

a

Month	Unit/Worker	Net Demand	Cum. Net Demand	Minimum Work Force
December	50	2500*	2500	50
January	50	5000	7500	75
February	50	4000	11500	76.66666667
March	50	2000	13500	67.5
April	50	1250**	14750	59

* 3000 - 500 (on hand)

** 1000 + 250 (desired)

Hence the minimum constant workforce is 77 workers

The cost of the resulting plan is:

Month	Starting Workforce	Hired	Fired	Ending Workforce	Starting Inventory	Produced	Demand	Ending Inventory
December	60	17	0	77	500	3850	3000	1350
January	77	0	0	77	1350	3850	5000	200
February	77	0	0	77	200	3850	4000	50
March	77	0	0	77	50	3850	2000	1900
April	77	0	0	77	1900	3850	1000	4750
TOTAL		17		385				8250
unit cost		\$1,600		\$1,500				\$3
		\$27,200		\$577,500				\$24,750

\$629,450

b

Month	Starting Workforce	Hired	Fired	Ending Workforce	Starting Inventory	Produced	Demand	Ending Inventory
December	60		10	50	500	2500	3000	0
January	50	50		100	0	5000	5000	0
February	100		20	80	0	4000	4000	0
March	80		40	40	0	2000	2000	0
April	40		15	25	0	1250	1000	250
TOTAL		50	85	295				250
unit cost		\$1,600	2000	\$1,500				\$3
		\$80,000	170000	\$442,500				\$750

\$693,250

c

Month	Starting Workforce	Hired	Fired	Ending Workforce	Starting Inventory	Produced	Demand	Ending Inventory	Ending Backlog
December	60		1	59	500	2950	3000	450	0
January	59			59	450	2950	5000	0	1600
February	59			59	-1600	2950	4000	0	2650
March	59			59	-2650	2950	2000	0	1700
April	59			59	-1700	2950	1000	250	0
TOTAL		0	1	295				700	5950
unit cost		\$1,600	2000	\$1,500				\$3	11
		\$0	2000	\$442,500				\$2,100	\$65,450

\$512,050

d

Decision Variables:

H_i : number of workers hired at the beginning of month i

F_i : number of workers fired at the beginning of month i

I_i : inventory at the end of month i

W_i : number of workers at the end of month i

O_i : overtime production in month i

P_i : total production (regular + overtime) in month i

Parameters:

D_i : number of units demanded in month i . (Given in the problem)

Formulation: (Part d)

Minimize
$$\sum_{i=1}^5 (1600 H_i + 2000 F_i + 3 I_i + 1500 W_i + 41.6 O_i)$$

Subject To:

$$W_{i-1} + H_i - F_i - W_i = 0 \quad \text{for } i: 1..5$$

$$50 W_i + O_i - P_i = 0 \quad \text{for } i: 1..5$$

$$I_{i-1} + P_i - I_i = D_i \quad \text{for } i:1..5$$

$$O_i - 5 W_i \leq 0 \quad \text{for } i:1..5$$

$$I_0 = 500$$

$$I_5 \geq 250$$

$$W_0 = 60$$

$$W_i \geq 0, P_i \geq 0, H_i \geq 0, F_i \geq 0 \quad \text{for } i:1..5$$

Minimize

$$\begin{aligned} \text{obj: } & 1600 H_1 + 1600 H_2 + 1600 H_3 + 1600 H_4 + 1600 H_5 \\ & + 2000 F_1 + 2000 F_2 + 2000 F_3 + 2000 F_4 + 2000 F_5 \\ & + 3 I_1 + 3 I_2 + 3 I_3 + 3 I_4 + 3 I_5 \\ & + 1500 W_1 + 1500 W_2 + 1500 W_3 + 1500 W_4 + 1500 W_5 \\ & + 41.6 O_1 + 41.6 O_2 + 41.6 O_3 + 41.6 O_4 + 41.6 O_5 \end{aligned}$$

