Single-Vehicle, Multiple-Destination Transportation Routing (Traveling Salesperson Problem)

[75 points]
This homework can be completed in groups, and is worth 75 points. There will be a 50-point homework quiz in class on Wednesday November 10. Note of course that all group members will be required to know all heuristics for the quiz.

Logistics is important not only for providers of goods, but also for providers of services. Transportation is often the primary concern for companies that provide on-site or in-person services. One such example is a traveling circus like Ringling Brothers and Barnum & Bailey, “The Greatest Show On Earth”.

The Ringling Brothers and Barnum & Bailey circus “cheats” by actually having two touring circuses, the “red” circus and the “blue” circus, which tour the country simultaneously and thus are able to cover more cities.

In this homework, we will be looking at the travels of a fictional circus with just a single touring unit. This Atlanta-based circus plans to visit 16 cities, listed in the table below:

<table>
<thead>
<tr>
<th>City</th>
<th>State</th>
<th>City</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta GA</td>
<td>Des Moines IA</td>
<td>Kansas City MO</td>
<td>Nashville TN</td>
</tr>
<tr>
<td>Birmingham AL</td>
<td>Dubuque IA</td>
<td>Little Rock AR</td>
<td>Oklahoma Cit OK</td>
</tr>
<tr>
<td>Columbus OH</td>
<td>Indianapolis IN</td>
<td>Louisville KY</td>
<td>Saint Louis MO</td>
</tr>
<tr>
<td>Dallas TX</td>
<td>Jackson MS</td>
<td>Memphis TN</td>
<td>Springfield IL</td>
</tr>
</tbody>
</table>

After visiting all of the cities, the circus will return to Atlanta.

As one might imagine, the travel costs associated with a circus are quite high. Special vehicles, equipment, foods, etc. are needed to transport the tigers, elephants and other circus animals. Other equipment, such as high-wire apparatus and set construction materials, are also expensive to transport. Therefore, the circus would like to minimize its transportation costs during its national tour.

Of course, a benefit of minimizing travel time is that the circus will have more time to perform, and thus generate more revenue. Since the special circus vehicles can only travel on major highways, we can assume a constant speed for every highway. Therefore, minimizing travel time is equivalent to minimizing travel distance in this case.

The goal of the circus is to find a good tour of all the cities; that is, to find a path that visits each city and then returns to its starting point. Of course, some tours are shorter than others, and the optimal tour is the one that requires the least total distance.
Part I

Use your own intuition to determine the shortest-distance circus tour you can find. Do not use any of the algorithms from class – don’t worry, you’ll have plenty of opportunity for that in the rest of the homework, and your grades will not depend on the quality of the solution you find in this part. For now, just try using your intuition and logic. Draw your best answer clearly on the attached map, and give the total distance.

Part II

Using the distances provided, use the Nearest Neighbor heuristic from class to find a tour for the circus to take. Use Atlanta as the starting node. Draw your answer clearly on the attached map, number the arcs in the sequence that you choose them, and give the total distance.

(Hint: Consider using Excel to calculate the closest node to p during each step)

Part III

Repeat Part II, but use the Twice-Around MST heuristic to find the tour. Use Prim’s algorithm to find the minimum spanning tree, starting from Atlanta. Draw your answer clearly on the attached map, showing both the tree and the final tour, and give the total distance.

Part IV

Repeat Part II, but use the Nearest Insertion heuristic instead of Nearest Neighbor. By the nature of the method, your tour should never contain any arc crossings. Start your tour by choosing first the arc with minimum cost. Draw your answer clearly on the attached map, number the nodes in the sequence that you insert them, and give the total distance.

(Hint: Since a tour built from an insertion heuristic will never contain arc crossings for geographic networks, you only need to calculate for insertion locations that would not create a crossing. Use Excel.)

Part V

Repeat Part II, but use the Farthest Insertion heuristic instead of Nearest Neighbor. By the nature of the method, your tour should never contain any arc crossings. Start your tour by choosing first the arc with maximum cost. Draw your answer clearly on the attached map, number the nodes in the sequence that you insert them, and give the total distance.

