8 Demand

8.1 Introduction

Suppose there are two goods \((X \text{ and } Y)\) and two people (1 and 2) in an economy. Person 1’s demand for good \(X\) is

\[
X_1 = D_X^1(P_X, P_Y, I_1),
\]

and person 2’s demand for good \(X\) is

\[
X_2 = D_X^2(P_X, P_Y, I_2).
\]

The aggregate (market) demand is

\[
X = X_1 + X_2 = D_X^1(P_X, P_Y, I_1) + D_X^2(P_X, P_Y, I_2) = MD_X(P_X, P_Y, I_1, I_2).
\]

The following graphic depicts the aggregation:

![Figure 34: Choice of Capacities](image)

Factors that could cause the demand curve to shift (either upward or downward) include new entrants into the market, a change in income of the individuals in the population, advertising, and a rise in the price of substitutes.
8.2 Elasticity of Demand

Given a demand function $B = f(A\ldots)$, the elasticity of good $B$ with respect to good $A$ is defined as

$$e_{B,A} = \frac{\% \text{change in } B}{\% \text{change in } A} = \frac{\Delta B}{B} \frac{A}{\Delta A} = \frac{\partial B}{\partial A} * \frac{A}{B}.$$

8.2.1 Price Elasticity of Demand

The price elasticity of demand ($e_{Q,P}$) is the percentage change in demand that accompanies a given percentage change in price. It is defined as follows:

$$e_{Q,P} = \frac{\partial Q}{\partial P} * \frac{P}{Q}.$$

Typically, there are three regions defined for price elasticity of demand:

$e_{Q,P} < -1 \rightarrow$ elastic (luxury)

$e_{Q,P} = 1 \rightarrow$ unit elastic

$e_{Q,P} > -1 \rightarrow$ inelastic (necessities)

The total expenditure on any good is the product of $P$ and $Q$. Therefore,

$$\frac{\partial PQ}{\partial P} = Q + P \frac{\partial Q}{\partial P} \rightarrow \frac{\partial PQ}{\partial Q} = 1 + \frac{\partial Q}{\partial P} * \frac{P}{Q} = 1 + e_{Q,P}.$$

Thus, for elastic goods, an increase in price results in a decrease in demand, while an increase in price results in an increase in demand. The reverse effect holds for inelastic goods, while changes in price do not effect the demand for unit elastic goods.

8.2.2 Income Elasticity of Demand

The income elasticity of demand ($e_{Q,I}$) is the percentage change in demand that accompanies a given percentage change in income. It is defined as follows:

$$e_{Q,I} = \frac{\partial Q}{\partial I} * \frac{I}{Q}.$$

If $e_{Q,I} > 1$, the good is termed a luxury good since purchases of these goods increase more rapidly than income does.

8.2.3 Cross-Price Elasticity

The cross-price elasticity ($e_{Q_{i},P_{j}}$) is the percentage change in quantity demanded with respect to the price change of another good. Hence, it is defined as

$$e_{Q_{i},P_{j}} = \frac{\partial Q_{i}}{\partial P_{j}} * \frac{P_{j}}{Q_{i}}.$$
Thus, $e_{Q_i,P_j}$ will be positive if $i$ and $j$ are substitutes, and negative if they are compliments. The following table lists some example elasticities:

<table>
<thead>
<tr>
<th>Item</th>
<th>Income Elasticity</th>
<th>Price Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.28</td>
<td>-0.21</td>
</tr>
<tr>
<td>Automobile</td>
<td>3.00</td>
<td>-1.20</td>
</tr>
<tr>
<td>Electricity</td>
<td>0.61</td>
<td>-1.14</td>
</tr>
<tr>
<td>Medical Services</td>
<td>0.22</td>
<td>-0.20</td>
</tr>
<tr>
<td>Charitable Giving</td>
<td>0.71</td>
<td>-1.29</td>
</tr>
<tr>
<td>Gasoline</td>
<td>1.06</td>
<td>-0.54</td>
</tr>
<tr>
<td>Rent Housing</td>
<td>1.00</td>
<td>-0.18</td>
</tr>
<tr>
<td>Ownership Housing</td>
<td>1.20</td>
<td>-1.20</td>
</tr>
</tbody>
</table>

Table 18: Example Elasticities

Notice that, from an income elasticity perspective, food and medical services are necessities, while automobiles are luxuries. Also, notice that most price elasticities are low; hence, price changes do not necessarily induce substantial proportional changes in quantities. The fact that electricity is price elastic became readily apparent during the sharp increase in electricity prices during the 1970’s. The demand for electricity, consequently, sharply decreased, and many power plants were forced to shut down.

