Administrivia

- Teaching assistant: Martin Smith
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- Office hours
  Tuesday 3:30-4:30
  Wednesday 2:00-3:00
  Location: ISYE 403

Recap

- Last class
  - Games in normal form
  - NBC vs. ABC, old vs. new technology, Prisoner’s dilemma
  - Dominant and dominated actions, best response
  - Equilibrium in dominated actions, Nash Equilibrium
  - Pure vs. mixed strategies
- This class
  - Games with continuous action sets
  - Duopoly games
  - Cournot, Bertrand
  - Comparison of duopoly games with monopoly

Example

- A scarce manufacturing resource is required by two departments, A and B
- \( Y_j \geq 0 \): quantity of the resource used by department j
- Payoff to department j from using one unit of the resource
  \[ 200-(Y_j+Y_{-j})^2 = 0 \]
- Department j’s maximization problem
  \[ \max Y_j[200-(Y_j+Y_{-j})^2] \]
- From FOC:
  \[ 200-(Y_j+Y_{-j})^2-2Y_j(Y_j+Y_{-j}) = 0 \]

Example (cont.)

\[
\begin{align*}
&200 - 3Y_A^2 - 4Y_A Y_B - Y_B^2 = 0 & \quad (1) \\
&200 - 3Y_B^2 - 4Y_A Y_B - Y_A^2 = 0 & \quad (2)
\end{align*}
\]

\[
\frac{(2)-(1)}{2}: \quad 2Y_A^2 - 2Y_B^2 = 0 \rightarrow Y_A = Y_B
\]

From (1):
\[
200 - 3Y_A^2 - 4Y_A Y_B - Y_B^2 = 0 \rightarrow 8Y_A^2 = 200 \rightarrow Y_A = Y_B = 5 \rightarrow \text{Payoff for department j: 500}
\]

A solution with higher payoff?
\[
Y_A = Y_B = 4 \rightarrow \text{Payoff for department j: 544}
\]

The Tragedy of the Commons

- Immigrant villages in New England in the 17th century
  - privately owned homesteads and gardens
  - community-owned pastures, called commons, where all of
    the villagers’ livestock could graze
  - Incentive to avoid overuse of their private lands, so
    they would remain productive in the future
  - Result? The commons were overgrazed and
    degenerated to the point that they were no longer able
    to support the villagers’ cattle
  - The failure of private incentives to provide adequate
    maintenance of public resources is known to economists
    as “the tragedy of the commons.”

Examples of the Tragedy of the Commons

- Congestion on urban highways
- Overpopulation
- Pollution
- The depletion of fish stocks in international waters
Duopoly models

- Two competing firms, selling a homogeneous good
- The marginal cost of producing each unit of the good: $c_1$ and $c_2$
- The market price, $P$ is determined by (inverse) market demand:
  - $P=a-bQ$ if $a>bQ$, $P=0$ otherwise.
- Both firms seek to maximize profits
- Cournot: Firms set quantities simultaneously
- Bertrand: Firm set prices simultaneously
- Stackelberg: Firms set quantities, firm 1 followed by firm 2

Cournot Competition

- The market price, $P$ is determined by (inverse) market demand:
  - $P=a-bQ$ if $a>bQ$, $P=0$ otherwise.
- Each firm decides on the quantity to sell (market share): $q_1$ and $q_2$
- $Q=q_1+q_2$ total market demand
- Both firms seek to maximize profits

Cournot Competition: Best response of Firm 1

- Suppose firm 2 produces $q_2$
- Firm 1’s profits, if it produces $q_1$ are:
  - $\pi_1 = (P-c_1)q_1 = (a-b(q_1+q_2))q_1 - c_1q_1$
    - (Residual) revenue – Cost
- How to choose $q_1$ to maximize $\pi_1$?
  - First note that $\pi_1$ is concave: $d^2\pi_1/dq_1^2 = -2b < 0$
  - First order conditions (FOC):
    - $d\pi_1/dq_1 = a - 2bq_1 - bq_2 - c_1 = 0$
    - Residual marginal revenue – Marginal cost
    - $q_1 = (a-c_1)/2b - q_2/2 = R_1(q_2)$

