x^{5}_{13,14}, x^{6}_{13,14}, x^{7}_{13,14}, x^{8}_{13,14}, x^{9}_{13,14}, x^{10}_{13,14}, x^{12}_{13,14}, x^{13}_{13,14}, x^{15}_{13,14}, x^{16}_{13,14}, x^{19}_{13,14}, x^{20}_{13,14}, x^{22}_{13,14}, x^{23}_{13,14} \geq 0$$
$$x^{5}_{14,13}, x^{6}_{14,13}, x^{7}_{14,13}, x^{8}_{14,13}, x^{9}_{14,13}, x^{10}_{14,13}, x^{12}_{14,13}, x^{13}_{14,13}, x^{15}_{14,13}, x^{16}_{14,13}, x^{19}_{14,13}, x^{20}_{14,13}, x^{22}_{14,13}, x^{23}_{14,13}, x_{14,13} \geq 0$$
$$x^{3}_{13,22}, x^{5}_{13,22}, x^{6}_{13,22}, x^{7}_{13,22}, x^{8}_{13,22}, x^{9}_{13,22}, x^{10}_{13,22}, x^{12}_{13,22}, x^{13}_{13,22}, x^{15}_{13,22}, x^{16}_{13,22}, x^{19}_{13,22}, x^{20}_{13,22}, x^{22}_{13,22}, x^{23}_{13,22} \geq 0$$
$$x^{5}_{22,13}, x^{6}_{22,13}, x^{7}_{22,13}, x^{8}_{22,13}, x^{9}_{22,13}, x^{10}_{22,13}, x^{12}_{22,13}, x^{13}_{22,13}, x^{15}_{22,13}, x^{16}_{22,13}, x^{19}_{22,13}, x^{20}_{22,13}, x^{22}_{22,13}, x^{23}_{22,13}, x_{22,13} \geq 0$$
$$x^{3}_{14,15}, x^{5}_{14,15}, x^{6}_{14,15}, x^{7}_{14,15}, x^{8}_{14,15}, x^{9}_{14,15}, x^{10}_{14,15}, x^{12}_{14,15}, x^{13}_{14,15}, x^{15}_{14,15}, x^{16}_{14,15}, x^{19}_{14,15}, x^{20}_{14,15}, x^{22}_{14,15}, x^{23}_{14,15} \geq 0$$
$$x^{5}_{15,14}, x^{6}_{15,14}, x^{7}_{15,14}, x^{8}_{15,14}, x^{9}_{15,14}, x^{10}_{15,14}, x^{12}_{15,14}, x^{13}_{15,14}, x^{15}_{15,14}, x^{16}_{15,14}, x^{19}_{15,14}, x^{20}_{15,14}, x^{22}_{15,14}, x^{23}_{15,14}, x_{15,14} \geq 0$$
$$x^{3}_{15,16}, x^{5}_{15,16}, x^{6}_{15,16}, x^{7}_{15,16}, x^{8}_{15,16}, x^{9}_{15,16}, x^{10}_{15,16}, x^{12}_{15,16}, x^{13}_{15,16}, x^{15}_{15,16}, x^{16}_{15,16}, x^{19}_{15,16}, x^{20}_{15,16}, x^{22}_{15,16}, x^{23}_{15,16} \geq 0$$
$$x^{5}_{16,15}, x^{6}_{16,15}, x^{7}_{16,15}, x^{8}_{16,15}, x^{9}_{16,15}, x^{10}_{16,15}, x^{12}_{16,15}, x^{13}_{16,15}, x^{15}_{16,15}, x^{16}_{16,15}, x^{19}_{16,15}, x^{20}_{16,15}, x^{22}_{16,15}, x^{23}_{16,15}, x_{16,15} \geq 0$$
$$x^{3}_{16,17}, x^{5}_{16,17}, x^{6}_{16,17}, x^{7}_{16,17}, x^{8}_{16,17}, x^{9}_{16,17}, x^{10}_{16,17}, x^{12}_{16,17}, x^{13}_{16,17}, x^{15}_{16,17}, x^{16}_{16,17}, x^{19}_{16,17}, x^{20}_{16,17}, x^{22}_{16,17}, x^{23}_{16,17} \geq 0$$
$$x^{5}_{17,16}, x^{6}_{17,16}, x^{7}_{17,16}, x^{8}_{17,16}, x^{9}_{17,16}, x^{10}_{17,16}, x^{12}_{17,16}, x^{13}_{17,16}, x^{15}_{17,16}, x^{16}_{17,16}, x^{19}_{17,16}, x^{20}_{17,16}, x^{22}_{17,16}, x^{23}_{17,16}, x_{17,16} \geq 0$$
$$x^{3}_{18,19}, x^{5}_{18,19}, x^{6}_{18,19}, x^{7}_{18,19}, x^{8}_{18,19}, x^{9}_{18,19}, x^{10}_{18,19}, x^{12}_{18,19}, x^{13}_{18,19}, x^{15}_{18,19}, x^{16}_{18,19}, x^{19}_{18,19}, x^{20}_{18,19}, x^{22}_{18,19}, x^{23}_{18,19} \geq 0$$
$$x^{5}_{19,18}, x^{6}_{19,18}, x^{7}_{19,18}, x^{8}_{19,18}, x^{9}_{19,18}, x^{10}_{19,18}, x^{12}_{19,18}, x^{13}_{19,18}, x^{15}_{19,18}, x^{16}_{19,18}, x^{19}_{19,18}, x^{20}_{19,18}, x^{22}_{19,18}, x^{23}_{19,18}, x_{19,18} \geq 0$$
$$x^{3}_{19,20}, x^{5}_{19,20}, x^{6}_{19,20}, x^{7}_{19,20}, x^{8}_{19,20}, x^{9}_{19,20}, x^{10}_{19,20}, x^{12}_{19,20}, x^{13}_{19,20}, x^{15}_{19,20}, x^{16}_{19,20}, x^{19}_{19,20}, x^{20}_{19,20}, x^{22}_{19,20}, x^{23}_{19,20} \geq 0$$
$$x^{5}_{20,19}, x^{6}_{20,19}, x^{7}_{20,19}, x^{8}_{20,19}, x^{9}_{20,19}, x^{10}_{20,19}, x^{12}_{20,19}, x^{13}_{20,19}, x^{15}_{20,19}, x^{16}_{20,19}, x^{19}_{20,19}, x^{20}_{20,19}, x^{22}_{20,19}, x^{23}_{20,19}, x_{20,19} \geq 0$$
$$x^{3}_{20,21}, x^{5}_{20,21}, x^{6}_{20,21}, x^{7}_{20,21}, x^{8}_{20,21}, x^{9}_{20,21}, x^{10}_{20,21}, x^{12}_{20,21}, x^{13}_{20,21}, x^{15}_{20,21}, x^{16}_{20,21}, x^{19}_{20,21}, x^{20}_{20,21}, x^{22}_{20,21}, x^{23}_{20,21} \geq 0$$

