

Homework #2
ISYE 6230 – Economic Decision Analysis II - Spring 2003
Due February 6, 2003 12:05pm

Show all your work.

Question 1: (5 points) Find an article from popular press (e.g., Business Week, Fortune, The New York Times, Economist, Business 2.0, etc.) which relates to the topics to be covered in this course. Write a short paragraph explaining how the situation discussed in the article is related to the course topics. Attach the article to your homework, and highlight the relevant sections.

Question 2: (6 points) Two firms are competing against each other based on product quality. Let q_j be the quality level set by firm j . The profit functions for the firms are:

$$\begin{aligned}\pi^1(q_1, q_2) &= 5q_1 + 2q_1q_2 - 5q_1^2 \\ \pi^2(q_1, q_2) &= 4q_2 + 11q_1q_2 - 4q_2^2\end{aligned}$$

Assuming that Firm 1 is the first mover, determine the equilibrium quality levels and the corresponding profits. If Firm 1 had the choice, would it prefer to be the first or the second mover? Explain.

Question 3: (8 points) Consider a game with two players and a game-master. At the beginning of the game, the game-master tosses a biased coin, which comes up “heads” 80% of the time. The bias of the coin is known to both players. The game-master shows the outcome of the coin flip to player 1. Player 1 then announces to player 2 “heads” or “tails.” Having heard what player 1 says, but not having seen the actual outcome of the coin flip, player 2 must then guess the actual outcome of the coin flip – “heads” or “tails.” Player 2’s payoff is \$1, if he correctly guesses the outcome of the coin flip, and \$0 otherwise.

Player 1 gets \$1 if she correctly reports the outcome of the coin flip to player 2; otherwise, she gets \$0. In addition, player 1 gets:

- \$2 if player 2 guesses “heads”
 - \$0 if player 2 guesses “tails”
- regardless of the actual outcome of the coin flip.

- (a) What are the action sets of the players?
- (b) Draw an extensive form representation of this game. Is this representation unique?
- (c) What are the equilibrium strategies of the players?

Question 4: (5 points) Consider the following game. Player 1 moves first and chooses X or Y. If player 1 chooses X, the game ends with a payoff (3,3). If player 1 chooses Y, then after observing player 1’s choice, player 2 chooses A or B: A leads to a payoff (5,2), and B leads to a payoff (2,0). Find all pure strategy Nash equilibria of this game. Which of these equilibria is credible? Explain.

Question 5: (10 points) A firm and a union are at dispute about the wages and try to reach an agreement with the help of an arbitrator. The firm and the union will simultaneously make wage offers, w_F and w_U , and after receiving these offers, the arbitrator will choose one of them for settlement. The arbitrator has an ideal settlement, say, x , she would like to impose, and she will choose the offer that is closest to x . If both offers are equidistant from x , then the arbitrator flips a fair coin and chooses one of the offers randomly. The arbitrator knows x but the firm and the union do not. The firm and the union believe that x is randomly distributed according to the uniform distribution $U \sim [L, U]$. The firm wants to minimize the expected wage settlement whereas the union wants to maximize it. Find the offers of the firm and the union at the Nash equilibrium. (Hint: You may assume $w_F \leq w_U$, which is indeed the case in equilibrium.)

Question 6: (10 points) Suppose there are M farmers, each of whom has the right to graze cows on the village common. The amount of milk a cow produces depends on the total number of cows, N , grazing on the green. The revenue of farmer j per cow from producing n_j cows is $v(n_j + \sum_{i \neq j} n_i)$ where $N = \sum_i n_i$, $v(N) = 0$ for $N > K$, and $v(0) > c$, $v' \leq 0$ and $v'' < 0$ for $N \leq K$. Each cow costs c and suppose that cows are perfectly divisible. Farmers simultaneously decide how many cows to purchase; all purchased cows graze on the common.

Find the first order conditions to be satisfied in the Nash equilibrium. Let N^* be the monopoly outcome. Show that $N > N^*$. How is this game related to the Cournot model with multiple firms?