# Homework #5 - Solutions

## Engineering Optimization (ISyE 4231) - Fall 2000

**Problem 1**.

**Question 3**. This schedule is feasible.

Aircraft 1 : AT(1,4) = AT(1,3) = 1 and TT(4,3) = 1
Aircraft 2 : AT(2,8) = AT(2,2) = 1 and TT(8,2) = 1
Aircraft 3 : AT(3,1) = 1
Aircraft 4 : AT(4,7) = AT(4,6) = 1 and TT(7,6) = 1.

Moreover, this schedule satisfies the maintenance restriction of aircraft 1 because *fly(4)* + *flight time from city 8 to city 9* + *fly(3)* = 150 + 60 +120 = 330 < 337 and the ending time of aircraft 1's schedule = 298 + 150 = 448 < 630.

Alternative feasible schedule : There are no alternative feasible schedules without subcontracting more aircraft. The reason is as follows.

Aircraft 3 : Aircraft 3 has to serve trip 1 since trip 1 is previously scheduled to aircraft 1. No trips can be served immediately after or immediately before trip 1 by the same aircraft because TT(2,1)=…=TT(8,1)=0 and TT(1,2)=…=TT(1,8)=0. So aircraft 3 can serve only trip 1 during this schedule period.

Aircraft 1 : From Question4, we know that we need to subcontract one extra aircraft to serve trip 3,5,6 and 8. So we have to serve trip 4 and trip 7 with our aircraft not to subcontract more aircraft. Since AT(1,4)=1 and AT(2,4)=AT(3,4)=AT(4,4)=0, Aircraft 1 has to serve trip4. Because of the maintenance restrict of aircraft 1, trip 3 is the only possible next trip that can be served by aircraft 1. Aircraft 1 has to serve trip 3.(Suppose not. We need to subcontract more aircraft because only aircraft 2 and 4 are available for serving trip 3, 5, 6 and 8.) So aircraft 1 has to serve trip 4 and trip 3.

Aircraft 4 : Aircraft 4 has to serve trip 7.(Suppose not we need to subcontract one more aircraft for trip 7 because AT(4,7)=1 and AT(1,7)=AT(2,7)=AT(3,7)=0). Only trip 6 can be served immediately after trip 7 out of trip 5,6,8 from the TT matrix. So aircraft 4 has to serve trip 6.( Suppose not we need to subcontract more aircraft because only aircraft 2 is available for trip 5,6 and 8.) So aircraft 4 has to serve trip 7 and trip 6.

Aircraft 2 : Aircraft 2 has to serve trip 2 since trip 2 is previously scheduled to aircraft 2. Aircraft 2 has to serve trip 8.( Suppose not No aircraft are available for trip 5 and 8 because TT(5,2)=0.) So aircraft 2 has to serve trip 8 and trip 2.

Therefore we have to subcontract one extra aircraft for serving trip 5.

Obviously, there are several alternative feasible solutions with subcontracting more aircraft.

**Question 4**. No. Consider trips 1, 3, 5 and 8. Since these trips all intersect each other (see figure), we need 4 different aircraft to serve them. Furthermore, trip 6 intersects trips 1, 3, and 5, so the aircraft serving trips 1, 3, and 5 cannot serve trip 6. Finally, there is not enough time to fly from the destination of trip 8 to the departure location of trip 6 on time, so if an aircraft serves trip 8 then it cannot serve trip 6. Therefore, among the trips 1, 3, 5, 6 and 8, at least one should be subcontracted.

More formally, we have

TT(1,3) = TT(1,5) = TT(1,6) = TT(1,8) = 0
TT(3,1) = TT(3,5) = TT(3,6) = TT(3,8) = 0
TT(5,1) = TT(5,3) = TT(5,6) = TT(5,8) = 0
TT(6,1) = TT(6,3) = TT(6,5) = TT(6,8) = 0
TT(8,1) = TT(8,3) = TT(8,5) = TT(8,6) = 0.

This means that we need different airplanes to serve each trip1, trip3, trip5, trip6 and trip8. So there are no feasible schedules without subcontracting extra aircraft (we have 4 airplanes and any two of above five trips cannot be served by one aircraft.)

**Problem 2**.

(a) Standard form

$$\begin{aligned} max \quad & 3x_1 + x_2 \\ s.t. \quad & -2x_1 + x_2 + x_3 = 2 \\ & x_1 + x_2 + x_4 = 6 \\ & x_1 + x_5 = 4 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

(b) Above standard-form linear program has the following coefficient data.

