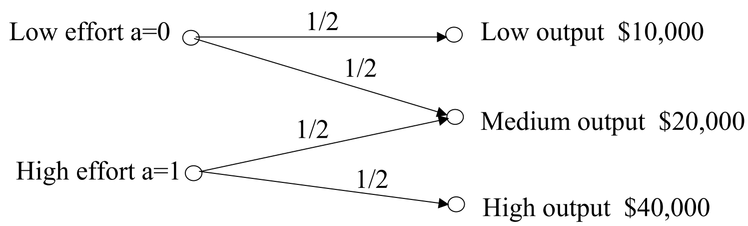
Example



- Cost of effort to the agent 10,000a
- Agent's reservation utility U=0
- Agent's expected payoff: E[wage – cost of effort]
- Principal's expected payoff: E[output – wage]

Example (cont)

- Under low effort
  - Agent's cost=0 → Offer w=U=0 to the agent
  - Principal's expected payoff = 0.5(10,000)+0.5(20,000)-0=15,000
- Under high effort
  - Principal's expected revenue = 0.5(20,000)+0.5(40,000)=30,000
  - Principal's expected payoff = E[revenue – wage]
  - For participation: E[wage] ≥ U+cost of effort=10,000
- For the principal to prefer high effort over low E[wage]<=15,000

Example: Bonus payment

- Principal's proposed contract
  - If the output is low or medium, w=0
  - If the output is high, w=24,000
- Participation constraint for high effort

0.5(0)+0.5(24,000)-10,000=2,000>0

- Incentive constraint

0.5(0)+0.5(24,000)-10,000=2,000>0.5(0)+0.5(0)=0

- Principal's payoff: 18,000 Agent's payoff: 2,000

- What is the best w that maximizes the principal's payoff?

0.5(0)+0.5w-10,000≥0 → w=20,000

Principal's payoff: 20,000 Agent's payoff: 0

Example: Revenue sharing

- Principal's proposed contract
  - If the output is less than x=18,000, w=0
  - If the output R ≥ x, w=R-x
- Participation constraint

    0.5(20,000-18,000)+0.5(40,000-18,000)-10,000=2,000>0

- Incentive constraint

  0.5(20,000-18,000)+0.5(40,000-18,000)-10,000=2,000> 0.5(0)+0.5(20,000-18,000)=1,000

- Principal's payoff: 18,000 Agent's payoff: 2,000
- What is the best x that maximizes the principal's payoff?

Example: Revenue sharing (cont.)

- $x>20{,}000$: Participation constraint is not satisfied
  - $0.5(0)+0.5(40{,}000-x)-10{,}000= 10{,}000-0.5x<0$ if $x>20{,}000$
- $10{,}000<x\leq 20{,}000$
  - Participation: $0.5(20{,}000-x)+0.5(40{,}000-x)-10{,}000= 20{,}000-x \geq 0$
  - Incentive: $0.5(20{,}000-x)+0.5(40{,}000-x)-10{,}000 \geq 0.5(0)+0.5(20{,}000-x)$
  - $x\leq 20{,}000 \rightarrow$ To maximize principal's payoff $x=20{,}000$
  - Principal's payoff: 20,000 Agent's payoff: 0
- $x \leq 10{,}000$:
  - Participation: $0.5(20{,}000-x)+0.5(40{,}000-x)-10{,}000= 20{,}000-x \geq 0$
  - Incentive: $0.5(20{,}000-x)+0.5(40{,}000-x)-10{,}000 \geq 0.5(10{,}000-x)+0.5(20{,}000-x)$
  - To maximize principal's payoff $x=10{,}000$
  - Principal's payoff: 10,000 Agent's payoff: 10,000
- Optimal $x=20{,}000$

Piece-rate system

- Principal's proposed contract: $w=\alpha R$ if the output is R
- Participation constraint for high effort

$0.5(\alpha 20{,}000)+0.5(\alpha 40{,}000)-10{,}000=30{,}000\alpha-10{,}000 \geq 0 \rightarrow \alpha\geq 1/3$

- Incentive constraint

$0.5(\alpha 20{,}000)+0.5(\alpha 40{,}000)-10{,}000=30{,}000\alpha-10{,}000 \geq 0.5(\alpha 10{,}000)+0.5(\alpha 20{,}000) \rightarrow \alpha\geq 2/3$

- To maximize the principal's payoff: $\alpha=2/3$
- Principal's payoff: 10,000 Agent's payoff: 10,000
- Is it optimal for the principal to induce high effort under this contract?