### 8.2.4 Sum of Elasticities

Suppose we assume that demand is homogeneous of degree 0. (A function is homogeneous of degree $m$ if

$$f(tx_1, tx_2, \ldots, tx_n = t^m(x_1, x_2, \ldots, x_n)).$$

Euler’s Theorem states that, if a function $f(x_1, \ldots, x_n)$ is homogeneous of degree $m$, then

$$f_1x_1 + \ldots + f_nx_n = mf(x_1, \ldots, x_n).$$

Thus, using this theorem, we have

$$\frac{\partial x}{\partial P_x} P_x + \frac{\partial x}{\partial P_y} P_y + \frac{\partial x}{\partial I} I = 0 \rightarrow \frac{\partial x}{\partial P_x} P_x + \frac{\partial x}{\partial P_y} P_y + \frac{\partial x}{\partial I} I = 0 \rightarrow e_{x,P_x} + e_{y,P_y} + e_{x,I} = 0.$$

Thus, the sum of demand elasticities with respect to price and income equals zero.

During the previous lecture, we began discussing demand. We concluded with the result that the sum of demand elasticities with respect to price and income equals zero. Now, we want to continue with a look at non-linear demand.
8.3 Nonlinear Demand

We have typically used linear demand functions (of type $Q = a + bP$) in our examples. In some ways, however, they are unsatisfactory since elasticity changes.

Figure 35: Price Elasticities

If $Q = a + bP$, as in the preceding diagram (with $b < 0$), then

$$e_{Q,P} = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = b \cdot \frac{P}{Q},$$

which means that as $\frac{P}{Q}$ increases, the curve becomes more elastic.

Now, suppose the demand function is not linear but, instead, is characterized by the form

$$Q = aP^b \rightarrow logQ = loga + blogP,$$

and

$$e_{Q,P} = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = \frac{baP^{b-1} + P}{aP^b} = b.$$

A generalized version of this is as follows:

$$X = aP^b P^c I^d \rightarrow logX = loga + blogP_x + clogP_y + dlogI.$$ 

It is, then, easy to show that

$$e_{x,P_x} = b, e_{x,P_y} = c, e_{x,I} = d.$$

8.4 Demand Estimation

Two ways firms can estimate demand are by running experimentations (e.g. experiments that Amazon.com ran by changing price before buckling under consumer pressure) or by
using historical data such as past sales and past economic data. Let’s look at an example case of estimating demand for bandwidth by Edell and Varaiya.

**Example 1: ISP’s**

Currently, internet service providers (ISP) tend to charge a flat rate (e.g. $25 a month unlimited access). Firms tend to do this instead of differentiating service because of both economic (it can be complicated to figure out the economics of differentiated pricing) and technical (the technology required to package the service in different forms may be difficult to come by) reasons. The goal of this study is to devise a pricing strategy. In order to do this, we must have an ability to characterize demand.

The study was done in 1998. There were 70 customers, consisting entirely of college faculty and students. Each customer participated in a sequence of service plans that lasted 6-10 weeks. Each service plan implemented a pricing model. The researchers ran several experiments. Here is an example of one:

<table>
<thead>
<tr>
<th>Connect Speeds</th>
<th>Average cents per min</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 kps</td>
<td>0</td>
</tr>
<tr>
<td>16 kps</td>
<td>0.3</td>
</tr>
<tr>
<td>32 kps</td>
<td>1.4</td>
</tr>
<tr>
<td>64 kps</td>
<td>4.5</td>
</tr>
<tr>
<td>96 kps</td>
<td>5.6</td>
</tr>
<tr>
<td>128 kps</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Table 19: Experiment 1

In the preceding experiment, customers could instantaneously shift between different service qualities. A spending meter on the screen informed the user of how much had been spent.
We began an in-depth discussion on a 1998 study that attempted to characterize demand of bandwidth. Before we continue with the example, however, let’s discuss the potential problem of waste, illustrated by the following diagram.