(Hint: See the hint for part III)

Part VI

Which tour, I-V, is the best tour? Can you guarantee that the method producing the best tour for this example will produce the best results, even if other cities needed to be visited?
Part I – “Intuition” Tour

Total Distance: ________

Any reasonable answer would work for this problem, with a brief explanation.
### City-to-City Distances

<table>
<thead>
<tr>
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<th>LiR</th>
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<th>OkC</th>
<th>KY</th>
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<td>612</td>
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<td>701</td>
<td>768</td>
<td>442</td>
<td>468</td>
<td>480</td>
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<td>174</td>
<td>533</td>
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<td>517</td>
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<td>616</td>
<td>243</td>
<td>0</td>
</tr>
</tbody>
</table>
Part II – Nearest Neighbor Tour

Total Distance: 3984

Steps

1. Nearest neighbor to Atlanta = Birmingham, 148 miles
2. Nearest neighbor to Birmingham = Nashville, 183 miles
3. Nearest neighbor to Nashville = Louisville, 161 miles
4. " Louisville = Indianapolis, 108 miles
5. " Indianapolis = Columbus, 175 miles
6. " Columbus = Springfield, 360 miles
7. " Springfield = St Louis, 81 miles
8. " St Louis = Kansas City, 231 miles
9. " Kansas City = Des Moines, 160 miles
10." Des Moines = Dubuque, 166 miles
11." Dubuque = Memphis, 517 miles
12." Memphis = Little Rock, 147 miles
13." Little Rock = Jackson, 215 miles
14." Jackson = Dallas, 390 miles
15." Dallas = OK City, 174 miles
16." OK City = Atlanta, 768 miles
Part III – Twice-around MST Tour

Total Distance: __3584__

PRIM’S STEPS

1. Nearest to Atlanta (A): Birmingham (B) [from (A)]
2. Nearest to {A, B}: Nashville (N) [from (B)]
3. Nearest to {A, B, N}: Louisville (L) [from (N)]
4. Nearest to {A, B, N, L}: Indianapolis (I) [from (L)]
5. Nearest to {A, B, N, L, I}: Columbus (C) [from (I)]
6. Nearest to {A, B, N, L, I, C}: Springfield (S) [from (I)]
7. Nearest to {A, B, N, L, I, C, S}: St Louis (SL) [from (S)]
8. Nearest to {A, B, N, L, I, C, S, SL}: Memphis (M) [from (N)]
9. Nearest to {A, B, N, L, I, C, S, SL, M}: Little Rock (LR) [from (M)]
10. Nearest to {A, B, N, L, I, C, S, SL, M, LR}: Jackson (J) [from (M)]
11. Nearest to {A, B, N, L, I, C, S, SL, M, LR, J}: Dubuque (U) [from (S)]
12. Nearest to {A, B, N, L, I, C, S, SL, M, LR, J, U}: Des Moines (DM) [from (U)]
13. Nearest to {A, B, N, L, I, C, S, SL, M, LR, J, U, DM}: Kansas City (K) [from (DM)]
14. Nearest to {A, B, N, L, I, C, S, SL, M, LR, J, U, DM, K}: Dallas (D) [from (LR)]
15. Oklahoma City (O) [from (D)]

WALK FROM ATLANTA
Part IV – Nearest Insertion Tour

Total Distance: **3319**

Initialize: \( T = \{ S, SL, S \}; \quad C(T) = 162 \)

Iteration: 1 Nearest node for insertion: \( I \); Insert node between: \( S \) and \( SL \)
Insertion cost: 335; \( T = \{ S, I, SL, S \}; \quad C(T) = 162 + 335 = 497 \)

Iteration: 2 Nearest node for insertion: \( L \); Insert node between: \( I \) and \( SL \)
Insertion cost: 137; \( T = \{ S, I, L, SL, S \}; \quad C(T) = 497 + 137 = 634 \)

Iteration: 3 Nearest node for insertion: \( N \); Insert node between: \( L \) and \( SL \)
Insertion cost: 159; \( T = \{ S, I, L, N, SL, S \}; \quad C(T) = 634 + 159 = 793 \)

Iteration: 4 Nearest node for insertion: \( C \); Insert node between: \( I \) and \( L \)
Insertion cost: 248; \( T = \{ S, I, C, L, N, SL, S \}; \quad C(T) = 793 + 248 = 1041 \)

Iteration: 5 Nearest node for insertion: \( B \); Insert node between: \( N \) and \( SL \)
Insertion cost: 332; \( T = \{ S, I, C, L, N, B, SL, S \}; \quad C(T) = 1041 + 332 = 1373 \)

Iteration: 6 Nearest node for insertion: \( A \); Insert node between: \( N \) and \( B \)
Insertion cost: 177; \( T = \{ S, I, C, L, N, A, B, SL, S \}; \quad C(T) = 1373 + 177 = 1550 \)

