Pricing for Environmental Compliance in the Auto Industry

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Julie L. Swann

Regulations

• Vehicles contribute to the environment
• California Air Resource Board
  – 10% of all vehicles sold will be partial or full Zero-Emissions Vehicles (ZEV)
• European Union
  – CO2 emissions will be reduced by 25% between 1995 and 2008
• United States Congress
  – Minimum Corporate Average Fuel Economy (CAFE) standards
CAFE Background

- **Purpose**
- **CAFE Target Values**
  - 27.5 mpg for car, 20.7 mpg for truck
  - Truck rising to 22.2 by 2007
- **Penalties**
- **Debits/Credits**

Manufacturer’s Solution

- Improve fuel efficiency of current vehicles
- Develop electric and hybrid vehicles
- Research alternative fuel sources
- **Problem:**
  - Technologies fail
  - Standards change
  - Market demand shifts
  - Consumer preferences run counter to environmental standards
Short Term Solution

- Use price to shift demand
- Alliance of Automobile Manufacturers
  - “In order to meet the target, we are going to have to change America’s buying habits”
- Project purpose:
  - To develop a tool to aid in making decisions that maximize profitability while meeting environmental standards

CAFE Model Complexity

- EPA Guidelines
- Fuel Economy
- Testing Groups
- Demand Data
- Capacity Limits
- Financial Data
- CAFE Target
CAFE Guidelines

- Vehicles are grouped by attributes according to three levels
- Each base level must be tested
- Highest volume vehicles are tested to represent other vehicles within a configuration
- Tested vehicles must represent at least 90% of the CAFE regulated volume

CAFE Calculation

- CAFE Calculation is a harmonic weighted average
- Vehicles with low gas mileage have more impact on the CAFE value

\[
CAFE = \frac{\sum_{all \_configs} \frac{1}{Config \_MPG} \ast Config \_Sales}{\text{Total } \_Sales}
\]
Current Decision Making Process

• Steps
  – Determine volume needed to meet CAFE
  – Achieve volume through incentives
• Goal: Comply with CAFE regulations

• Missing Pieces
  – Demand/price relation not fully known
  – Testing calculation considered after the fact
  – Profit absent

Model Formulation

• Maximize Profitability
• subject to:
  – CAFE target
  – Capacity
  – Testing Rules
  – Demand Curves
  – Min/Max Price and Volume
• Variables: Volume/Price
Mathematical Model

Variables:
- \( V_i \) Volume of vehicle \( i \)
- \( P_i \) Price of vehicle \( i \)

Sets:
- \( I \) Set of all vehicles (each representing a configuration)
- \( E \) Set of all vehicle engines
- \( T \) Set of all vehicle transmissions
- \( A \) Set of all vehicle assembly types

Input Parameters:
- \( T_i \) Test value for vehicle \( i \): 1 indicates vehicle is tested, 0 otherwise
- \( M_i \) MPG of vehicle \( i \)
- \( \text{Beta}_i \) Demand intercept for vehicle \( i \)
- \( \alpha_i \) Demand multiplier for vehicle \( i \)
- \( A_{ik} \) Vector of attributes; one value for each vehicle and each attribute (1..k)
- \( c_e \) Capacity of engine \( e \)
- \( c_t \) Capacity of transmission \( t \)
- \( c_a \) Capacity of assembly type \( a \)
- \( M \) CAFE target value
- \( K_i \) Production cost for vehicle \( i \)

Objective Function:
\[
\text{maximize Profit} \quad \sum_{i \in I} V_i \cdot (P_i - K_i)
\]

Subject to:
- CAFE constraint:
\[
\sum_{i \in I} \frac{V_i \cdot T_i}{m_i} \leq \frac{1}{M} \sum_{i \in I} V_i
\]
- Engine Capacity:
\[
\sum_{i \in I} V_i \leq c_e \quad \forall e \in E
\]
- Transmission Capacity:
\[
\sum_{i \in I} V_i \leq c_t \quad \forall t \in T
\]
- Assembly Capacity:
\[
\sum_{i \in I} V_i \leq c_a \quad \forall a \in A
\]
- Calc. Volume:
\[
V_i = \alpha_i P_i + \beta_i \quad \forall i \in I
\]
- Min/Max:
\[
\gamma_i P_i \leq \gamma_i P_i \quad \forall i \in I
\]
- + Testing Constraints
Visual CAFE

• Tool for pricing and production decisions
• Tracks top volume configurations
• Allows user to adjust volume or price
•Calculates profitability and CAFE compliance
• Offers “Excel Solver” to optimize profitability of all vehicles simultaneously

Optimal Pricing

Estimated Profitability under CAFE

![Graph showing estimated profitability under CAFE](image)
Expansion: Demand Diversions

• If the price of a Blazer increases, demand diverts to Explorer
• Model demand diversions in calculation of volumes & prices
• Estimate volume changes more accurately

Issues

• Cross-elasticities not available
  – Change in quantity for vehicle i depends on the price of vehicle j
• Diversions based on 2nd choice product data
  – Ex: If Sally decides not to purchase an Explorer, she wants a Blazer instead
• Diversion data independent of price
• Data at name plate/body style only (ex: 2 door Explorer)
Initial Results

- Incorporated in linear demand curves
- “Cheating” occurred
- Insights:
  - Change prices in “mileage groups” together
  - Give range for valid volume/price changes
  - Determine sophisticated diversion formula