![Choice of Capacities](image)

**Figure 36: Choice of Capacities**

In the preceding diagram, if the ISP prices at $c$, then the consumer would consume $x_u$. This would be cost per unit consumption (e.g. MB data transferred). If the ISP charges a flat rate, however, then the consumer will procure $x_f$ since cost of additional consumption past $x_u$ is 0. Thus, the area that corresponds to waste is the cost of providing $x_f - x_u$ units. The consumer surplus, therefore, is lowered by the waste. Of course, if there was one customer type, then the firm could set the rate to the area of the triangle and capture all of the surplus. Waste increases for individuals with high rates of demand because the demand curve is shifted out, resulting in a larger triangle. It also increases when the price elasticity of demand increases, since the demand curve will have a longer tail.

In 1996, America Online (AOL) changed from charging $9.95 per month — including five hours connect time plus $2.95 per hour — to $19.95 per month. The average connect time went from 6.4 hours to 22.1 hours by 1998, resulting in a decrease in the company’s operating margin. (Of course, there was also a major increase in competition.)

The idea of a **user cross-subsidy** is that, due to flat-rate pricing, all users pay the same fee, resulting in light users subsidizing heavier users. (This could also potentially result in adverse selection.)

Returning to the example from last time, one way to estimate demand is to use the constant price elasticity function

$$x_i = e^{b_i\sum_{j=1}^{n} p_j^{a_{ij}}}$$

where

$x_i$ is the number of minutes of connect time for speed $i$, $p_j$ is the cents paid per minute for speed $j$, $a_{ij}$ is the coefficient solved for by regression, and $b_i$ is a constant. We can linearize this by taking the log to get

$$\log x_i = b_i + \sum_j a_{ij} \log p_j,$$
where $i, j \in \{16, 32, 64, 96, 128\}$. The data was collected for 70 users over approximately six months. After running a regression model, the following results were obtained:

<table>
<thead>
<tr>
<th>Bandwidth</th>
<th>P128</th>
<th>P96</th>
<th>P64</th>
<th>P32</th>
<th>P16</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>-2.0</td>
<td>0.80</td>
<td>0.25</td>
<td>-0.02</td>
<td>-0.16</td>
</tr>
<tr>
<td>96</td>
<td>1.70</td>
<td>-3.10</td>
<td>0.43</td>
<td>0.19</td>
<td>0.18</td>
</tr>
<tr>
<td>64</td>
<td>0.77</td>
<td>1.80</td>
<td>-2.90</td>
<td>0.59</td>
<td>0.21</td>
</tr>
<tr>
<td>32</td>
<td>0.81</td>
<td>-1.00</td>
<td>1.00</td>
<td>-1.40</td>
<td>0.18</td>
</tr>
<tr>
<td>16</td>
<td>0.20</td>
<td>-0.29</td>
<td>0.04</td>
<td>1.20</td>
<td>-1.30</td>
</tr>
</tbody>
</table>

Table 20: Price and Cross-Price Elasticities from Experiment

The cross-price elasticities in bold type were significant by the t-statistic. The implied elasticities are quite large. For example, a 1% increase in the price of 96 kps leads to a 3% decrease in its demand. A 1% increase in price for 128 kps leads to a 1.7% increase in demand for 96k. This implies that if users are offered differentiated quality of service, they would demand more than one service quality.

The $b_i$ coefficient indicates the overall change in demand. After running a regression model with respect to $b_i$, the following results were obtained:

<table>
<thead>
<tr>
<th></th>
<th>P128</th>
<th>P96</th>
<th>P64</th>
<th>P32</th>
<th>P16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_i$</td>
<td>4.1</td>
<td>2.6</td>
<td>2.7</td>
<td>2.33</td>
<td>0.52</td>
</tr>
<tr>
<td>$e^{b_i}$</td>
<td>60.3</td>
<td>13.5</td>
<td>14.9</td>
<td>10.3</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 21: Change in Overall Demand

Thus, increasing the speed from 16 to 128 results in an over 30-fold increase in demand, while going from 32 to 128 increases the demand approximately six-fold.