Iteration: 7 Nearest node for insertion: \( M \); Insert node between: \( B \) and \( SL \)
Insertion cost: 52; \( T = \{ S, I, C, L, N, A, B, M, SL, S \}; \quad C(T) = 1550 + 52 = 1602 \)

Iteration: 8 Nearest node for insertion: \( LR \); Insert node between: \( M \) and \( SL \)
Insertion cost: 188; \( T = \{ S, I, C, L, N, A, B, M, LR, SL, S \}; \quad C(T) = 1602 + 188 = 1790 \)

Iteration: 9 Nearest node for insertion: \( J \); Insert node between: \( B \) and \( M \)
Insertion cost: 201; \( T = \{ S, I, C, L, N, A, B, J, M, LR, SL, S \}; \quad C(T) = 1790 + 201 = 1991 \)
Iteration: 10 Nearest node for insertion: U; Insert node between: S and I
Iteration: 11 Nearest node for insertion: DM; Insert node between: S and U
Iteration: 12; Nearest node for insertion: K; Insert node between: S and DM
Iteration: 13; Nearest node for insertion: D; Insert node between: LR and SL
Iteration: 14; Nearest node for insertion: O; Insert node between: D and SL
Part V – Farthest Insertion Tour

Total Distance: 3033

Initialization: \( T=\{D,C,D\}; \ C(T)=1822 \)
Iteration: 1; Farthest node for insertion: DM; Insert node between: D and C
Insertion cost: 281; \( T=\{D,DM,C,D\}; \ C(T)=1822+281=2103 \)
Iteration: 2; Farthest node for insertion: B; Insert node between: C and D
Insertion cost: 163; \( T=\{D,DM,C,B,D\}; \ C(T)=2103+163=2266 \)
Iteration: 3; Farthest node for insertion: LR; Insert node between: B and D
Insertion cost: 34; \( T=\{D,DM,C,B,LR,D\}; \ C(T)=2266+34=2300 \)
Iteration: 4; Farthest node for insertion: SL; Insert node between: DM and C
Insertion cost: 94; \( T=\{D,DM,SL,C,B,LR,D\}; \ C(T)=2300+94=2394 \)
Iteration: 5; Farthest node for insertion: J; Insert node between: B and LR
Insertion cost: 96; \( T=\{D,DM,SL,C,B,J,LR,D\}; \ C(T)=2394+96=2490 \)
Iteration: 6; Farthest node for insertion: N; Insert node between: C and B
Insertion cost: 26; \( T=\{D,DM,SL,C,N,B,J,LR,D\}; \ C(T)=2490+26=2516 \)
Iteration: 7; Farthest node for insertion: I; Insert node between: SL and C
Insertion cost: 2; \( T=\{D,DM,SL,I,C,N,B,J,LR,D\}; \ C(T)=2516+2=2518 \)
Iteration: 8; Farthest node for insertion: O; Insert node between: D and DM
Insertion cost: 33; \( T=\{D,O,DM,SL,I,C,N,B,J,LR,D\}; \ C(T)=2518+33=2551 \)
Iteration: 9; Farthest node for insertion: U; Insert node between: DM and SL
Insertion cost: 159; \( T=\{D,O,DM,U,SL,I,C,N,B,J,LR,D\}; \ C(T)=2551+159=2710 \)
Iteration: 10; Farthest node for insertion: K; Insert node between: O and DM
Insertion cost: 6; \( T=\{D,O,K,DM,U,SL,I,C,N,B,J,LR,D\}; \ C(T)=2710+6 =2716 \)
Iteration: 11; Farthest node for insertion: A; Insert node between: N and B
Insertion cost: 177; \( T = \{D,O,K,DM,U,SL,I,C,N,A,B,J,LR,D\} \); \( C(T) = 2716 + 177 = 2893 \)

Iteration: 12; Farthest node for insertion: M; Insert node between: J and LR
Insertion cost: 121; \( T = \{D,O,K,DM,U,SL,I,C,N,A,B,J,M,LR,D\} \); \( C(T) = 2893 + 121 = 3014 \)

Iteration: 13; Farthest node for insertion: L; Insert node between: C and N
Insertion cost: 6; \( T = \{D,O,K,DM,U,SL,I,C,L,N,A,B,J,M,LR,D\} \); \( C(T) = 3014 + 6 = 3020 \)

Iteration: 14; Farthest node for insertion: S; Insert node between: U and SL