Show all your work.

**Question 1** (10 points)

(a) Discuss the advantages and disadvantages of revenue sharing. In what kind of environments would a revenue sharing agreement be beneficial for a supplier?

(b) Discuss how the risk of demand uncertainty is shared between the supplier and retailer under the following contracts: revenue sharing, buyback, wholesale price, two-part tariff, and quantity discount. (Assume that the retail price is exogenous and there is only one retailer.)

**Question 2** (25 points)

You are in charge of stocking Rolling Stone magazine at a local convenience store. You have been informed that Britney Spears will be on the next cover, and you know how that leads to unusually high magazine sales. You have compiled data on past sales when other such popular stars were on the cover, and you estimate that the demand will follow a uniform distribution with a range of 1000 to 2000. It costs you $1.50 to buy each magazine from the supplier and you sell them for $3.95 at your store. It costs the manufacturer $0.50 to produce and deliver each magazine to you. If any magazines are left over at the end of the month, you donate them to a local physician’s association. If you run out of magazines, you estimate a loss of goodwill cost of $2.00 per stockout.

(a) **Decentralized model, no coordination:** Using the newsvendor model, show that the optimal order quantity is \( Q = 1748 \) magazines. Show that the manufacturer’s profit is $1748, your profit is $3114, and so the supply chain profit is $4862. Also show that your expected overstock level under this arrangement is 279.75 magazines.

(b) **Centralized supply chain:** Show that the optimal number of magazines you should order to maximize total supply chain profit is \( Q^* = 1915 \).

(c) **Buyback contract:** Suppose the manufacturer offers you a buyback price of $0.75 per left over magazine, and you accept. Does this new agreement coordinate the supply chain? What are your profit and the manufacturer’s profit under this agreement?

(d) **Buyback contract with capacity limits:** Suppose that you only have capacity for at most 1748 magazines. Determine the manufacturer’s new expected profit, your new expected profit, and the supply chain’s new expected profit if you order the optimal amount under your capacity limit. Is the buyback contract a good idea for the supplier in this case?
(e) How does the capacity limit affect the effectiveness of a buyback contract? Explain why it is important for the manufacturer and retailer to communicate and understand the implications of capacity limits when attempting to use coordination mechanisms to maximize supply chain profit.

**Question 3** (30 points + 10 Bonus)

Consider a *quantity flexibility* (QF) contract, where the supplier charges a wholesale price \( w \) per unit and allows the retailer to change the quantity ordered after observing the demand. If the retailer orders \( N \) units, the manufacturer commits to providing \( K = (1+\alpha)N \) and the retailer is committed to purchase at least \( q = (1-\beta)N \). Both \( \alpha \) and \( \beta \) are between 0 and 1. Let \( Q \) denote the retailer’s purchase quantity. The retailer can purchase up to \( K \) units depending on the demand observed.

(i) (2 points) How is the inventory risk (due to demand uncertainty) shared in this agreement between the retailer and the supplier?

(ii) (1 points) Suppose that the cost of returns are high (e.g., due to high shipping costs) and the supplier bears that cost. In that case, do you think this quantity flexibility contract more or less desirable for the supplier?

(iii) (2 points) Do you think the retailer would order more, or less, under a quantity flexibility contract compared to a standard wholesale price contract? Explain your answers clearly.

The supplier’s unit production cost is \( c_s \) and the supplier charges \( w \) per unit to the retailer. The retailer sells to end consumers at a price of \( P \) per unit. The salvage value for any leftover inventory is zero. Assume that the demand can be modeled by a random variable \( D \), uniformly distributed between 800 and 1200.

(iv) (2 points) For the following three ranges of actual demand, how much would the retailer purchase under the QF agreement? \([0,q]\) \((q,K]\) \((K,\infty]\).

(v) (3 points) What is the expected quantity purchased by the retailer? Write this expression as a function of the parameters \( q \) and \( K \).

(vi) (3 points) What is the expected quantity sold by the retailer? What is the expected overstock at the manufacturer?

(vii) (2 points) What are the expected profits of the retailer and the supplier?

(viii) (5 points) Model this problem as a centralized supply chain. Argue that in that case we would have \( K=Q \). What is the optimal value of \( Q \)? What is the optimal supply chain profit?
(iv) (10 points) If the demand is uniformly distributed between $L$ and $U$, what is the optimal order quantity of the retailer ($Q$), as a function of the contract parameters $w$, $\alpha$ and $\beta$?

(x) (10 points Bonus) Suppose $w=100$, $c_s=10$, $P=200$, and $\alpha = \beta$. Compute the expected order size, sales, and the profits of the manufacturer and the retailer for the following values of $\alpha$: 0, 0.2, 0.4, 0.5, 0.6, 0.8. (Use a spreadsheet.) Do the same computation for two other values of the wholesale price, e.g., 90 and 120. If the supplier would choose $w$ and $\alpha$ based on these experiments, what would you recommend?