Origin of Diverted Demand

<table>
<thead>
<tr>
<th>Proportion of Demand Contribution</th>
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<tbody>
<tr>
<td>Demand</td>
</tr>
<tr>
<td>Diverted Demand</td>
</tr>
<tr>
<td>“Own” Demand</td>
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</table>

<table>
<thead>
<tr>
<th>Vehicles</th>
<th>Demand</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>4</td>
<td>200,000</td>
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<td>7</td>
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<td>19</td>
<td></td>
</tr>
<tr>
<td>22</td>
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</tr>
</tbody>
</table>
Discrete Choice Demand Models

- Based on choice of customer from options
- Model utility (or preference)
  \[ U_i = V_i + \varepsilon_i \]
- Uses product characteristics and importance of attribute
  \[ V_i = x_{i1} \beta_{i1} + x_{i2} \beta_{i2} + \ldots + x_{ik} \beta_{ik} \]
- Estimate parameters and use to generate elasticities

Logit Models

- Assumes Weibull distribution for error terms (McFadden 1973)
- Red Bus/Blue Bus (Independence of Irrelevant Alternatives)
- Nested Multinomial Logit Model
- Auto Applications (Berry et al 1998)
Segmented Choice Logit Model

- **Segments**: for each pair of vehicles (i, j), customers choose i, j, or drop out of market
- **Input**: diversion choices, own elasticities, base price and volume, and market elasticity
- **Output**: parameters for utility functions for each segment

Single Segment Example

- **Utilities and probability of a choice**
  \[
  x_i = \beta_i - \alpha_i P_i \\
  x_j = \beta_j - \alpha_j P_j
  \]
  \[
  \text{Pr}(i \text{ is chosen}) = \frac{e^{x_i}}{\sum_{j \in C} e^{x_j}}
  \]

- **Given prices and number in a segment**, \( Q_i(P_i, P_j) \) = number that buy i = \( N \left( \frac{e^{x_i}}{e^{x_i} + e^{x_j}} \right) \)

  \( Q_j(P_i, P_j) \) = number that buy j = \( N \left( \frac{e^{x_j}}{e^{x_i} + e^{x_j}} \right) \)
Single Segment Example

- Elasticity definitions and previous equations

\[
E_i = -\frac{\partial Q_i}{\partial P_i} P_i = \frac{\alpha_i P_i (1 + e^{r_i})}{e^{r_i} + e^{r_j} + 1} \\
E_j = -\frac{\partial Q_j}{\partial P_j} P_j = \frac{\alpha_j P_j (1 + e^{r_j})}{e^{r_i} + e^{r_j} + 1}
\]

- Given an overall price level of \(\delta\) within a segment,

\[
U_i = \beta_i - \alpha_i (\delta P_i) + \varepsilon_i
\]

Single Segment Example

- Total demand and total elasticity for the segment are:

\[
Q_T = Q_i + Q_j = N(e^{r_i} + e^{r_j}) \quad E_T = -\frac{\partial Q_T}{\partial \delta} \frac{\delta}{Q_T}_{\delta=1}
\]

- More algebra gives us:

\[
E_T = \frac{\alpha_i P_i e^{r_i} + \alpha_j P_j e^{r_j}}{(e^{r_i} + e^{r_j})(e^{r_i} + e^{r_j} + 1)}
\]

- 5 unknowns: \(N, B_i, B_j, \alpha_i, \alpha_j\); 5 boxed equations
Single Segment Example

- From initial equations we have
  
  Demand for i from (i, j) segment = $Q^{(i,j)}$

  Demand for j from (i, j) segment = $Q^{(i,j)}$

- We want the demand for i over all possible segments

  
  \[ D_i(P_i, P_j, P_k) = Q^{(i,j)}(P_i, P_j) + Q^{(i,k)}(P_i, P_k) \]

SCLM Demand Model

- Uses available information
- Reduces the effect of the IIA property
- Minimal computational effort
- Customer diversions depend on prices of other vehicles
Potential Extensions

- Non-tested vehicle groups
- Vehicle Testing Decision
- Multi-year aspect
- Other environmental standards

Mathematical Model

Testing Constraints: Add index \((b,c)\) for base and configuration levels of each vehicle to \(T\) and \(V\)

subject to:

\[
\sum_{i \in I} \sum_{c \in C} T_{i}^{b,c} \geq 1 \quad \forall b \in B \quad \text{Test each base level}
\]

\[
T_{i}^{b,c} \cdot V_{i}^{b,c} \geq T_{i}^{b,c} \cdot V_{j}^{b,c} \quad \forall b, c \in C, i \in I \quad \text{Test highest volume}
\]

\[
\sum_{i \in I} \sum_{c \in C} T_{i}^{b,c} \cdot V_{i}^{b,c} \geq 0.9 \sum_{i \in I} \sum_{c \in C} V_{i}^{b,c} \quad \forall b \in B \quad \text{Test 90%}
\]

\[
T_{i}^{b,c} = 0 \text{ or } 1 \quad \forall b, c \in C, i \in I \quad \text{Binary}
\]
Final Implementation Results

- Expanded Visual CAFE
  - Non-linear solver
  - Additional vehicle groups
  - SCLM model
- Implemented in Access with Visual Basic solver
- Future environmental standards

FOR MORE INFO...

Julie Swann
ISyE, Georgia Tech
http://www.isye.gatech.edu/~jswann
jswann@isye.gatech.edu