Example: Cournot Competition

- $P = 130-(q_1+q_2)$, so $a=130$, $b=1$
- $c_1 = c_2 = c = 10$
- Suppose Firm 2 thinks that Firm 1 will set $q_1=40$
  - Residual demand of Firm 2: $P= 90-q_2$
  - Residual revenue of Firm 2: $RR=90q_2-q_2^2$
  - Residual marginal revenue:
    - $RMR=90-2q_2$
- Setting $RMR=MC=10$
  - $90-2q_2=10$ → $q_2=40$

Cournot Competition: Best response of Firm 2

- Suppose firm 1 produces $q_1$
- Firm 2’s profits, if it produces $q_2$ are:
  - $\pi_2 = (P-c_2)q_2 = (a-b(q_1+q_2))q_2 - c_2q_2$
    - (Residual) revenue – Cost
- First order conditions:
  - $d\pi_2/dq_2 = a - 2bq_1 - bq_2 - c_2 = 0$
  - Residual marginal revenue – Marginal cost
  - $q_2 = (a-c_2)/2b - q_1/2 = R_2(q_1)$

Cournot Competition: Graphical solution

- $P=130-Q$
- $RMR=90-2q$
- $MC=10$
- $q=40$, $P=90$
Cournot Equilibrium

- \( q_1 = \frac{(a-c_1)}{2b} - \frac{q_2}{2} \)
- \( q_2 = \frac{(a-c_2)}{2b} - \frac{q_1}{2} \)
- Solving together for \( q_1 \) and \( q_2 \):
  \[
  q_1^C = \frac{(a-2c_1+c_2)}{3b} \quad q_2^C = \frac{(a-2c_2+c_1)}{3b}
  \]
- Market demand and price:
  \[
  Q^C = q_1^C + q_2^C = \frac{(2a-c_1-c_2)}{3b}
  \]
  \[
  P^C = a - bQ^C = \frac{(a+c_1+c_2)}{3}
  \]

Example: Cournot Competition

- \( P = 130-(q_1+q_2) \), so \( a=130, b=1 \)
- \( c_1 = c_2 = c = 10 \)
- The firms' best response functions:
  \[
  q_1 = (a-bq_1-c)/2b = (130-q_1-10)/2 = 60-q_1/2
  \]
  \[
  q_2 = (a-bq_2-c)/2b = (130-q_2-10)/2 = 60-q_2/2
  \]
- Solving for \( q_1 \) and \( q_2 \):
  \[
  q_1 = q_2 = 40
  \]
  \[
  Q = 80 \quad P = 50
  \]
- Firms' profits:
  \[
  \pi_1 = \pi_2 = (50-10)(40) = 1600
  \]

Cournot Equilibrium with \( N \) firms

- Optimization problem:
  \[
  \max_i \pi_i(q_i,q_{-i}) = [a-bq_i-h\sum_{j\neq i} q_j]q_i - c_i q_i
  \]
- First order conditions:
  \[
  a - 2bq_i - h\sum_{j\neq i} q_j - c_i = 0 \quad \forall i = 1,\ldots, N
  \]
- Substitute \( Q = \sum q_i \):
  \[
  a - bQ - hQ - c_i = 0 \quad \forall i = 1,\ldots, N
  \]
- Sum over \( N \):
  \[
  Na - bQ - hNQ - \sum_i c_i = 0
  \]

If each firm has the same cost \( c = c \):

- \( q_i^C = \frac{Q^C}{N} = \frac{a-c}{(N+1)b} \)
- \( p^C = \frac{a + nc}{N+1} \)

Cournot Equilibrium with \( N \) firms

- Bertrand Equilibrium Model
  - Firms set prices rather than quantities
  - \( P = a-bQ \)
  - Customers buy from the firm with the cheapest price
  - The market is split evenly if firms offer the same price
Best response

- Firm 1’s profit function:
  \[ \pi(P_1) = (P_1 - c_1) q_1 \]
  - To ensure \( q_1 > 0 \) (recall: \( P = a - bQ \) and \( Q = (a - P)/b \))
  \( P_1 \leq a \)
  - To ensure nonnegative profits
  \( P_1 \geq c_1 \)
  - Firm 1 should choose
  \( c_1 \leq P_1 \leq a \)
  - Similarly, firm 2 should choose
  \( c_2 \leq P_2 \leq a \)

Best response (cont.)