$$
\begin{array}{lcl}
x^{5}_{21,20}, x^{6}_{21,20}, x^{7}_{21,20}, x^{8}_{21,20}, x^{9}_{21,20}, x^{10}_{21,20}, x^{12}_{21,20}, x^{13}_{21,20}, x^{15}_{21,20}, x^{16}_{21,20}, x^{19}_{21,20}, x^{20}_{21,20}, x^{22}_{21,20}, x^{23}_{21,20}, x_{21,20} & \geq & 0 \\
x^{3}_{20,23}, x^{5}_{20,23}, x^{6}_{20,23}, x^{7}_{20,23}, x^{8}_{20,23}, x^{9}_{20,23}, x^{10}_{20,23}, x^{12}_{20,23}, x^{13}_{20,23}, x^{15}_{20,23}, x^{16}_{20,23}, x^{19}_{20,23}, x^{20}_{20,23}, x^{22}_{20,23}, x^{23}_{20,23} & \geq & 0 \\
x^{5}_{23,20}, x^{6}_{23,20}, x^{7}_{23,20}, x^{8}_{23,20}, x^{9}_{23,20}, x^{10}_{23,20}, x^{12}_{23,20}, x^{13}_{23,20}, x^{15}_{23,20}, x^{16}_{23,20}, x^{19}_{23,20}, x^{20}_{23,20}, x^{22}_{23,20}, x^{23}_{23,20}, x_{23,20} & \geq & 0 \\
x^{3}_{21,22}, x^{5}_{21,22}, x^{6}_{21,22}, x^{7}_{21,22}, x^{8}_{21,22}, x^{9}_{21,22}, x^{10}_{21,22}, x^{12}_{21,22}, x^{13}_{21,22}, x^{15}_{21,22}, x^{16}_{21,22}, x^{19}_{21,22}, x^{20}_{21,22}, x^{22}_{21,22}, x^{23}_{21,22} & \geq & 0 \\
x^{5}_{22,21}, x^{6}_{22,21}, x^{7}_{22,21}, x^{8}_{22,21}, x^{9}_{22,21}, x^{10}_{22,21}, x^{12}_{22,21}, x^{13}_{22,21}, x^{15}_{22,21}, x^{16}_{22,21}, x^{19}_{22,21}, x^{20}_{22,21}, x^{22}_{22,21}, x^{23}_{22,21}, x_{22,21} & \geq & 0 \\
x^{3}_{22,23}, x^{5}_{22,23}, x^{6}_{22,23}, x^{7}_{22,23}, x^{8}_{22,23}, x^{9}_{22,23}, x^{10}_{22,23}, x^{12}_{22,23}, x^{13}_{22,23}, x^{15}_{22,23}, x^{16}_{22,23}, x^{19}_{22,23}, x^{20}_{22,23}, x^{22}_{22,23}, x^{23}_{22,23} & \geq & 0 \\
x^{5}_{23,22}, x^{6}_{23,22}, x^{7}_{23,22}, x^{8}_{23,22}, x^{9}_{23,22}, x^{10}_{23,22}, x^{12}_{23,22}, x^{13}_{23,22}, x^{15}_{23,22}, x^{16}_{23,22}, x^{19}_{23,22}, x^{20}_{23,22}, x^{22}_{23,22}, x^{23}_{23,22}, x_{23,22} & \geq & 0
\end{array}
$$

