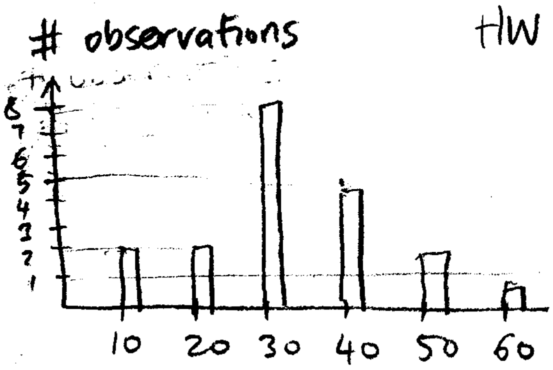
HW # 8 Solutions



a) $\bar{X} = 33$

$s = 12.63$

$s^2 = 159.55$

b) $c_u = 2000 - 1250 = 750$

$c_o = 1250 - 300 = 950$

$$\text{Critical ratio} = \frac{750}{750 + 950} = 0.441$$

$$\Phi(z) = 0.441 \rightarrow \Phi^{-1}(0.441) = -0.15 = z$$

$$Q^* = \sigma z + \mu = 12.63(-0.15) + 33 = 31.105 \cong 31.$$

c)

Value	# Times Obs	Relative freq	Cum freq.
10	2	0.1	0.1
20	2	0.1	0.2
30	8	0.4	0.6
40	5	0.25	0.85
50	2	0.1	0.95
60	1	0.05	1.00

critical ratio 0.441

The optimal value is 30. (the smallest number cumulative probability exceeds critical ratio)

d) Expected total profit with Normal approximation

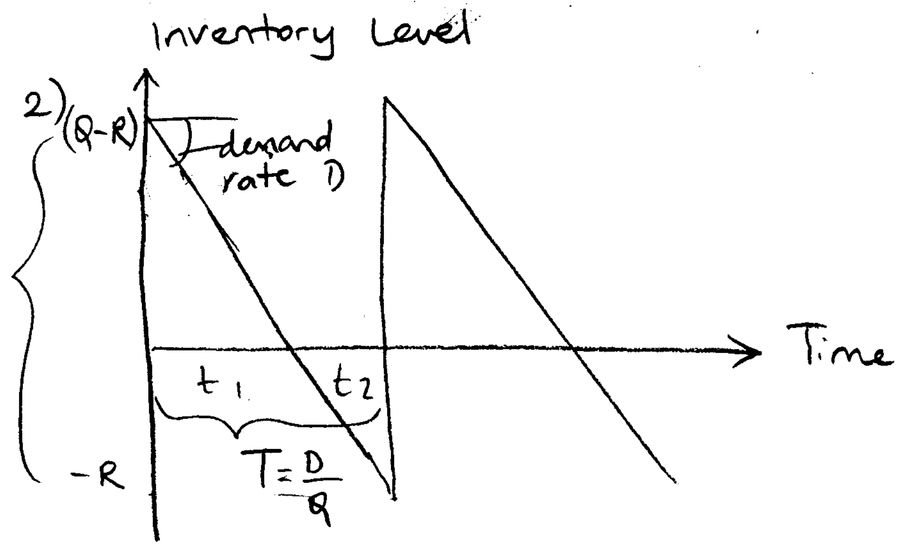
$\bar{X} = 33 \rightarrow$ 2 units under with 31. (\$66000 − \$1500)

Expected total profit with discrete distribution

$\bar{X} = 33 \rightarrow$ 3 units under with 30, (\$66000 − \$2250)

Comparing expected profits, Normal approximation works well.

2)



R: maximum shortage level

b: annual per unit cost of shortage

b': unit shortage cost

a) Total Cost: Ordering cost + Holding Cost + Backorder Costs + Purchasing

$$\text{Average inventory} = \frac{I_{max}}{2} \cdot \frac{t_1}{T} = \frac{Q-R}{2} \cdot \frac{Q-R}{Q} = \frac{(Q-R)^2}{2Q}$$

$$\text{Average backorders} = \frac{S_{max}}{2} \cdot \frac{t_2}{T} = \frac{R}{2} \cdot \frac{R}{Q} = \frac{R^2}{2Q}$$

$$\text{\# units backordered/year} = S_{max} \cdot \text{\# of cycles} = R\,\frac{D}{Q}$$

$$\text{Ordering Cost} = \frac{D}{Q}K$$

$$\text{Holding Cost} = h \cdot \frac{(Q-R)^2}{2Q} \qquad \text{Purchasing cost} = p \cdot D$$

$$\text{Backorder Cost}(b) = b\,\frac{R^2}{2Q}$$

$$\text{Backorder Cost}(b') = b'\,\frac{D}{Q}$$

$$TC(Q,R) = \frac{D}{Q}K + h\,\frac{(Q-R)^2}{2Q} + b\,\frac{R^2}{2Q} + b'\,\frac{RD}{Q} + pD$$

b) To find Q^* and R^* we simultaneously solve

$$\frac{\partial TC}{\partial Q} = 0 \quad \text{and} \quad \frac{\partial TC}{\partial R} = 0$$

$$\frac{\partial TC}{\partial Q} = -\frac{D}{Q^2}K + h\left(\frac{1}{2} - \frac{R^2}{2Q^2}\right) + \frac{bR^2}{2Q^2} - b'\frac{RD}{Q^2}$$

$$\frac{\partial TC}{\partial R} = \frac{h}{Q}(R-Q) + \frac{bR}{Q} + b'\frac{D}{Q}$$