Subject To:

$$c1: W_0 + H_1 - F_1 - W_1 = 0$$

$$c2: W_1 + H_2 - F_2 - W_2 = 0$$

$$c3: W_2 + H_3 - F_3 - W_3 = 0$$

$$c4: W_3 + H_4 - F_4 - W_4 = 0$$

$$c5: W_4 + H_5 - F_5 - W_5 = 0$$

$$c6: 50 W_1 + O_1 - P_1 = 0$$

$$c7: 50 W_2 + O_2 - P_2 = 0$$

$$c8: 50 W_3 + O_3 - P_3 = 0$$

$$c9: 50 W_4 + O_4 - P_4 = 0$$

$$c10: 50 W_5 + O_5 - P_5 = 0$$

$$C11: I_0 + P_1 - I_1 = 3000$$

$$c12: I_1 + P_2 - I_2 = 5000$$

$$c13: I_2 + P_3 - I_3 = 4000$$

$$c14: I_3 + P_4 - I_4 = 2000$$

$$c15: I_4 + P_5 - I_5 = 1000$$

$$c16: I_0 = 500$$

$$c17: I_5 \geq 250$$

$$c18: W_0 = 60$$

$$c19: O_1 - 5 W_1 \leq 0$$

$$c20: O_2 - 5 W_2 \leq 0$$

$$c21: O_3 - 5 W_3 \leq 0$$

$$c22: O_4 - 5 W_4 \leq 0$$

$$c23: O_5 - 5 W_5 \leq 0$$

Bounds

$$0 \leq W_1$$

$$0 \leq W_2$$

$$0 \leq W_3$$

$$0 \leq W_4$$

$$0 \leq W_5$$

$$0 \leq P_1$$

$$0 \leq P_2$$

$$0 \leq P_3$$

$$0 \leq P_4$$

$$0 \leq P_5$$

End

Solver Output:

LP problem filename: /tmp/cpxweb.1754.lp

$$\begin{aligned} \text{min } & +1600 H_1 + 1600 H_2 + 1600 H_3 + 1600 H_4 + 1600 H_5 \\ & + 2000 F_1 + 2000 F_2 + 2000 F_3 + 2000 F_4 + 2000 F_5 \\ & + 3 I_1 + 3 I_2 + 3 I_3 + 3 I_4 + 3 I_5 \\ & + 1500 W_1 + 1500 W_2 + 1500 W_3 + 1500 W_4 + 1500 W_5 \\ & + 41.6 O_1 + 41.6 O_2 + 41.6 O_3 + 41.6 O_4 + 41.6 O_5 \end{aligned}$$

subject to

$$(c1) \quad +1 H_1 -1 F_1 -1 W_1 +1 W_0 = 0$$

$$(c2) \quad +1 H_2 -1 F_2 +1 W_1 -1 W_2 = 0$$

$$(c3) \quad +1 H_3 -1 F_3 +1 W_2 -1 W_3 = 0$$

(c4) +1 H4 -1 F4 +1 W3 -1 W4 = 0
(c5) +1 H5 -1 F5 +1 W4 -1 W5 = 0
(c6) +50 W1 +1 O1 -1 P1 = 0
(c7) +50 W2 +1 O2 -1 P2 = 0
(c8) +50 W3 +1 O3 -1 P3 = 0
(c9) +50 W4 +1 O4 -1 P4 = 0
(c10) +50 W5 +1 O5 -1 P5 = 0
(c11) -1 I1 +1 P1 +1 I0 = 3000
(c12) +1 I1 -1 I2 +1 P2 = 5000
(c13) +1 I2 -1 I3 +1 P3 = 4000
(c14) +1 I3 -1 I4 +1 P4 = 2000
(c15) +1 I4 -1 I5 +1 P5 = 1000
(c16) +1 I0 = 500
(c17) +1 I5 >= 250
(c18) +1 W0 = 60
(c19) -5 W1 +1 O1 <= 0
(c20) -5 W2 +1 O2 <= 0
(c21) -5 W3 +1 O3 <= 0
(c22) -5 W4 +1 O4 <= 0
(c23) -5 W5 +1 O5 <= 0
0 <= H1 <= +infinity
0 <= H2 <= +infinity
0 <= H3 <= +infinity
0 <= H4 <= +infinity
0 <= H5 <= +infinity
0 <= F1 <= +infinity
0 <= F2 <= +infinity
0 <= F3 <= +infinity
0 <= F4 <= +infinity
0 <= F5 <= +infinity
0 <= I1 <= +infinity
0 <= I2 <= +infinity
0 <= I3 <= +infinity
0 <= I4 <= +infinity
0 <= I5 <= +infinity
0 <= W1 <= +infinity
0 <= W2 <= +infinity
0 <= W3 <= +infinity
0 <= W4 <= +infinity
0 <= W5 <= +infinity
0 <= O1 <= +infinity
0 <= O2 <= +infinity
0 <= O3 <= +infinity
0 <= O4 <= +infinity
0 <= O5 <= +infinity
0 <= W0 <= +infinity
0 <= P1 <= +infinity
0 <= P2 <= +infinity
0 <= P3 <= +infinity
0 <= P4 <= +infinity
0 <= P5 <= +infinity
0 <= I0 <= +infinity