The solution to this formulation yields the shortest path trees into (out of) all odd degree nodes, which enable us to compute the shortest path distances between each pair of odd degree nodes.

For odd degree nodes $i, j \in O$, let ℓ_{ij} denote the shortest path distance between i and j, as given in

Node	2	3	5	6	7	8	9	10	12	13	15	16	19	20	22	23
2	0	74	65	68	75	33	37	69	52	98	86	108	91	116	116	121
3	74	0	49	78	59	107	75	113	60	82	70	76	99	87	89	47
5	65	49	0	65	50	98	102	134	109	56	21	45	138	113	74	96
6	68	78	65	0	115	35	105	71	120	99	64	40	104	129	117	125
7	75	59	50	115	0	108	99	137	84	23	58	82	105	80	41	71
8	33	107	98	35	108	0	70	36	85	131	99	75	69	94	133	134
9	37	75	102	105	99	70	0	38	15	122	123	145	54	79	118	84
10	69	113	134	71	137	36	38	0	53	115	135	111	33	58	97	98
12	52	60	109	120	84	85	15	53	0	107	130	136	39	64	103	69
13	98	82	56	99	23	131	122	115	107	0	35	59	82	57	18	60
15	86	70	21	64	58	99	123	135	130	35	0	24	117	92	53	95
16	108	76	45	40	82	75	145	111	136	59	24	0	141	116	77	119
19	91	99	138	104	105	69	54	33	39	82	117	141	0	25	64	65
20	116	87	113	129	80	94	79	58	64	57	92	116	25	0	39	40
22	116	89	74	117	41	133	118	97	103	18	53	77	64	39	0	42
23	121	47	96	125	71	134	84	98	69	60	95	119	65	40	42	0

Table 1: Shortest path distances between each pair of odd degree nodes.

Table 1. Next we formulate a matching problem to find the least cost matching of odd degree nodes.
Decision variables:

Let $z_{ij} = \begin{cases} 1 & \text{if we match } i \text{ and } j \text{ (duplicate the edges on the shortest path between } i \text{ and } j) \\ 0 & \text{o/w} \end{cases}$

Then the formulation of the problem is as follows:

$$
\begin{array}{lrcll}
\min & & & \sum_{i \in O} \sum_{j \in O, j>i} \ell_{ij} z_{ij} & \\
\text{s.t.} & & & & \\
& \sum_{\{j \in O\,:\,j<i\}} z_{ji} + \sum_{\{j \in O\,:\,j>i\}} z_{ij} & = & 1 & \forall\, i \in O \\
& z_{ij} & \in & \{0,1\} & \forall\, i,j \in O,\ i<j
\end{array}
$$