- Firm i’s demand depends on the relationship between \( P_i \) and \( P_j \):
  \[
  q_i = \begin{cases} 
    0, & \text{if } P_i > P_j \\
    \frac{a - P_i}{b}, & \text{if } P_i < P_j \\
    \frac{a - P}{2b}, & \text{if } P_i = P_j 
  \end{cases} 
  \]
  - Firm 1 should choose \( c_1 \leq P_1 \leq P_2 \) (if possible)
  - Firm 2 should choose \( c_2 \leq P_2 \leq P_1 \) (if possible)

Bertrand equilibrium

- For both firms to sell positive quantities profitably
  \( c_1 \leq P_1 \leq P_2 \) and \( c_2 \leq P_2 \leq P_1 \)
  - Suppose \( c_1 = c_2 = c \)
    \( P = c \) \quad q_1 = q_2 = (a - c)/2b 
  - Suppose \( c_1 < c_2 \)
    \( P_1 = c_2 - \epsilon \quad P_2 \geq c_2 \)
    \( q_1 = (a - c_2 + \epsilon)/b \quad q_2 = 0 \)

Example

- \( P = 130 - (q_1 + q_2) \) (\( a = 130, b = 1 \))
- \( c_1 = c_2 = c = 10 \)
- \( P = 10 \)
- \( q_1 = q_2 = (a - P)/2b = 60 \)
- \( Q = 120 \)
- Firms’ profits:
  \( \pi_1 = \pi_2 = 0 \)

Quantity-setting monopolist

- Single firm (monopolist), selling a single good
- The marginal cost of producing each unit of the good: \( c \)
- The firm decides on the quantity to sell: \( Q \) (market demand)
- The market price, \( P \) is determined by (inverse) market demand:
  \( P = a - bQ \) if \( a > bQ \), \( P = 0 \) otherwise.
- The firm seeks to maximize profits

Quantity-setting monopolist

- The firm’s profits, if it produces \( Q \) are:
  \[ \pi = (P - c)Q = (a - bQ)Q - cQ \]
  \( = \) Revenue \( - \) Cost
- How to choose \( Q \) to maximize \( \pi \)?
- First note that \( \pi \) is concave: \( d^2\pi/dQ^2 = -2b < 0 \)
- First order conditions (FOC):
  \[
  \frac{d\pi}{dQ} = a - 2bQ - c = \text{Marginal revenue} - \text{Marginal cost} \\
  0 \rightarrow Q = (a - c)/2b \\
  P = (a + c)/2 \]
Example

- \( P = 130 - Q \) \( (a=130, b=1) \)
- \( c = 10 \)
- \( Q = \frac{(a-c)}{2b} = 60 \)
- \( P = \frac{(a+c)}{2} = 70 \)
- Monopolist’s profits:
  \[ \pi = (70-10)60 = 3600 \]

Monopoly vs. Cournot vs. Bertrand

<table>
<thead>
<tr>
<th></th>
<th>Competitive</th>
<th>Bertrand</th>
<th>Cournot</th>
<th>Monopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>10</td>
<td>10</td>
<td>50</td>
<td>70</td>
</tr>
<tr>
<td>Quantity</td>
<td>120</td>
<td>120</td>
<td>80</td>
<td>60</td>
</tr>
<tr>
<td>Total Firm Profits</td>
<td>0</td>
<td>0</td>
<td>3200</td>
<td>3600</td>
</tr>
</tbody>
</table>

- Firm profits and prices:
  Competitive ≤ Bertrand ≤ Cournot ≤ Monopoly