1. (a) Same as Problem 2 HW6 part a. \( c = \frac{2}{11} \)
(b) Same as Problem 2 HW6 part e. the first moment of \( X \): \( E[X] = \frac{2}{11} \)
   the second moment of \( X \): \( E[X^2] = \sum_k k^2 P(X = k) = (-2)^2 \frac{2}{11} + (-1)^2 \frac{7}{11} + \frac{2}{11} = \frac{9}{11} \)
(c) \( \text{Var}(X) = E[X^2] - (E[X])^2 = \frac{9}{11} - (\frac{2}{11})^2 = \frac{95}{121} \)
(d) \( \sigma = \sqrt{\text{Var}(X)} = \frac{\sqrt{95}}{11} \)

2. (a) Same as Problem 3 HW6 part a. \( c = \frac{1}{3} \)
(b) Same as Problem 3 HW6 part e. The first moment of \( Y \): \( E[Y] = \frac{5}{4} \)
   the second moment of \( Y \): \( E[Y^2] = \int_0^\infty s^2 f(s) ds = \int_0^1 s^2 \cos^4 ds = \frac{13}{6} \)
(c) \( \text{Var}(Y) = E[Y^2] - (E[Y])^2 = \frac{13}{6} - (\frac{5}{4})^2 = \frac{51}{24} \)
(d) \( \sigma = \sqrt{\text{Var}(Y)} = \frac{\sqrt{51}}{12} \)

3. (a) \( E[X] = \sum_k k = 16kP(X = k) = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = \frac{7}{2} \)
(b) \( E[X^2] = \sum_k k^2 P(X = k) = \frac{1}{6}(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) = \frac{91}{6} \)
(c) \( \text{Var}(X) = E[X^2] - (E[X])^2 = \frac{91}{6} - (\frac{7}{2})^2 = \frac{35}{12} \)
(d) \( \sigma = \sqrt{\text{Var}(X)} = \frac{\sqrt{35}}{12} \)

4. (a) cdf of \( X \):
   For all \( t < 0 \) we have: \( P(X \leq t) = P(3ln(1-U) \leq t) = P(ln(1-U) \leq \frac{t}{3}) = P(1-U \leq e^{\frac{t}{3}}) = P(1-e^{\frac{t}{3}} \leq U) = 1 - P(U \leq 1 - e^{\frac{t}{3}}) = 1 - (1 - e^{\frac{t}{3}}) = e^{\frac{t}{3}} \)
   for \( t \geq 0 \) we have: \( P(X \leq t) = P(3ln(1-U) \leq t) = 1 \)

pdf of \( X \):
   For \( t < 0 \) we have: \( f(t) = \frac{d}{dt} e^{\frac{t}{3}} = \frac{1}{3} e^{\frac{t}{3}} \)
   for \( t \geq 0 \) we have: \( f(t) = \frac{d}{dt} 1 = 0 \)

(b) The distribution of \( X \) is expontial with \( \lambda = \frac{1}{3} \)

5. (a) pmf of \( Y \):
   \( P(Y = k) = P(k - 1 < x \leq k) = F_x(k) - F_x(k - 1) = e^\frac{k}{3} - e^\frac{k-1}{3} = e^\frac{k}{3}(1 - e^\frac{-1}{3}) \)
   (b) The distribution of \( Y = 1 \) is geometric with parameter \( p = 1 - e^\frac{-1}{3} \)

6. (a) Let \( v \) = the speed that the worker walks, \( r \) = time to retrieve the item. Then, \( T = \frac{v}{v} + r = \frac{2v}{v} + 12 \)
(b) Given \( E[L] = 20 \)
   \( E[T] = E[\frac{2v}{v} + 12] = \frac{2}{v} E[L] + E[12] = 20 \)
(c) Since \( \text{Var}(aX + b) = a^2 \text{Var}(X) \), given \( \text{Var}(L) = 100 \)
   we have \( \text{Var}(T) = \text{Var}(\frac{2}{v} L + 12) = (\frac{2}{v})^2 \text{Var}(L) = 16 \)

7. (a) pdf of \( L \):
   \( f(L) = 0.85 \frac{1}{120} \frac{L}{120} \) for \( 0 \leq L \leq 10 \)
   \( f(L) = 0.15 \frac{1}{120} \frac{L}{120} \) for \( 10 \leq L \leq 50 \)
   then \( E[L] = \int_0^{50} L f(L) dL = \int_0^{10} 0.85 \frac{1}{120} L dL + \int_{10}^{50} 0.15 \frac{1}{120} L dL = 8.75 \)
   \( E[L^2] = \int_0^{50} L^2 f(L) dL = \int_0^{10} 0.85 \frac{1}{120} L^2 dL + \int_{10}^{50} 0.15 \frac{1}{120} L^2 dL = 183.33 \)
   \( \text{Var}(L) = E[L^2] - (E[L])^2 = 106.77 \)
8. (a) Let’s call sides of the rectangle a and b. We’ll find maximum rectangular area A such that sum of the lengths of three sides of the rectangle are equal to X: \(2a + b = X\). So, we have the following problem:

\[
\max ab \quad \text{such that} \quad 2a + b = X
\]

We have \(b = X - 2a\) and then \(A = ab = a(X - 2a) = ax - 2a^2\) to find the optimal length of a:

\[
\frac{d}{da} A = X - 4a \quad \text{and} \quad X - 4a = 0 \quad \text{for} \quad a = \frac{1}{4} X
\]

Then \(b = \frac{1}{2} X\)

(b) \(A = ab = \frac{1}{4} X \frac{1}{2} X = \frac{X^2}{8}\)

(c) First we’ll find cdf of A: \(F_A(t) = P(\frac{\pi X^2}{8} \leq t) = P(X^2 \leq 8t) = P(X \leq \sqrt{8t}) = 1 - e^{-\frac{1}{\sqrt{8t}}}\)

Then pdf of A is \(f(t) = \frac{\sqrt{2e}}{10\sqrt{t}}\)

(d) \(E[A] = E[\frac{X^2}{8}] = \frac{1}{8} E[X^2] = \frac{200}{8} = 25\) since \(E[X^2] = \text{Var}[X] + E[X]^2 = 100 + 100 = 200\) (using the fact that X is exponentially distributed with mean 10)

9. (a) cdf of the area: \(F_A(t) = P(\pi X^2 \leq t) = P(X \leq \sqrt{\frac{t}{\pi}})\) Since X is exponentially distributed we have

\[
F_X(\sqrt{\frac{t}{\pi}}) = 1 - e^{-\frac{1}{\sqrt{t}}} = \frac{1}{\sqrt{\pi}}
\]

Then pdf of the area: \(f(t) = \frac{\sqrt{2e}}{4\sqrt{\pi t}}\)

(b) Mean of the area: \(E[A] = E[\pi X^2] = \pi E[X^2] = 8\pi\) since \(E[X^2] = \text{Var}[X] + E[X]^2 = 4 + 4 = 8\) (using the fact that X is exponentially distributed with mean 2)