Sample Exam #2

Engineering Optimization (ISyE 4231) - Fall 2000

This is a closed book, closed notes exam. You are allowed to use one letter size, single-sided page of notes.

Question 1

The NCAA is making plans for distributing tickets to the upcoming regional basketball championships. Up to 10,000 available seats will be divided between the media, the competing universities, and the general public. Media people are admitted free, but the NCAA receives $45 per ticket from universities and $100 per ticket from the general public. At least 500 tickets must be reserved for the media and at least half as many tickets should go to the competing universities as to the general public. Within these restrictions, the NCAA wishes to find the allocation that raises the most money.

(a) Briefly explain how this problem can be modeled by the following LP:

$$\begin{aligned} \text{Max} \quad & 45x_2+100x_3 \\ \text{Subject to} \quad & \\ & x_1 + x_2 + x_3 \leq 10{,}000 \\ & x_2 - 0.5x_3 \geq 0 \\ & x_1 \geq 500 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(b) Convert the problem into standard form.

Answer the following questions using the matrix notation:

(c) Consider the following solution to this problem: $(x_1, x_2, x_3)=(500, 19{,}000/6, 19{,}000/3)$. What are the values of the slack variables in this solution? (Use the standard form of part b). Which variables are basic/nonbasic? What is the basis matrix B?

(d) Is the solution in part (c) the optimum solution? (Hint: The inverse of the basis matrix is **).

(e) What is the marginal cost to the NCAA of each seat guaranteed to the media? How much could NCAA save if it did not have to reserve seats for the media?

(f) Suppose there is an alternative dome where the games could be played that can provide 15,000 seats. How much additional revenue would be gained if there were 15,000 seats? (Assume there is enough demand to fill all the seats.)

(g) To accommodate high demand from student supporters of participating universities, the NCAA is considering marketing a new “scrunch seat” that counts fully against the “university ≥ half public” rule. Could an optimal solution allocate any such seats at a ticket price of $35? At a price of $25?

(h) Using the matrix notation, create the simplex tableau that corresponds to the solution given in part (c).

Question 2

Find the dual of the problem in Question 1. Compute the dual optimum solution and the dual objective function value using the primal basis B. Interpret the dual variables.

To answer the following questions, you may use the dual, or x_3’s coefficient in the top row of the simplex tableau (which you may compute using the matrix notation) of the primal. NCAA considers reducing the price of public tickets to $50. How much revenue would be lost in that case? What if the price of a public ticket dropped to $30?

Question 3
Paper can be made from new wood pulp, from recycled office paper or from recycled newsprint. New pulp, recycled office paper and recycled newsprint cost \$100, \$50 and \$20, respectively. Four processes are available for making paper. The following table shows how much material (in tons) each process would use to make one ton of paper.

	Pulp	Recycled office paper	Recycled newsprint
Process 1	3	-	-
Process 2	1	4	-
Process 3	1	-	12
Process 4	-	8	-

Currently, only 80 tons of pulp is available. The company wants to produce 100 tons of paper at minimum cost.

(a) Explain why the problem can be modeled as the following LP:

Min $100x1+50x2+20x3$
Subject to
$x1-3y1-y2-y3 = 0$
$x2-4y2-8y4=0$
$x3-12y3 = 0$
$x1 \leq 80$
$y1+\ldots+y4 \geq 100$
$x1, \ldots, x3, y1, \ldots, y4 \geq 0$

(b) Find the dual of this linear program.

Answer the following questions using the Lindo output:

(c) What is the optimum solution and the optimum objective function value? What are the basic and nonbasic variables?
(d) What is the optimum dual solution? Verify that the dual objective is equal to the primal objective.
(e) What would be the additional cost of producing 10 additional tons of paper?
(f) A supplier is offering to sell to this manufacturer additional pulp. How much additional pulp (if any) should this manufacturer buy, at what price?
(g) How would the optimal solution change if the price of pulp increased to \$150 per ton?
(h) How would the optimal solution change if the price of recycled office paper decreased by \$20 per ton? What if it decreased by \$30 per ton?
(i) Would the basis change if the company wanted to produce only 50 tons of paper?
(j) There is a shortage of recycled newsprint and only 30 tons are available. Would this additional constraint change the optimum solution?
(k) Would the solution change if the machine on which process 1 runs fails? What if the machine that runs process 2 fails?

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 32000.00

VARIABLE	VALUE	REDUCED COST
X1	80.000000	.000000
X2	480.000000	.000000
X3	.000000	3.333333
Y1	.000000	200.000000
Y2	80.000000	.000000
Y3	.000000	.000000
Y4	20.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	-200.000000
3)	.000000	-50.000000
4)	.000000	-16.666670
5)	.000000	100.000000
6)	.000000	-400.000000

NO. ITERATIONS= 3

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	100.000000	100.000000	INFINITY
X2	50.000000	10.000000	25.000000
X3	20.000000	INFINITY	3.333334
Y1	.000000	INFINITY	200.000000
Y2	.000000	40.000010	INFINITY
Y3	.000000	INFINITY	40.000010
Y4	.000000	INFINITY	100.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	.000000	80.000000	20.000000
3	.000000	INFINITY	480.000000
4	.000000	.000000	960.000000
5	80.000000	20.000000	80.000000
6	100.000000	INFINITY	20.000000

Question 4

Consider the following linear program:

Max $2x1 + 2.5\,x2$
Subject to

$$x1 + 2x2 \leq 350$$
$$2x1 + x2 \leq 400$$
$$x1, x2 \geq 0$$

(a) Solve this problem graphically.
(b) Convert this problem into standard form and solve using the Simplex method in the tableau form.
(c) Do one iteration of the simplex method in standard form (i.e. by finding feasible and improving directions at each iteration).
(d) Show how a basic feasible solution at each iteration corresponds to an extreme point.