Show all your steps to get full credit.

Reading assignment: Read Chapter 5.

1. (10 points) Answer questions 1 and 2 of the case.

1. (5 points) Convert the following LPs into standard form.

   (a) max \( 15x_1 + 3x_2 - 5x_3 \)
       \[ \text{s.t.} \quad \begin{align*}
       x_1 + 10x_2 - 7x_3 & \leq 50 \\
       3x_1 - x_2 & \geq 0 \\
       x_1 - 2x_2 + 4x_3 & = 21 \\
       x_1, x_2, x_3 & \geq 0 \\
       x_3 & \text{unrestricted in sign}
       \end{align*} \]
   
   (b) min \( 3x + 5y + 7z \)
       \[ \text{s.t.} \quad \begin{align*}
       x - 3y + 2z & \leq 19 \\
       2x + 4y - 2z & = -12 \\
       x - 2y + 5z & \geq 6 \\
       x, z & \geq 0 \\
       y & \text{unrestricted in sign}
       \end{align*} \]

2. (25 points) Consider the following linear program:

   max \( 3x_1 + 2x_2 - x_3 \)
   \[ \text{s.t.} \quad \begin{align*}
   x_1 + 10x_2 - 5x_3 & \leq 15 \\
   2x_1 - x_2 & \geq 0 \\
   x_1 - 2x_2 + 4x_3 & \leq 12 \\
   x_1, x_2, x_3 & \geq 0
   \end{align*} \]

Do two iterations of simplex search. At each iteration,
(1) identify feasible simplex directions,
(2) identify a feasible and improving simplex direction (use the gradient test to show that your direction is improving),
(3) determine the maximum step size for the direction you chose in (3), and
(4) compute the new point to start the next iteration.
At each iteration indicate the basic and nonbasic variables and state which variable enters and which variable leaves the basis.
3. (10 points): The following plot shows several feasible points in a linear program and contours of its objective function.

Determine whether each of the following sequences of solutions could have been followed by the simplex algorithm applied to the corresponding LP in standard form. Explain.

(a) A, H, D  
(b) G, A, B, C, D  
(c) A, B, D

(d) A, B, E  
(e) F, G, A