With the given data, the formulation is

$$
\begin{aligned}
\min \quad & \{74z_{2,3} + 65z_{2,5} + 68z_{2,6} + 75z_{2,7} + 33z_{2,8} + 37z_{2,9} + 69z_{2,10} + 52z_{2,12} \\
& +98z_{2,13} + 86z_{2,15} + 108z_{2,16} + 91z_{2,19} + 116z_{2,20} + 116z_{2,22} + 121z_{2,23} + 49z_{3,5} \\
& +78z_{3,6} + 59z_{3,7} + 107z_{3,8} + 75z_{3,9} + 113z_{3,10} + 60z_{3,12} + 82z_{3,13} + 70z_{3,15} \\
& +76z_{3,16} + 99z_{3,19} + 87z_{3,20} + 89z_{3,22} + 47z_{3,23} + 65z_{5,6} + 50z_{5,7} + 98z_{5,8} \\
& +102z_{5,9} + 134z_{5,10} + 109z_{5,12} + 56z_{5,13} + 21z_{5,15} + 45z_{5,16} + 138z_{5,19} + 113z_{5,20} \\
& +74z_{5,22} + 96z_{5,23} + 115z_{6,7} + 35z_{6,8} + 105z_{6,9} + 71z_{6,10} + 120z_{6,12} + 99z_{6,13} \\
& +64z_{6,15} + 40z_{6,16} + 104z_{6,19} + 129z_{6,20} + 117z_{6,22} + 125z_{6,23} + 108z_{7,8} + 99z_{7,9}
\end{aligned}
$$

