1. The sample space is \( S = \{1, 2, 3, 4, 5, 6\} \times \{H, T\} \times \{1, 2, 3, 4, 5, 6\} \). Here “x” denotes the Cartesian product: \( S \) is the set of ordered triples \((a, b, c)\), where \( a \) and \( c \) are elements of \( \{1, 2, 3, 4, 5, 6\} \), and \( b \) is an element of \( \{H, T\} \). All \( 6 \times 2 \times 6 = 72 \) elements of \( S \) are equally likely if the die and coin are fair.

2. Since the events \( A_k \) partition the sample space (that is, they are pairwise disjoint events and their union is all of \( S \)), we must have \( \sum_k P(A_k) = 1 \). Therefore \( c(1 + 2 + 3 + 4 + 5) = 1 \), and we find that \( c = \frac{1}{15} \).

As given, \( P(A_k) = \frac{k}{15} \) for \( k \in \{1, 2, 3, 4, 5\} \). Since the events \( A_k \) are disjoint, we can add their probabilities to determine the probability of their unions:

- \( P(B_1) = P(S) = 1 \), \( P(B_2) = P(A_2) + P(A_3) + P(A_4) + P(A_5) = \frac{14}{15} \), \( P(B_3) = P(A_3) + P(A_4) + P(A_5) = \frac{12}{15} \), \( P(B_4) = P(A_4) + P(A_5) = \frac{9}{15} \).

3. The events \( A_k \) partition the sample space, so we have \( \sum_{k=0}^{\infty} P(A_k) = 1 \), which means that \( c \sum_{k=0}^{\infty} \frac{6^k}{k!} = 1 \).

Recall that by Taylor series we obtain \( \sum_{k=0}^{\infty} \frac{6^k}{k!} = e^6 \). Therefore \( c = e^{-6} \). As given, \( P(A_0) = e^{-6} \) (note that \( 0! = 1 \)), and since the events \( A_k \) are disjoint \( P(B_1) = P(A_0) + P(A_1) = 7e^{-6} \).

4. The events \( A_k \) partition the sample space, so we have \( \sum_{k=1}^{\infty} P(A_k) = 1 \), which means that \( c \sum_{k=1}^{\infty} \left(\frac{1}{6}\right)^k = 1 \).

Recall that if \( G_n = \sum_{k=1}^{n} r^k \), then \( (1-r)G_n = r - r^{n+1} \), and for \( |r| < 1 \) and \( n \to \infty \) we have \( G_\infty = r/(1-r) \). Therefore \( c = 5 \), and as given \( P(A_1) = \frac{5}{6} \).