Solutions for homework #4

Pb.#1

a) Decision variables: X_{ikl} = tons of paper type k produced on machine type l at mill i;

Y_{ijk} = tons of paper type k shipped to customer type k from mill i;

$$\begin{aligned}
min \quad & \sum_{i=1}^{10}\sum_{k=1}^{5}\sum_{l=1}^{3} P_{ikl}\cdot X_{ikl} + \sum_{i=1}^{10}\sum_{j=1}^{725}\sum_{k=1}^{5} T_{ijk}\cdot Y_{ijk} & & \\
\text{s. t.} \quad & \sum_{i=1}^{10} Y_{ijk} \geq D_{jk} & \forall j = 1\ldots725, k = 1\ldots5 \quad & (1)\\
& \sum_{k=1}^{5}\sum_{l=1}^{3} r_{klm}\cdot X_{ikl} \leq R_{im} & \forall i = 1\ldots10, m = 1\ldots4 \quad & (2)\\
& \sum_{k=1}^{5} c_{kl}\cdot X_{ikl} \leq C_{il} & \forall i = 1\ldots10, l = 1\ldots3 \quad & (3)\\
& \sum_{l=1}^{3} X_{ikl} = \sum_{j=1}^{725} Y_{ijk} & \forall i = 1\ldots10, k = 1\ldots5 \quad & (4)\\
& X_{ikl} \geq 0 & \forall i = 1\ldots10, k = 1\ldots5, l = 1\ldots3 \quad & (5)\\
& Y_{ijk} \geq 0 & \forall i = 1\ldots10, j = 1\ldots725, k = 1\ldots5 \quad & (6)
\end{aligned}$$

(1) = the total quantity of paper type k that has been shipped from mill i to customer j has to meet the quantity demanded for each paper type k and customer j;

(2) = the total units of raw material m used to produce paper type k on machine l has to be less than the available units for each raw material m at each mill i;

(3) = the total capacity used to produce paper type k on machine l has to be less than the available capacity for each machine l at each mill i;

(4) = the quantity of paper type k shipped to all customers from mill i

has to be equal to the total quantity of paper type k produced at mill i on all machines;

(5), (6) = nonnegativity constraints.

b) # of variables = # of X variables + # of Y variables = (# of i) · (# of k) · (# of l) + (# of i) · (# of j) · (# of k) = $10 \cdot 5 \cdot 3 + 10 \cdot 725 \cdot 5 =$ 36,400;

# of constraints = (# of constraints (1)) + (# of constraints (2)) + (# of constraints (3)) + (# of constraints (4)) + (# of constraints (5)) + (# of constraints (6))= (# of j) · (# of k) + (# of i) · (# of m) + (# of i) · (# of l) + (# of i) · (# of k) + (# of i) · (# of k) · (# of l) + (# of i) · (# of j) · (# of k) = $725 \cdot 5 + 10 \cdot 4 + 10 \cdot 3 + 10 \cdot 5 + 10 \cdot 5 \cdot 3 + 10 \cdot 725 \cdot 5 =$ 40, 145

Pb.#2

a) LP:

$$\begin{aligned} max \quad & 2x_1 + 0.5x_2 \\ \text{s. t.} \quad & x_1 + x_2 \leq 1 \\ & 4x_1 + x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

All constraints and the objective function are linear.

b) ILP:

$$\begin{aligned} max \quad & x_1 + x_2 + x_3 \\ \text{s. t.} \quad & x_1 - x_2 + x_3 \leq 1 \\ & x_1, x_2, x_3 \in \{0, 1, 2\} \end{aligned}$$

The constraint and the objective function are linear; the variables are integer.

c) NLP:

$$\begin{aligned} max \quad & x_1^2 - x_2^2 + x_3^2 \\ \text{s. t.} \quad & x_1 + \sqrt{x_2} + x_3 \leq 1 \end{aligned}$$

The constraint and the objective function are nonlinear.

d) INLP:

$$\begin{aligned} max \quad & x_1^2 - x_2 \\ \text{s. t.} \quad & x_1 + x_2^2 \leq 1 \\ & x_1, x_2 \in Z \end{aligned}$$

The constraint and the objective function are nonlinear; the variables are integer.

Pb.#3

a) (4,1) is feasible because it satisfies all constraints, but no sort of optimum because it can be improved in the neighborhood (e.g., at (4,0.75));

(1,0) is feasible because it satisfies all constraints, but no sort of optimum because it can be improved in the neighborhood (e.g., at (1,0.5));

(0,1) is infeasible and thus no sort of optimum because it violates constraint $x_1 \geq 1$;

(5,0) is feasible and only a local maximum because all constraints are satisfied and no nearby point has better objective value, but there are better such as (1,3).

b) Sequence 1: cannot be followed by an improving search algorithm because the value of the objective function for (2,1) is strictly higher than the value of the objective function for (4,1);

Sequence 2: can be followed by an improving search algorithm because the value of the objective function is strictly increasing along the sequence.