$$
\begin{array}{rcl}
& & +137z_{7,10} + 84z_{7,12} + 23z_{7,13} + 58z_{7,15} + 82z_{7,16} + 105z_{7,19} + 80z_{7,20} + 41z_{7,22} \\
& & +71z_{7,23} + 70z_{8,9} + 36z_{8,10} + 85z_{8,12} + 131z_{8,13} + 99z_{8,15} + 75z_{8,16} + 69z_{8,19} \\
& & +94z_{8,20} + 133z_{8,22} + 134z_{8,23} + 38z_{9,10} + 15z_{9,12} + 122z_{9,13} + 123z_{9,15} + 145z_{9,16} \\
& & +54z_{9,19} + 79z_{9,20} + 118z_{9,22} + 84z_{9,23} + 53z_{10,12} + 115z_{10,13} + 135z_{10,15} + 111z_{10,16} \\
& & +33z_{10,19} + 58z_{10,20} + 97z_{10,22} + 98z_{10,23} + 107z_{12,13} + 130z_{12,15} + 136z_{12,16} + 39z_{12,19} \\
& & +64z_{12,20} + 103z_{12,22} + 69z_{12,23} + 35z_{13,15} + 59z_{13,16} + 82z_{13,19} + 57z_{13,20} + 18z_{13,22} \\
& & +60z_{13,23} + 24z_{15,16} + 117z_{15,19} + 92z_{15,20} + 53z_{15,22} + 95z_{15,23} + 141z_{16,19} + 116z_{16,20} \\
& & +77z_{16,22} + 119z_{16,23} + 25z_{19,20} + 64z_{19,22} + 65z_{19,23} + 39z_{20,22} + 40z_{20,23} + 42z_{22,23}\} \\
\text{s.t.} & & z_{2,3} + z_{2,5} + z_{2,6} + z_{2,7} + z_{2,8} + z_{2,9} + z_{2,10} + z_{2,12} \\
& & +z_{2,13} + z_{2,15} + z_{2,16} + z_{2,19} + z_{2,20} + z_{2,22} + z_{2,23} \quad = \quad 1 \\
& & z_{2,3} + z_{3,5} + z_{3,6} + z_{3,7} + z_{3,8} + z_{3,9} + z_{3,10} + z_{3,12} \\
& & +z_{3,13} + z_{3,15} + z_{3,16} + z_{3,19} + z_{3,20} + z_{3,22} + z_{3,23} \quad = \quad 1 \\
& & z_{2,5} + z_{3,5} + z_{5,6} + z_{5,7} + z_{5,8} + z_{5,9} + z_{5,10} + z_{5,12} \\
& & +z_{5,13} + z_{5,15} + z_{5,16} + z_{5,19} + z_{5,20} + z_{5,22} + z_{5,23} \quad = \quad 1 \\
& & z_{2,6} + z_{3,6} + z_{5,6} + z_{6,7} + z_{6,8} + z_{6,9} + z_{6,10} + z_{6,12} \\
& & +z_{6,13} + z_{6,15} + z_{6,16} + z_{6,19} + z_{6,20} + z_{6,22} + z_{6,23} \quad = \quad 1 \\
& & z_{2,7} + z_{3,7} + z_{5,7} + z_{6,7} + z_{7,8} + z_{7,9} + z_{7,10} + z_{7,12} \\
& & +z_{7,13} + z_{7,15} + z_{7,16} + z_{7,19} + z_{7,20} + z_{7,22} + z_{7,23} \quad = \quad 1 \\
& & z_{2,8} + z_{3,8} + z_{5,8} + z_{6,8} + z_{7,8} + z_{8,9} + z_{8,10} + z_{8,12} \\
& & +z_{8,13} + z_{8,15} + z_{8,16} + z_{8,19} + z_{8,20} + z_{8,22} + z_{8,23} \quad = \quad 1 \\
& & z_{2,9} + z_{3,9} + z_{5,9} + z_{6,9} + z_{7,9} + z_{8,9} + z_{9,10} + z_{9,12} \\
& & +z_{9,13} + z_{9,15} + z_{9,16} + z_{9,19} + z_{9,20} + z_{9,22} + z_{9,23} \quad = \quad 1 \\
& & z_{2,10} + z_{3,10} + z_{5,10} + z_{6,10} + z_{7,10} + z_{8,10} + z_{9,10} + z_{10,12} \\
& & +z_{10,13} + z_{10,15} + z_{10,16} + z_{10,19} + z_{10,20} + z_{10,22} + z_{10,23} \quad = \quad 1 \\
& & z_{2,12} + z_{3,12} + z_{5,12} + z_{6,12} + z_{7,12} + z_{8,12} + z_{9,12} + z_{10,12} \\
& & +z_{12,13} + z_{12,15} + z_{12,16} + z_{12,19} + z_{12,20} + z_{12,22} + z_{12,23} \quad = \quad 1 \\
& & z_{2,13} + z_{3,13} + z_{5,13} + z_{6,13} + z_{7,13} + z_{8,13} + z_{9,13} + z_{10,13} \\
& & +z_{12,13} + z_{13,15} + z_{13,16} + z_{13,19} + z_{13,20} + z_{13,22} + z_{13,23} \quad = \quad 1 \\
& & z_{2,15} + z_{3,15} + z_{5,15} + z_{6,15} + z_{7,15} + z_{8,15} + z_{9,15} + z_{10,15} \\
& & +z_{12,15} + z_{13,15} + z_{15,16} + z_{15,19} + z_{15,20} + z_{15,22} + z_{15,23} \quad = \quad 1 \\
& & z_{2,16} + z_{3,16} + z_{5,16} + z_{6,16} + z_{7,16} + z_{8,16} + z_{9,16} + z_{10,16} \\
& & +z_{12,16} + z_{13,16} + z_{15,16} + z_{16,19} + z_{16,20} + z_{16,22} + z_{16,23} \quad = \quad 1 \\
& & z_{2,19} + z_{3,19} + z_{5,19} + z_{6,19} + z_{7,19} + z_{8,19} + z_{9,19} + z_{10,19} \\
& & +z_{12,19} + z_{13,19} + z_{15,19} + z_{16,19} + z_{19,20} + z_{19,22} + z_{19,23} \quad = \quad 1 \\
& & z_{2,20} + z_{3,20} + z_{5,20} + z_{6,20} + z_{7,20} + z_{8,20} + z_{9,20} + z_{10,20} \\
& & +z_{12,20} + z_{13,20} + z_{15,20} + z_{16,20} + z_{19,20} + z_{20,22} + z_{20,23} \quad = \quad 1 \\
& & z_{2,22} + z_{3,22} + z_{5,22} + z_{6,22} + z_{7,22} + z_{8,22} + z_{9,22} + z_{10,22} \\
& & +z_{12,22} + z_{13,22} + z_{15,22} + z_{16,22} + z_{19,22} + z_{20,22} + z_{22,23} \quad = \quad 1 \\
& & z_{2,23} + z_{3,23} + z_{5,23} + z_{6,23} + z_{7,23} + z_{8,23} + z_{9,23} + z_{10,23} \\
& & +z_{12,23} + z_{13,23} + z_{15,23} + z_{16,23} + z_{19,23} + z_{20,23} + z_{22,23} \quad = \quad 1 \\
& & z_{2,3}, z_{2,5}, z_{2,6}, z_{2,7}, z_{2,8}, z_{2,9}, z_{2,10}, z_{2,12}, z_{2,13}, z_{2,15}, z_{2,16}, z_{2,19}, z_{2,20}, z_{2,22}, z_{2,23}, z_{3,5} \quad \in \quad \{0,1\} \\
& & z_{3,6}, z_{3,7}, z_{3,8}, z_{3,9}, z_{3,10}, z_{3,12}, z_{3,13}, z_{3,15}, z_{3,16}, z_{3,19}, z_{3,20}, z_{3,22}, z_{3,23}, z_{5,6}, z_{5,7}, z_{5,8} \quad \in \quad \{0,1\} \\
& & z_{5,9}, z_{5,10}, z_{5,12}, z_{5,13}, z_{5,15}, z_{5,16}, z_{5,19}, z_{5,20}, z_{5,22}, z_{5,23}, z_{6,7}, z_{6,8}, z_{6,9}, z_{6,10}, z_{6,12}, z_{6,13} \quad \in \quad \{0,1\} \\
& & z_{6,15}, z_{6,16}, z_{6,19}, z_{6,20}, z_{6,22}, z_{6,23}, z_{7,8}, z_{7,9}, z_{7,10}, z_{7,12}, z_{7,13}, z_{7,15}, z_{7,16}, z_{7,19}, z_{7,20}, z_{7,22} \quad \in \quad \{0,1\} \\
& & z_{7,23}, z_{8,9}, z_{8,10}, z_{8,12}, z_{8,13}, z_{8,15}, z_{8,16}, z_{8,19}, z_{8,20}, z_{8,22}, z_{8,23}, z_{9,10}, z_{9,12}, z_{9,13}, z_{9,15}, z_{9,16} \quad \in \quad \{0,1\} \\
& & z_{9,19}, z_{9,20}, z_{9,22}, z_{9,23}, z_{10,12}, z_{10,13}, z_{10,15}, z_{10,16}, z_{10,19}, z_{10,20}, z_{10,22}, z_{10,23}, z_{12,13}, z_{12,15}, z_{12,16}, z_{12,19} \quad \in \quad \{0,1\} \\
& & z_{12,20}, z_{12,22}, z_{12,23}, z_{13,15}, z_{13,16}, z_{13,19}, z_{13,20}, z_{13,22}, z_{13,23}, z_{15,16}, z_{15,19}, z_{15,20}, z_{15,22}, z_{15,23}, z_{16,19}, z_{16,20} \quad \in \quad \{0,1\} \\
& & z_{16,22}, z_{16,23}, z_{19,20}, z_{19,22}, z_{19,23}, z_{20,22}, z_{20,23}, z_{22,23} \quad \in \quad \{0,1\}
\end{array}
$$