Here are the results from the regression model associated with this handout. $R^2$ for the model is 0.3. If we include a dummy variable for customer type $k$,

$$log x_i^k = b_i^k + \sum_j a_{ij} log p_j \to R^2 = 0.98.$$  

Thus the parameters stayed the same, even though the equation changed.

We will end today’s lecture by introducing the Rand study for medical care, which we will discuss more in-depth on next time.

**8.4.1 Medical Care Demand Estimation**

The Rand Study was an $80 million study initiated by the Federal Government in 1974. There has been a substantial increase in medical care costs in the United States over the years, going from 4.4% of GDP in 1950, doubling to 8.8% of GDP in 1980, and increasing to
13.2% of GDP in 2000. About half of these costs are covered by the government. A crucial issue is how has insurance played a role in this. Major factors include an increasingly aging population, better technology (i.e. medicines), supply (doctors), and waste (insurance). A key question involves the sensitivity of healthcare to the copayment structure. We will discuss these and other issues on next time.

The headline in *The New York Times* on Sunday, March 17, 2002 was ”Many Doctors Say They Are Refusing Medicare Patients.” The popular view is that to increase access and utilization, both Medicare (care for the elderly) and Medicaid (care for the poor) must raise rates. During today’s lecture, we will attempt to analyze the demand process in order to answer this question.

Medicaid started in 1965 for people with low-income as a part of President Lyndon Johnson’s Great Society package. Since then, the expenditures have increased in the following manner:

<table>
<thead>
<tr>
<th>Year</th>
<th>Expenditure Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965</td>
<td>$0.6 billion</td>
</tr>
<tr>
<td>1975</td>
<td>$7.4 billion</td>
</tr>
<tr>
<td>1985</td>
<td>$22.6 billion</td>
</tr>
<tr>
<td>1995</td>
<td>$86.2 billion</td>
</tr>
<tr>
<td>2000</td>
<td>$117.6 billion</td>
</tr>
</tbody>
</table>

Table 22: Medicaid Expenditures

For comparison purposes, the expenditures for Medicare in 2000 totalled $224 billion. Total expenditures in health care nationally was approximately $1.3 trillion dollars; the budget allotment was $1.73 trillion. Thus, Medicare and Medicaid combined make up approximately 20% of the budget.

In 1996, approximately 36 million people received Medicaid, which accounted for 22% of in-patient hospital services and 25% of nursing home care.

Today, based on data obtained from the Centers for Disease Control and Prevention, Division of Oral Health, we will attempt to answer the following question: In the state of Louisiana, does it make sense to increase Medicaid payments for oral health?
First, let’s discuss the dental markets. In the private market depicted in the graphic below, dentists are price setters.

Figure 37: Private Dental Market

In the Medicaid market below, dentists are price takers.

Figure 38: Medicaid Dental Market
Combining these two markets yields the following combination market:

\[ P \quad Q \\]

\[ \text{Supply} \quad \text{Marginal Revenue} \]

Figure 39: Combined Market

There are two possible scenarios for the Medicaid market. If \( Q > Q^* \), then increasing the supply will cause an increase in output (i.e. increase utilization). If \( Q < Q^* \), however, then medical demand is less than supply, and increasing supply will have no effect on demand. Therefore, increasing Medicaid reimbursements will only serve the purpose of making dentists richer in the latter case.

**Example 1**

Let’s now discuss the handout. First, the column headings correspond to the following:

- **PARISH** = identifier of parish.
- **POP** = population of parish.
- \( U \) = number of individuals in the parish with at least one dental visit.
- **DENT** = number of dentists in the parish.
- **TOTINC** = total income per person in the parish.
- **FLUOR** = number of persons in the parish with access to fluoridated water.
- **FLIG** = number of people eligible for Medicaid in the parish.

