HW #7 Solution

① 16 hours/day : 250 days/year = 4000 hours/year
$k = \$80/\text{hour}$ $i = 10\%$ production capacity

(a) Material Cost	(b) Annual Demand	(c) Production Time	(d) Setup Time	(e) = 4000/(c) Production Rate
\$625.00	2500	0.30	8	13333.33
\$382.50	5000	0.15	4.5	26666.66
\$275.00	7500	0.10	2	40000
\$250.00	12000	0.10	2	40000

a) First verify that problem is feasible

$$\Sigma(\lambda_j/p_j) = \frac{2500}{13333.33} + \frac{5000}{26666.67} + \frac{7500}{40000} + \frac{12000}{40000}$$
$$= 0.1875 + 0.1875 + 0.1875 + 0.3$$
$$= 0.8625 < 1 \text{ so feasible}$$

Setup Costs (k_j)	Modified holding costs (h'_j)
$8 \times 80 = 640$	$625 \times 0.1 \times (1-0.1875) = 50.78$
$4.5 \times 80 = 360$	$382.5 \times 0.1 \times (1-0.1875) = 31.08$
$2 \times 80 = 160$	$275 \times 0.1 \times (1-0.1875) = 22.34$
$2 \times 80 = 160$	$275 \times 0.1 \times (1-0.3) = 17.50$
(setup time x rate)	$h' = i \cdot c\,(1-\lambda_j/p_j)$

$$T^* = \sqrt{\frac{2\,\Sigma k_j}{\Sigma h'_j \lambda_j}} = \sqrt{\frac{2(640+360+160+160)}{2500(50.78)+5000(31.08)+7500(22.34)+12000(17.5)}}$$
$$= \sqrt{\frac{2640}{659900}} = 0.06325$$

$Q_j = \lambda_j T^*$

$2500 \times 0.06325 = 158$
$5000 \times 0.06325 = 316$
$7500 \times 0.06325 = 474$
$12000 \times 0.06325 = 759$

The plant would use these lot sizes and repeat the sequence every 16 days.

This solution can be implemented only if T^* is at least T_{min}.

$$T \geq \frac{\sum s_j}{1 - \sum_{j=1}^{n} (\lambda_j / P_j)} = T_{min} \qquad \frac{8+4.5+2+2}{1-0.8625} = 120 \text{ hours}$$

$$= \frac{120}{250 \times 16} = 0.03 \text{ years}$$

Since $0.06325 \geq 0.03$ so solution can be implemented

b) $T_j = Q_j / P_j$

$158/13333.33 = 0.01185$

$316/26666.67 = 0.01185$

$474/40000 = 0.01185$

$759/40000 = 0.018975$

Total production time $= 0.054525$

Total idle time $= 0.06325 - 0.054525$

$= 0.008725 \cong 13.8\%$

c) Annual holding and setup costs;

$$G(T) = \sum_{j=1}^{n} \left(K_j / T + h'_j \lambda_j T/2 \right)$$

$$= \frac{640 + 360 + 160 + 160}{0.06325} + 659900 \times \frac{0.06325}{2}$$

$= \$41738.9$

② Storage space = 12000 sq. feet (W)

$i = 10\%$

item	Annual Demand	Cost/order	Unit price	(w_i) Square feet/unit
1	12500	150	24.00	5.0
2	15000	80	34.50	4.0
3	15000	80	12.80	4.0

a) EOQ?

First calculate the EOQ's and see if the space limit is violated.

$$EOQ_i = \sqrt{\frac{2 \cdot K_i \cdot \lambda_i}{2 h_i}}$$

$h_1 = 2.4$

$h_2 = 3.45$

$h_3 = 1.28$

$$EOQ_1 = \sqrt{\frac{2 \times 150 \times 12500}{24}} = 1250$$

$$EOQ_2 = \sqrt{\frac{2 \times 80 \times 15000}{3.45}} = 834$$

$$EOQ_3 = \sqrt{\frac{2 \times 80 \times 15000}{1.28}} = 1369.$$

Total space $= 1250 \times 5 + 834 \times 4 + 1369 \times 4$

$\sum EOQ_i w_i = 15062 > 12000$

Since $\frac{w_i}{h_i}$ are not proportional, use Lagrangian method

$$Q^*_i = \sqrt{\frac{2 K_i \lambda_i}{h_i + 2\theta w_i}}$$ where θ is such that $\sum_{i=1}^{n} w_i Q_i^* = W$

Determine upper and lower bounds on θ, assuming equal ratios.

$$m = W / \sum(EOQ_i w_i) = \frac{12000}{15062} = 0.7967$$

The resulting lot sizes are 996, 665 and 1091

The three values of θ that result in these lot sizes are

$$\theta_1 : \sqrt{\frac{2\times 150\times 12500}{2.4+10\theta}} = 996 \Rightarrow \theta_1 = 0.138$$

$$\theta_2 : \sqrt{\frac{2\times 80\times 15000}{3.45+8\theta}} = 665 \Rightarrow \theta_2 = 0.247$$

$$\theta_3 : \sqrt{\frac{2\times 80\times 15000}{1.28+8\theta}} = 1091 \Rightarrow \theta_3 = 0.092$$

The true value of θ will be between 0.092 and 0.247

Start with $\theta = 0.17 \left(= \frac{0.092+0.247}{2}\right)$

$Q_1 = 956$, $Q_2 = 706$, $Q_3 = 953$

$\sum w_i q_i = 956\times 5 + (706+953)4 = 11416 < 12000$

So $\theta < 0.17$, try $\theta = 0.13 \left(\frac{0.092+0.17}{2}\right)$

$Q_1 = 1006$ $Q_2 = 731$ $Q_3 = 1017$

$\sum w_i q_i = 5\times 1006 + 4(731+1017) = 12022 > 12000$

so $\theta > 0.13$ / try 0.132

$Q_1 = 1004$ $Q_2 = 730$, $Q_3 = 1014$

$\sum w_i Q_i = 1004\times 5 + (730+1014)\times 4 = 11996$

So, $Q_1^* = 1004$, $Q_2^* = 730$, $Q_3^* = 1014$

b) The value of θ found in part (a) can be interpreted as the decrease in the average annual cost that would result from adding an additional unit of resource. (See book pg. 224, paragraph 2) The resource here is the storage space

2c) No storage limit $c_2 = \$12.80$

Budget constraint 50000

Calculate EOQ, see if violates budget constraint

From part (a) $EOQ_1 = 1250$

$EOQ_2 = EOQ_3 = 1369$

$$\sum c_i EOQ_i = 1250 \times 24 + 2 \times 1369 \times 12.80$$
$$= 65046.4 > 50000$$

Since $\frac{c_i}{h_i}$ are proportional ($i = 10\%$), we can use simple scaling

$$Q_1^* = 1250 \times \frac{50000}{65046} = 961$$

$$Q_2^* = Q_3^* = 1369 \times \frac{50000}{65046} = 1052$$