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | |
|---|---|---|---|---|---|---|
| Max **c** | 3 | 1 | 0 | 0 | 0 | **b** |
| | -2 | 1 | 1 | 0 | 0 | 2 |
| **A** | 1 | 1 | 0 | 1 | 0 | 6 |
| | 1 | 0 | 0 | 0 | 1 | 4 |

$x^{(0)}$ = (0, 0, 2, 6, 4) and $x_3$, $x_4$ and $x_5$ are basic.

*First iteration*

Simplex directions ( $A\Delta x = 0$ )

Simplex direction increasing $x_1$ when $\Delta x_1$ is fixed =1.:

$$-2(1) + 1(0) + 1\Delta x_3 + 0\Delta x_4 + 0\Delta x_5 = 0$$
$$+1(1) + 1(0) + 0\Delta x_3 + 1\Delta x_4 + 0\Delta x_5 = 0$$
$$+1(1) + 0(0) + 0\Delta x_3 + 0\Delta x_4 + 1\Delta x_5 = 0$$

By solving above equation system we have $\Delta x = (1,0,2,-1,-1)$

Simplex direction increasing $x_2$ when $\Delta x_2$ is fixed =1.:

$$-2(0) + 1(1) + 1\Delta x_3 + 0\Delta x_4 + 0\Delta x_5 = 0$$
$$+1(0) + 1(1) + 0\Delta x_3 + 1\Delta x_4 + 0\Delta x_5 = 0$$
$$+1(0) + 0(1) + 0\Delta x_3 + 0\Delta x_4 + 1\Delta x_5 = 0$$

By solving above equation system we have $\Delta x = (0,1,-1,-1,0)$

Improving simplex directions

$\bar{c}_1 = (3,1,0,0,0) \bullet (1,0,2,-1,-1) = 3 > 0$ , $\Delta x = (1,0,2,-1,-1)$ is a improving simplex direction

$\bar{c}_2 = (3,1,0,0,0) \bullet (0,1,-1,-1,0) = 1 > 0$ , $\Delta x = (0,1,-1,-1,0)$ is a improving simplex direction

Entering variable : candidate entering variables are $x_1$ and $x_2$. Let's pick $x_1$.

Leaving variable : min{6/1, 4/1} = 4. minimized for $x_5$. $x_5$ leaves the basis.

New basic feasible solution :

$x^{(1)} \leftarrow x^{(0)} + l\,\Delta x$ = (0, 0, 2, 6, 4) + 4(1, 0, 2, -1, -1) = (4, 0, 10, 2, 0)

and $x_1$, $x_3$ and $x_4$ are basic

*Second iteration*

Simplex directions ( $A\Delta x = 0$ )

Simplex direction increasing $x_2$ when $\Delta x_2$ is fixed =1.:

$$-2\Delta x_1 + 1(1) + 1\Delta x_3 + 0\Delta x_4 + 0(0) = 0$$
$$+1\Delta x_1 + 1(1) + 0\Delta x_3 + 1\Delta x_4 + 0(0) = 0$$
$$+1\Delta x_1 + 0(1) + 0\Delta x_3 + 0\Delta x_4 + 1(0) = 0$$

By solving above equation system we have $\Delta x = (0,1,-1,-1,0)$

Simplex direction increasing $x_5$ when $\Delta x_5$ is fixed =1.:

$$-2\Delta x_1 + 1(0) + 1\Delta x_3 + 0\Delta x_4 + 0(1) = 0$$
$$+1\Delta x_1 + 1(0) + 0\Delta x_3 + 1\Delta x_4 + 0(1) = 0$$
$$+1\Delta x_1 + 0(0) + 0\Delta x_3 + 0\Delta x_4 + 1(1) = 0$$

By solving above equation system we have $\Delta x = (-1,0,-2,1,1)$

Improving simplex directions

$\bar{c}_2 = (3,1,0,0,0) \bullet (0,1,-1,-1,0) = 1 > 0$, $\Delta x = (0,1,-1,-1,0)$ is a improving simplex direction

$\bar{c}_5 = (3,1,0,0,0) \bullet (-1,0,-2,1,1) = -3 < 0$, $\Delta x = (-1,0,-2,1,1)$ is **not** a improving simplex direction

Entering variable : $x_2$.

