Introduction to Game Theory

A collection of tools for predicting outcomes for a group of interacting agents where an action of a single agent directly effects the payoffs of the other participating agents.

What is a game?

- Many types of games: board games, card games, video games, field games (e.g., football), etc.
  - A zero-sum game is one in which the players' interests are in direct conflict, e.g. in football, one team wins and the other loses.
  - A game is non-zero sum, if players interests are not always in direct conflict, so that there are opportunities for both to gain.
- We focus on games where:
  - There are 2 or more *players*.
  - There is some choice of action where *strategy* matters.
  - The game has one or more *outcomes*, e.g. someone wins, someone loses.
  - The outcome depends on the strategies chosen by all players; there is *strategic interaction*.
- What does this rule out?
  - Games of pure chance, e.g., lotteries, slot machines. (Strategies don't matter).
  - Games without strategic interaction between players, e.g. Solitaire.

Elements of a game

- The *players*
  - how many players are there?
  - does nature/chance play a role?
- A complete description of what the players can do – *the set of all possible actions*.
- A description of *the payoff consequences* for each player for every possible combination of actions chosen by all players playing the game.

Example: NBC vs. ABC

NBC

ABC	News	Sitcom
News	55% 45%	52% 48%
Sitcom	50% 50%	45% 55%

- Players: NBC and ABC
- Set of all possible actions: sitcom, news
- Payoffs: market shares for each outcome

Normal form game: Notation

- A set of N players $I = \{1, \ldots, N\}$
- Each player $i \in N$ has an action set $A^i = \{a_1^i, a_2^i, \ldots, a_{k_i}^i\}$ such that $a^i \in A^i$ is a particular action by i
- Each player i has a payoff $\pi^i(a)$ where $a = \{a^1, a^2, \ldots, a^N\}$ is the game outcome
- $a^{-i} = \{a^1, a^2, \ldots, a^{i-1}, a^{i+1}, \ldots, a^N\}$ denotes the actions of all players except i in the outcome

Assumptions

- Payoffs are known and fixed.
- Players are risk neutral, i.e., maximize expected payoffs.
  - Example: a risk neutral person is indifferent between
    - $25 for certain or
    - a 25% chance of earning $100 and a 75% chance of earning 0.
- All players behave rationally. They understand and seek to maximize their own payoffs.
- The rules of the game are common knowledge
  - Each player knows the set of players, strategies and payoffs from all possible combinations of strategies: call this information "X." *Common knowledge* means that each player knows that all players know X, that all players know that all players know X, that all players know that all players know that all players know X and so on,..., *ad infinitum*.

Classroom game

Player 2

Player 1

	X	Y
X	8, 7	-100, 6
Y	7, 6	6, 5

- Player 1
  - X: 8 Y: 12
- Player 2
  - X: 16 Y: 4
- Outcomes
  - (X,X): 8 (X,Y): 1 (Y,X): 8 (Y,Y): 3

Classroom game

Player 2

Player 1

	X	Y
X	8, 7	5, 6
Y	7, 6	6, 5

- Player 1
  - X: 10 Y: 10
- Player 2
  - X: 17 Y: 3
- Outcomes
  - (X,X): 10 (X,Y): 1 (Y,X): 5 (Y,Y): 4