Show all your work.

Question 1: (5 points) Find an article from popular press (e.g., Business Week, Fortune, The New York Times, Economist, Business 2.0, etc.) which relates to the topics to be covered in this course. Write a short paragraph explaining how the situation discussed in the article is related to the course topics. Attach the article to your homework, and highlight the relevant sections.

Question 2: (6 points) Two firms are competing against each other based on product quality. Let \( q_j \) be the quality level set by firm \( j \). The profit functions for the firms are:

\[
\pi_1(q_1, q_2) = 5q_1 + 2q_1q_2 - 5q_1^2 \\
\pi_2(q_1, q_2) = 4q_2 + 11q_1q_2 - 4q_2^2
\]

Assuming that Firm 1 is the first mover, determine the equilibrium quality levels and the corresponding profits. If Firm 1 had the choice, would it prefer to be the first or the second mover? Explain.

Question 3: (8 points) Consider a game with two players and a game-master. At the beginning of the game, the game-master tosses a biased coin, which comes up “heads” 80% of the time. The bias of the coin is known to both players. The game-master shows the outcome of the coin flip to player 1. Player 1 then announces to player 2 “heads” or “tails.” Having heard what player 1 says, but not having seen the actual outcome of the coin flip, player 2 must then guess the actual outcome of the coin flip – “heads” or “tails.” Player 2’s payoff is $1, if he correctly guesses the outcome of the coin flip, and $0 otherwise. Player 1 gets $1 if she correctly reports the outcome of the coin flip to player 2; otherwise, she gets $0. In addition, player 1 gets:

- $2 if player 2 guesses “heads”
- $0 if player 2 guesses “tails”

regardless of the actual outcome of the coin flip.

(a) What are the action sets of the players?
(b) Draw an extensive form representation of this game. Is this representation unique?
(c) What are the equilibrium strategies of the players?

Question 4: (5 points) Consider the following game. Player 1 moves first and chooses X or Y. If player 1 chooses X, the game ends with a payoff (3,3). If player 1 chooses Y, then after observing player 1’s choice, player 2 chooses A or B: A leads to a payoff (5,2), and B leads to a payoff (2,0). Find all pure strategy Nash equilibria of this game. Which of these equilibria is credible? Explain.
**Question 5:** (10 points) A firm and a union are at dispute about the wages and try to reach an agreement with the help of an arbitrator. The firm and the union will simultaneously make wage offers, $w_F$ and $w_U$, and after receiving these offers, the arbitrator will choose one of them for settlement. The arbitrator has an ideal settlement, say, $x$, she would like to impose, and she will choose the offer that is closest to $x$. If both offers are equidistant from $x$, then the arbitrator flips a fair coin and chooses one of the offers randomly. The arbitrator knows $x$ but the firm and the union do not. The firm and the union believe that $x$ is randomly distributed according to the uniform distribution $U \sim [L, U]$. The firm wants to minimize the expected wage settlement whereas the union wants to maximize it. Find the offers of the firm and the union at the Nash equilibrium. (Hint: You may assume $w_F \leq w_U$, which is indeed the case in equilibrium.)

**Question 6:** (10 points) Suppose there are $M$ farmers, each of whom has the right to graze cows on the village common. The amount of milk a cow produces depends on the total number of cows, $N$, grazing on the green. The revenue of farmer $j$ per cow from producing $n_j$ cows is $v(n_j + \sum_{i \neq j} n_i)$ where $N = \sum_i n_i$, $v(N) = 0$ for $N > K$, and $v(0) > c$, $v' \leq 0$ and $v'' < 0$ for $N \leq K$. Each cow costs $c$ and suppose that cows are perfectly divisible. Farmers simultaneously decide how many cows to purchase; all purchased cows graze on the common. Find the first order conditions to be satisfied in the Nash equilibrium. Let $N^*$ be the monopoly outcome. Show that $N > N^*$. How is this game related to the Cournot model with multiple firms?