The optimal solution is $z^*_{2,8} = 1$, $z^*_{3,23} = 1$, $z^*_{5,15} = 1$, $z^*_{6,16} = 1$, $z^*_{7,13} = 1$, $z^*_{9,12} = 1$, $z^*_{10,19} = 1$, $z^*_{20,22} = 1$, all other $z^*_{i,j} = 0$, that is, it is optimal to duplicate the edges on the shortest paths between 2 and 8, 3 and 23, 5 and 15, 6 and 16, 7 and 13, 9 and 12, 10 and 19, and 20 and 22.
Thus we duplicate the edges on 2–8, 3–11–23, 5–15, 6–17–16, 7–13, 9–12, 10–18–19, and 20–21–22.
An Euler tour on the new augmented network can be obtained with the depth first procedure covered in class.
An Optimal Euler tour = 1 – 2 – 8 – 6 – 4 – 3 – 11 – 7 – 13 – 14 – 15 – 5 – 4 – 16 – 17 – 6 – 17 – 16 – 15 – 5 – 1 – 3 – 11 – 23 – 22 – 21 – 20 – 19 – 12 – 9 – 10 – 18 – 19 – 18 – 10 – 8 – 2 – 9 – 12 – 11 – 23 – 20 – 21 – 22 – 13 – 7 – 1
Optimal Cost = 1220