The utilization \( U \), therefore, is defined as

\[ U = f(\text{POP}, \text{DENT}, \text{TOTINC}, \text{FLUOR}, \text{FLIG}). \]

The parishes were classified as either rural or urban, according to their population. Running the analyses yields an adjusted \( R^2 \) value of 0.9618 for rural parishes and an adjusted \( R^2 \) value of 0.9818 for urban parishes.
Their parameters and p-values are listed in the tables below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>67.51</td>
<td>0.4868</td>
</tr>
<tr>
<td>POP</td>
<td>0.0282</td>
<td>0.1094</td>
</tr>
<tr>
<td>DENT</td>
<td>-11.103</td>
<td>0.4367</td>
</tr>
<tr>
<td>ELIG</td>
<td>0.096</td>
<td>0.0021</td>
</tr>
<tr>
<td>TOTINC</td>
<td>-0.0003</td>
<td>0.7535</td>
</tr>
<tr>
<td>FLUOR</td>
<td>-0.0043</td>
<td>0.3606</td>
</tr>
</tbody>
</table>

Table 23: Parameters and P-values for Rural Parishes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>18.09</td>
<td>0.9652</td>
</tr>
<tr>
<td>POP</td>
<td>0.0045</td>
<td>0.8016</td>
</tr>
<tr>
<td>DENT</td>
<td>18.284</td>
<td>0.125</td>
</tr>
<tr>
<td>ELIG</td>
<td>0.497</td>
<td>0.0001</td>
</tr>
<tr>
<td>TOTINC</td>
<td>-0.0013</td>
<td>0.1774</td>
</tr>
<tr>
<td>FLUOR</td>
<td>0.00132</td>
<td>0.5604</td>
</tr>
</tbody>
</table>

Table 24: Parameters and P-values for Urban Parishes

Note that, in both cases, the only variable that is significant is the number of people eligible for Medicaid; hence, there are no supply side factors! This tells us that $q < q^*$, and Medicaid reimbursement rates should not be raised.

Now, we return to the Rand study that we began discussing at the close of last lecture. Health insurance accounts for most of the sustained rise in health expenditures; therefore, it is important to understand the demand response to changes in insurance. The stated goals of the Rand study were as follows:

- Determine how demand responds to insurance-induced changes in price.
- Determine if this demand response differs for the poor, who are typically covered by government programs.
- Determine if elasticities differ for different types of services (e.g. outpatient, preventive care, etc.).
- Determine how changes in the consumption of medical services affects healthcare.
- What was responsible for lower costs in HMO’s (e.g. cherry picking, lost services, efficiency).

The experiment was conducted over a three-year period from 1974-1977. There were six sites, and 20000 people participated in the study. Families were randomly assigned to different fee-for-service insurance plans.

The dependent variable in this study was the measure of medical services consumed.
The independent variables were the five possible insurance plans that the families were randomly assigned to. One was the free plan, which consisted of no out-of-pocket payments for outpatient nor inpatient services. There were three co-insurance plans with 25%, 50%, and 95% coverage. There was also a plan that was 95% co-insurance for outpatient services and free for inpatient services. Each plan had a maximum dollar expenditure on annual out-of-pocket expenditures of $1000. The following table lists the findings.

<table>
<thead>
<tr>
<th>Plan</th>
<th>Likelihood of Use (Dental)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>86.7% 68.7%</td>
</tr>
<tr>
<td>25% co-insurance</td>
<td>78.8% 53.6%</td>
</tr>
<tr>
<td>50% co-insurance</td>
<td>74.3% 54.1%</td>
</tr>
<tr>
<td>95% co-insurance</td>
<td>68.0% 47.1%</td>
</tr>
<tr>
<td>Individual</td>
<td>72.6% 48.9%</td>
</tr>
</tbody>
</table>

Table 25: Results from Rand Study

The plans with 0-25% co-insurance had an elasticities of -0.13, while the remaining plans (25-95%) had elasticities of -0.21.

On next time, in preparation for our discussion on combinatorial auctions, please read the Rothkopf paper.