Managerial compensation under Cournot competition

- Cournot competition
  - The market price, P is determined by (inverse) market demand:
    - P=a-Q if a>bQ, P=0 otherwise.
  - The *marginal cost* of producing each unit of the good is c
  - Each firm decides on the quantity to sell (market share): q_1 and q_2
  - Q= q_1+q_2 total market demand
  - Both firms seek to maximize profits

Revenue of firm i : $R_i = pq_i = (a-Q)q_i$

Profit of firm i : $\pi_i = R_i - cq_i$

The owner of firm i offers the following compensation

to her manager : $M_i = \mu_i[\alpha_i \pi_i + (1-\alpha_i)R_i]$

Managerial compensation under Cournot competition (cont.)

1. The owner of firm i needs to choose μ_i and α_i to maximize firm profits net of managerial compensation :

$$\pi^{Owner} = \max \pi_i - M_i$$

2. For given q_j and α_i, the manager of firm i chooses q_i to maximize $M_i = \mu_i[\alpha_i \pi_i + (1-\alpha_i)R_i]$.

$$M_i = \mu_i[\alpha_i(R_i - cq_i) + (1-\alpha_i)R_i] = \mu_i[R_i - \alpha_i c q_i]$$

$$M_i = \mu_i[(a - q_i - q_j)q_i - \alpha_i c q_i]$$

$$F.O.C.: \; a - 2q_i - q_j - \alpha_i c = 0 \;\; \rightarrow \;\; q_i = \frac{a - q_j}{2} - \frac{\alpha_i c}{2}, \; i = 1,2$$

Managerial compensation under Cournot competition (cont.)

In equilibrium:

$$q_i(\alpha_1,\alpha_2)=\frac{a+\alpha_j c-2\alpha_i c}{3},\quad i=1,2$$

$$Q(\alpha_1,\alpha_2)=\frac{a+\alpha_2 c-2\alpha_1 c}{3}+\frac{a+\alpha_1 c-2\alpha_2 c}{3}=\frac{2a-\alpha_1 c-\alpha_2 c}{3}$$

$$P(\alpha_1,\alpha_2)=\frac{a+\alpha_1 c+\alpha_2 c}{3}$$

Managerial compensation under Cournot competition (cont.)

Stage 1: Owners choose μ_i and α_i

How to choose μ_i?

To simplify, assume $\mu_i = M_i = 0$

Owner's objective:

$$\pi^{\text{Owner}}=[p(\alpha_1,\alpha_2)-c]q_i$$
$$=\frac{[a+c(\alpha_i+\alpha_j-3)]}{3}\frac{[a+c(\alpha_j-2\alpha_i)]}{3}$$

From F.O.C.: $\alpha_i=\dfrac{6c-a-c\alpha_j}{4c}$

Managerial compensation under Cournot competition (cont.)

In equilibrium:

$$\alpha = \alpha_1 = \alpha_2 = \frac{6c-a}{5c}$$

α increases in c, i.e., owners will compensate managers to place more weight on maximizing profits when costs are high.

$$q_1 = q_2 = \frac{a-\alpha c}{3} = \frac{2(a-c)}{5} > \frac{a-c}{3} = q_i^C$$

When owners are different than managers, output exceeds the Cournot equilibrium output.

Managerial compensation under Cournot competition (cont.)

Suppose the owner of Firm 2 is also the manager. What is α_2?

$$\alpha_2 = 1.$$

$$\alpha_1 = \frac{6c-a-c\alpha_2}{4c} = \frac{5c-a}{4c}$$

$$q_1 = \frac{a+\alpha_2 c - 2\alpha_1 c}{3} = \frac{a-c}{2}$$