Problem 2

The following network represents the network of one way streets in some city. These streets have to be cleared each time it snows. Streets are cleared by a specially equipped snowplow that cruises along the streets, scraping the snow to the sides. The snowplow can cruise along the streets only in the designated directions. The cost of operating the snowplowing service (including the cost of the plow, fuel and crew) is represented by costs on the edges of the network. Design a snowplowing tour for the plow that will traverse each street at least once, that will return the plow to its origin (plowhouse), and that will incur the minimum total cost. As a first step to designing plow routes, consider the case with a single snowplow, and ignore other constraints such as turn constraints at intersections, and route duration constraints. Write a complete LP formulation, explicitly with all applicable data values, of all problems you solve. (Hint: Use an LP solver of your choice, such as LINDO, GAMS, CPlex, XPRESS-MP, AIMMS, AMPL, to solve the optimization problems encountered along the way.) (25)

Answer: Let $N = \{1, 2, \ldots, 23\}$ denote the set of nodes. Let $A = \{(1,2), (1,3), (5,1), (1,7), (2,8), (2,9), (3,4), (3,11), (4,5), (6,4), (4,16), (5,15), (6,8), (17,6), (7,11), (7,13), (8,10), (9,10), (9,12), (10,18), (11,12), (11,23), (12,19), (13,14), (22,13), (14,15), (15,16), (16,17), (18,19), (19,20), (20,21), (20,23), (21,22), (23,22)\}$ denote the set of arcs. Let $G = (N, A)$ denote the given directed network.

Let c_{ij} denote the cost associated with arc (i, j).

For each node $i \in N$, let d_i denote the degree of i = outdegree - indegree of node i.

The first problem is to determine how many additional times (in addition to the requirement of at least one traversal) to traverse each arc to obtain a balanced network with minimum cost. Decision variables: Let x_{ij} denote the number of additional times to traverse arc (i, j).

Then the formulation of the problem is as follows:

$$
\begin{array}{lrcll}
\min & \sum_{(i,j)\in A} c_{ij}x_{ij} & & & \\
\text{s.t.} & & & & \\
& \sum_{\{j\in N\,:\,(j,i)\in A\}} x_{ji} - \sum_{\{j\in N\,:\,(i,j)\in A\}} x_{ij} & = & d_i & \forall\, i \in N \\
& x_{ij} & \geq & 0 & \forall\, (i,j) \in A
\end{array}
$$

With the given data, the formulation is

$$
\begin{array}{lrcl}
\min & \{45x_{1,2} + 29x_{1,3} + 20x_{5,1} + 30x_{1,7} + 33x_{2,8} + 37x_{2,9} & & \\
& +44x_{3,4} + 19x_{3,11} + 31x_{4,5} + 34x_{6,4} + 32x_{4,16} + 21x_{5,15} & & \\
& +35x_{6,8} + 14x_{17,6} + 43x_{7,11} + 23x_{7,13} + 36x_{8,10} + 38x_{9,10} & & \\
& +15x_{9,12} + 16x_{10,18} + 41x_{11,12} + 28x_{11,23} + 39x_{12,19} + 22x_{13,14} & & \\
& +18x_{22,13} + 13x_{14,15} + 24x_{15,16} + 26x_{16,17} + 17x_{18,19} + 25x_{19,20} & & \\
& +27x_{20,21} + 40x_{20,23} + 12x_{21,22} + 42x_{23,22}\} & & \\
\text{s.t.} & x_{5,1} - x_{1,2} - x_{1,3} - x_{1,7} & = & 2 \\
& x_{1,2} - x_{2,8} - x_{2,9} & = & 1 \\
& x_{1,3} - x_{3,4} - x_{3,11} & = & 1 \\
& x_{3,4} + x_{6,4} - x_{4,5} - x_{4,16} & = & 0 \\
& x_{4,5} - x_{5,1} - x_{5,15} & = & 1 \\
& x_{17,6} - x_{6,4} - x_{6,8} & = & 1 \\
& x_{1,7} - x_{7,11} - x_{7,13} & = & 1 \\
& x_{2,8} + x_{6,8} - x_{8,10} & = & -1 \\
& x_{2,9} - x_{9,10} - x_{9,12} & = & 1 \\
& x_{8,10} + x_{9,10} - x_{10,18} & = & -1 \\
& x_{3,11} + x_{7,11} - x_{11,12} - x_{11,23} & = & 0 \\
& x_{9,12} + x_{11,12} - x_{12,19} & = & -1
\end{array}
$$