$$\frac{\partial TR}{\partial R} = 0 \Rightarrow \frac{hR}{Q} - h + \frac{bR}{Q} + b'\frac{D}{Q} = 0$$

$$(h+b)R - hQ + b'D = 0$$

$$R^* = \frac{hQ^* - b'D}{(h+b)}$$

$$\frac{\partial TC}{\partial Q} = 0 \Rightarrow -2DK + hQ^2 - hR^2 - bR^2 - 2b'RD = 0$$

$$hQ^2 = 2DK + (h+b)R^2 + 2b'RD$$

(Substitute R^*) $$hQ^2 = 2DK + (h+b)\frac{(hQ - b'D)^2}{(h+b)^2} + 2b'D\,\frac{(hQ - b'D)}{(h+b)}$$

$$hQ^2 = 2DK + \frac{h^2Q^2 + b'^2D^2 - 2hQb'D}{h+b} + \frac{2hQbD - 2(b'D)^2}{h+b}$$

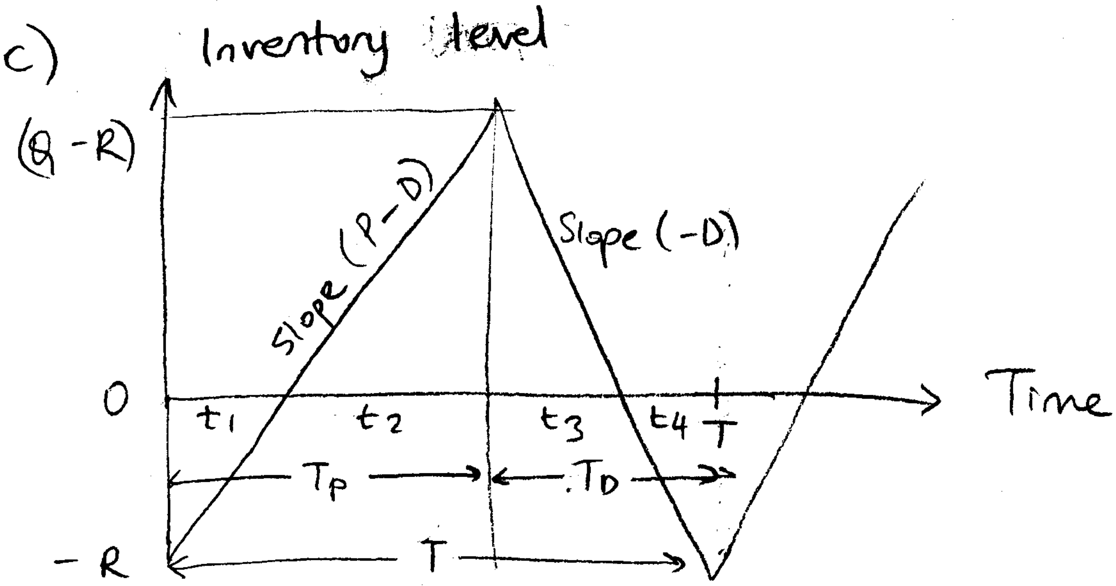
$$hQ^2 = 2DK + \frac{h^2Q^2 - b'^2D^2}{h+b}$$

$$(h^2 + hb - h^2)Q^2 = 2DK(h+b) - (b'D)^2$$

$$Q^2 = \frac{2DK}{h}\left(\frac{h+b}{b}\right) - \frac{(b'D)^2}{h(h+b)}\left(\frac{h+b}{b}\right)$$

$$Q^* = \sqrt{\frac{2DK}{h} - \frac{(b'D)^2}{h(h+b)}}\sqrt{\frac{h+b}{b}}$$

c)



$T = \frac{Q}{D}$ Cycle Time

$T_P = \frac{Q}{P}$ Time to produce Q units

$T_D = \frac{Q}{D}$ Time to deplete maximum inventory

$$Q = T_P(P - D) = \frac{Q}{P}(P - D) = Q\left(1 - \frac{D}{P}\right)$$

$I_{max} = Q - R = Q\left(1 - \frac{D}{P}\right) - R$ (Maximum Inventory)

$t_1 = \frac{R}{P - D}$ (time to recover from backlog)

$t_2 = \frac{Q - R}{P - D}$ (time to generate Q - R)

$t_3 = \frac{Q - R}{D}$ (time to deplete Q - R)

$t_4 = \frac{R}{D}$ (time generate backlog of R)

To obtain TC, we need annual average inventory ($\bar{I}$) and backorder ($\bar{B}$)

$$\bar{I} = \frac{1}{2T} I_{max}(t_2 + t_3) = \frac{\left[Q\left(1 - \frac{D}{P}\right) - R\right]^2}{2Q(1 - D/P)}$$

$$\bar{B} = \frac{1}{2T} R(T_1 + T_4) = \frac{R^2}{2Q(1 - \frac{D}{\psi})}$$

$$\text{Average annual holding cost} = h\bar{I} = \frac{h[Q(1-\frac{D}{p}) - R]^2}{2Q(1 - D/p)}$$

$$\text{Total shortage cost per cycle} = b'R + bT\bar{B}$$

$$\text{Annual average shortage cost} = \frac{1}{T}[b'R + bT\bar{B}]$$

$$= \frac{b'RD}{Q} + \frac{bR^2}{2Q(1-D/p)}$$

Purchasing cost   ordering cost

$$TC(Q,b) = pD + \frac{KD}{Q} + \frac{h[Q(1-D/p) - R]^2}{2Q(1-D/p)} + \frac{b'RD}{Q} + \frac{bR^2}{2Q(1-D/p)}$$