Homework 1
August 23, 2005
due on Tuesday, 24 August

1. Let the universal set \( S = \{1, 2, \ldots \} \). Let \( A \) denote the even integers in \( S \), and \( B = \{8, \ldots, 15\} \). What are \( \bar{A}, A \cap B, A \cup B, \) and \( \bar{B} \cap A \) (where \( \bar{B} \) denotes the complement of \( B \))?

2. Let the universal set \( S = [0, 8) \). Let \( A = [0, 2) \) and \( B = [7/5, 3) \). What are \( \bar{A}, A \cap B, A \cup B, \) and \( \bar{A} \cap B \)?

3. Compute \( \sum_{n=0}^{\infty} (1/8)^n \), and \( \sum_{n=0}^{\infty} (1/8)^n/n! \).

4. Compute \( 4! \) and \( \binom{n}{k} \) where \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \).

5. Is \( \sum_{n=2}^{\infty} 3/n \) finite, infinite, or undefined? Is \( \sum_{n=2}^{\infty} 3/n^2 \) finite, infinite or undefined?

6. Compute \( 2 \sum_{n=0}^{\infty} 1^n \) and \( 2 \sum_{n=0}^{\infty} (-1)^n \).

7. Compute \( \int_0^2 5x^4 \, dx \), \( \int_0^\infty (1/6)e^{-(1/6)x} \, dx \) and \( \int_0^\infty (1/6)xe^{-(1/6)x} \, dx \).

8. Compute \( \int_{x=0}^2 \int_{y=0}^x 4xy \, dy \, dx \) and \( \int_{x=0}^\infty \int_{y=0}^\infty e^{-(8x+9y)} \, dy \, dx \).

9. Find the value of \( x \) such that \( e^{-x/5} = 1/3 \).

10. Without integrating first, differentiate the following with respect to \( x \): \( \int_{x=0}^{x^4} xt \, dt \) using Leibniz’s formula. We will need this formula when analyzing the news vendor problem. You can look for Leibniz’s formula in calculus books or on the web at:

    http://web.mit.edu/18.02-esg/www/18.02IS/notes.html