Administivia
- Homework #2 posted on the course page
  - Due next Thursday (Jan. 6) at the beginning of class
- Exam #1
  - February 11
  - Closed book, closed notes
  - You are allowed to use
    - One page of notes, single-sided
    - Basic calculators

Recap
- Last class (1/28/03)
  - Infinitely repeated Prisoner’s Dilemma
  - Feasible payoffs
  - Friedman’s theorem
  - Infinitely repeated Cournot game
  - Trigger strategies
- Today (1/30/03)
  - Infinitely repeated Cournot game
  - Example: wage-setting
  - Extensive form representation

Repeated Cournot game
- Trigger strategy
  - Produce half the monopoly quantity, \(q^M/2\), in the first stage. In the \(t^\text{th}\) stage, produce \(q^T/2\) if both firms have produced \(q^T/2\) in all previous stages; otherwise, produce \(q_C\).
  - Playing the trigger strategy is SPNE if \(\delta \geq 9/17\)
- What if \(\delta < 9/17\)?

Repeated Cournot Game (cont.)
- Trigger strategy
  - Produce \(q^*,\) in the first stage. In the \(t^\text{th}\) stage, produce \(q^*\) if both firms have produced \(q^*\) in all previous stages; otherwise, produce \(q_C\).
  - Profit of one firm
    - If both produce \(q^*\): \((a-2q^*-c)q^* = \pi^*\)
    - If both produce \(q_C\): \((a-c)^2/9 = \pi_C^*\)
    - If firm \(j\) produces \(q^*\) and firm \(i\) deviates:
      \[
      \max (a - q_i - q_j^* - c) q_i \rightarrow q_i = (a - q^*-c)/2
      \]
      \[
      \pi_D^* = (a - q^*-c)^2/4
      \]

Repeated Cournot Game (cont.)
- Best response of firm \(i\):
  - If the last stage outcome is other than \((q^*, q^*)\)
    - Play \(q^*\) forever
  - If all previous stages’ outcomes are \((q^*, q^*)\)
    - Deviate: \(V = \pi^* + \pi^*_\delta/(1-\delta)\)
    - Play \(q^*\): \(V = \pi^* + \pi^*_\delta \rightarrow V^* = \pi^*/(1-\delta)\)
    - Playing the trigger strategy is NE iff
      \[
      \pi^*/(1-\delta) \geq \pi_D^* + \pi_C^* \delta/(1-\delta)
      \]
    - Substitute and solve for \(q^*\):
      \[
      q^* = (9-5\delta)(a-c)/3(9-\delta)
      \]
Recall: \(q^0=(a-c)/3\) \(q^M=(a-c)/2\)

Example: Wage setting
- Stage game
  - One firm, one worker
  - The firm offers the worker a wage, \(w\)
  - The worker accepts or rejects the firm’s offer
    - Reject: the worker becomes self-employed at wage \(w_0\)
    - Accept: Work (disutility \(e\)), or Shirk (disutility \(0\))
      - If the worker works (supplies effort): Output is \(y\)
      - If the worker shirks: Output is high with probability \(p\), and low=0 with probability \(1-p\)
  - The firm does not observe the worker’s effort decision
  - The output of the worker is observed by both parties
Payoffs (Firm, Worker)
- Work (Supply effort)
  - High output: \((y-w, w)\)
  - Low output: \((-w, w)\)
- Shirk
  - High output: \((y-w, w)\)
  - Low output: \((-w, w)\)

What is the subgame-perfect equilibrium in this stage game?
- For any \(w \geq w_0\), worker accepts employment and shirks
- Firm offers \(w=0\) (or any other \(w<w_0\))

Suppose firm offers \(w^* \geq w_0\)
- Worker accepts
- Work (Supply effort)
  \[ V_e = (w^*-e) + \delta V_e \rightarrow V_e = (w^*-e)/(1-\delta) \]
- Shirk
  \[ V_s = w^* + \delta(pV_e + (1-p)w_0/(1-\delta)) \rightarrow V_s = [(1-\delta)w^* + \delta(1-p)w_0]/(1-\delta) \]
- Worker should supply effort if \(V_e \geq w_0 + e +(1-\delta)/\delta(1-p)\)
  \[ w^* \geq w_0 + e + e(1-\delta)/(\delta(1-p)) \]

When is it the best response for the firm to offer \(w^*\)?
- From worker’s best response
  \[w^* \geq w_0 + e + e(1-\delta)/(\delta(1-p)) \]
- \(y \geq w^*\)
  \[ y \geq w_0 + e + e(1-\delta)/(\delta(1-p)) \]

The strategies induce a NE if (1) and (2) hold.

Is this a SPNE?

What are the subgames?
- Subgames beginning after a high-wage, high-output history
- Subgames beginning after all other histories

The set of players
The order of moves
The players’ payoffs as a function of the moves that were made
The set of actions available to the players when they move
Each player’s information when he makes his move
The probability distributions over any exogenous events (Nature)
Example 1

Player 1 moves first. After observing player 1’s action, player 2 moves
Player 1 action set: {U,D} Player 2 action set: {L,R}
Player 1 strategies: {U,D} Player 2 strategies: {(L,L), (L,R), (R,L), (R,R)}

Normal form representation of extensive-form games

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(L,L)</td>
</tr>
<tr>
<td>U</td>
<td>2,1</td>
</tr>
<tr>
<td>D</td>
<td>-1,1</td>
</tr>
</tbody>
</table>

- Player 2's strategies correspond to a contingent plan made in advance

Example

- Player 1 chooses an action from the feasible set {L,R}
- Player 2 observes player 1’s action and then chooses an action from the feasible set {L',R'}
- Player 3 observes whether or not the history of actions is (R,R') and then chooses an action from the feasible set {L'',R''}

Example (cont.)

Information set

- An information set for a player is a collection of decision nodes satisfying:
  - The player has the move at every node in the information set
  - When the play of the game reaches a node in the information set, the player with the move does not know which node in the information set has (or has not) been reached
Example (cont.)

Player 2 has two information sets, both singletons.
Player 3 has two information sets, one of them is singleton.