Leaving variable : min{10/1, 2/1} = 2. minimized for $x_4$. $x_4$ leaves the basis.

New basic feasible solution :

$x^{(2)} \leftarrow x^{(1)} + l\Delta x$ = (4, 0, 10, 2, 0) + 2(0, 1, -1, -1, 0) = (4, 2, 8, 0, 0)

and $x_1$, $x_2$ and $x_3$ are basic.

*Third iteration*

Simplex directions ( $A\Delta x = 0$ )

Simplex direction increasing $x_4$ when $\Delta x_4$ is fixed =1.:

$$-2\Delta x_1 + 1\Delta x_2 + 1\Delta x_3 + 0(1) + 0(0) = 0$$
$$+1\Delta x_1 + 1\Delta x_2 + 0\Delta x_3 + 1(1) + 0(0) = 0$$
$$+1\Delta x_1 + 0\Delta x_2 + 0\Delta x_3 + 0(1) + 1(0) = 0$$

By solving above equation system we have $\Delta x = (0,-1,1,1,0)$

Simplex direction increasing $x_5$ when $\Delta x_5$ is fixed =1.:

$$-2\Delta x_1 + 1\Delta x_2 + 1\Delta x_3 + 0(0) + 0(1) = 0$$
$$+1\Delta x_1 + 1\Delta x_2 + 0\Delta x_3 + 1(0) + 0(1) = 0$$
$$+1\Delta x_1 + 0\Delta x_2 + 0\Delta x_3 + 0(0) + 1(1) = 0$$

By solving above equation system we have $\Delta x = (-1,1,-3,0,1)$

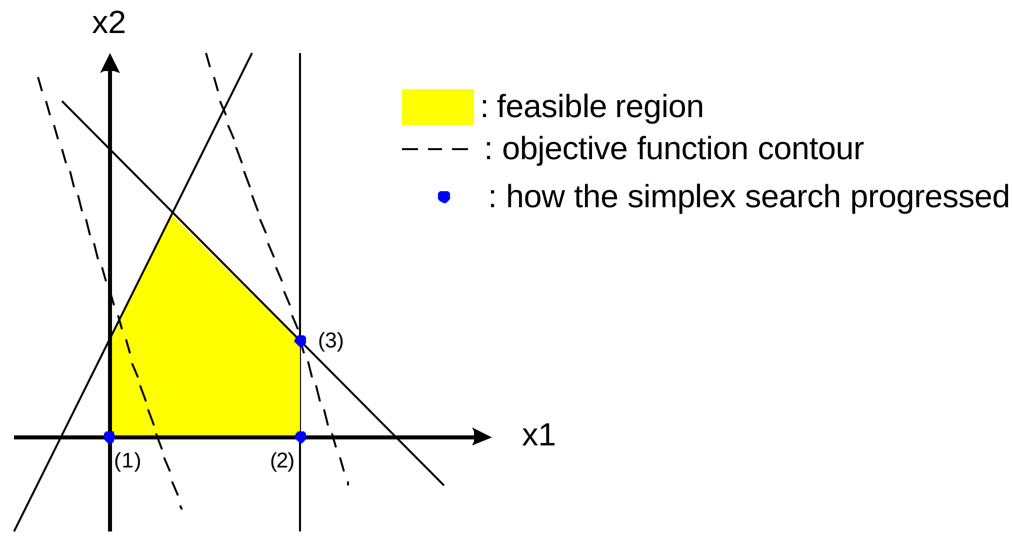
Improving simplex directions

$\bar{c}_2 = (3,1,0,0,0) \bullet (0,-1,1,1,0) = -1 < 0$, $\Delta x = (0,-1,1,1,0)$ is **not** a improving simplex direction

$\bar{c}_5 = (3,1,0,0,0) \bullet (-1,1,-3,0,1) = -2 < 0$, $\Delta x = (-1,1,-3,0,1)$ is **not** a improving simplex direction

There are no improving simplex directions. So the current solution is optimal.

(c)




Problem 3.

(a) Yes
(b) No. (E is not a extreme point.)
(c) No. (H is not a extreme point.)
(d) No. (Objective function values of sequence of solutions followed by simplex algorithm is non-decreasing in maximization problem and non-increasing in minimization problem. However, objective function values of this sequence is not monotone.)
(e) No. (From A, F can not be visited without visiting G in simplex algorithm)