$$
\begin{aligned}
x_{7,13} + x_{22,13} - x_{13,14} &= -1 \\
x_{13,14} - x_{14,15} &= 0 \\
x_{5,15} + x_{14,15} - x_{15,16} &= -1 \\
x_{4,16} + x_{15,16} - x_{16,17} &= -1 \\
x_{16,17} - x_{17,6} &= 0 \\
x_{10,18} - x_{18,19} &= 0 \\
x_{12,19} + x_{18,19} - x_{19,20} &= -1 \\
x_{19,20} - x_{20,21} - x_{20,23} &= 1 \\
x_{20,21} - x_{21,22} &= 0 \\
x_{21,22} + x_{23,22} - x_{22,13} &= -1 \\
x_{11,23} + x_{20,23} - x_{23,22} &= -1 \\
x_{1,2}, x_{1,3}, x_{5,1}, x_{1,7}, x_{2,8}, x_{2,9}, x_{3,4}, x_{3,11}, x_{4,5} &\geq 0 \\
x_{6,4}, x_{4,16}, x_{5,15}, x_{6,8}, x_{17,6}, x_{7,11}, x_{7,13}, x_{8,10}, x_{9,10} &\geq 0 \\
x_{9,12}, x_{10,18}, x_{11,12}, x_{11,23}, x_{12,19}, x_{13,14}, x_{22,23}, x_{14,15}, x_{15,16} &\geq 0 \\
x_{16,17}, x_{18,19}, x_{19,20}, x_{20,21}, x_{20,23}, x_{21,22}, x_{23,22} &\geq 0
\end{aligned}
$$

The optimal solution is $x^*_{1,2} = 2$, $x^*_{1,3} = 1$, $x^*_{5,1} = 6$, $x^*_{1,7} = 1$, $x^*_{2,9} = 1$, $x^*_{4,5} = 7$, $x^*_{6,4} = 7$, $x^*_{17,6} = 8$, $x^*_{8,10} = 1$, $x^*_{10,18} = 2$, $x^*_{12,19} = 1$, $x^*_{13,14} = 6$, $x^*_{22,13} = 5$, $x^*_{14,15} = 6$, $x^*_{15,16} = 7$, $x^*_{16,17} = 8$, $x^*_{18,19} = 2$, $x^*_{19,20} = 4$, $x^*_{20,21} = 3$, $x^*_{21,22} = 3$, $x^*_{23,22} = 1$, all other $x^*_{i,j} = 0$. This gives the number of additional times (in addition to the requirement of at least one traversal) to traverse each arc to obtain a balanced network with minimum cost. An Euler tour on the new augmented network can be obtained with the depth first procedure covered in class.

An Optimal Euler tour = 1 – 2 – 8 – 10 – 18 – 19 – 20 – 21 – 22 – 13 – 14 – 15 – 16 – 17 – 6 – 8 – 10 – 18 – 19 – 20 – 23 – 22 – 13 – 14 – 15 – 16 – 17 – 6 – 4 – 16 – 17 – 6 – 4 – 5 – 15 – 16 – 17 – 6 – 4 – 5 – 1 – 2 – 9 – 12 – 19 – 20 – 21 – 22 – 13 – 14 – 15 – 16 – 17 – 6 – 4 – 5 – 1 – 2 – 9 – 10 – 18 – 19 – 20 – 21 – 22 – 13 – 14 – 15 – 16 – 17 – 6 – 4 – 1 – 3 – 11 – 12 – 19 – 20 – 21 – 22 – 13 – 14 – 15 – 16 – 17 – 6 – 4 – 5 – 1 – 3 – 4 – 5 – 1 – 7 – 11 – 23 – 22 – 13 – 14 – 15 – 16 – 17 – 6 – 4 – 5 – 1 – 7 – 13 – 14 – 15 – 16 – 17 – 6 – 4 – 5 – 1

Optimal Cost = 2918