No dictionary printed (LP representation is not canonical).

Optimal solution found after 13 iterations.
Objective Value: 559536

Solutions:

	Primal solution	Dual solution
Variable	Value	Reduced costs
H1:	9.69697	0
H2:	0	1035
H3:	0	2235
H4:	0	3600
H5:	0	3250

F1:	0	3600
F2:	0	2565
F3:	0	1365
F4:	29.697	0
F5:	0	350
I1:	1333.33	0
I2:	166.667	0
I3:	0	35.8727
I4:	0	29
I5:	1000	0
W1:	69.697	0
W2:	69.697	0
W3:	69.697	0
W4:	40	0
W5:	40	0
O1:	348.485	0
O2:	348.485	0
O3:	348.485	0
O4:	0	18.6
W0:	60	0
P1:	3833.33	0
P2:	3833.33	0
P3:	3833.33	0
P4:	2000	0
O5:	0	3
P5:	2000	0
I0:	500	0

Row	Slack	Dual prices
c1:	0	1600
c2:	0	565
c3:	0	-635
c4:	0	-2000
c5:	0	-1650
c6:	0	49.8727
c7:	0	52.8727
c8:	0	55.8727
c9:	0	23
c10:	0	-3
C11:	0	49.8727
c12:	0	52.8727
c13:	0	55.8727
c14:	0	23
c15:	0	-3
c16:	0	-49.8727
c17:	-750	-0
c18:	0	-1600
c19:	0	-8.27273
c20:	0	-11.2727
c21:	0	-14.2727
c22:	200	0
c23:	200	0

Sensitivity:

Objective Function

Variable	Lower Bound	Current	Upper Bound
H1:	235	1600	3152.5
H2:	565	1600	+infinity
H3:	-635	1600	+infinity
H4:	-2000	1600	+infinity
H5:	-1650	1600	+infinity
F1:	-1600	2000	+infinity
F2:	-565	2000	+infinity
F3:	635	2000	+infinity
F4:	1070	2000	3450
F5:	1650	2000	+infinity
I1:	-30.8182	3	15.4091

I2:	-18.4091	3	27.8182
I3:	-32.8727	3	+infinity
I4:	-26	3	+infinity
I5:	0	3	10
W1:	135	1500	3052.5
W2:	135	1500	8205
W3:	135	1500	3547.5
W4:	50	1500	2430
W5:	50	1500	1850
O1:	-330.4	41.6	50.1313
O2:	-231.4	41.6	53.225
O3:	-231.4	41.6	56.3188
O4:	23	41.6	+infinity
W0:	-infinity	0	+infinity
P1:	-33.8182	0	12.4091
P2:	-24.8182	0	16.9091
P3:	-24.8182	0	21.4091
P4:	-29	0	35.8727
O5:	-3	0	+infinity
P5:	-3	0	7
I0:	-infinity	0	+infinity

Right Hand Side

Row	Lower Bound	Current	Upper Bound
c1:	-9.69697	0	+infinity
c2:	-9.09091	0	89.0909
c3:	-4.54545	0	44.5455
c4:	-infinity	0	29.697
c5:	-infinity	0	15
c6:	-1600	0	500
c7:	-1600	0	500
c8:	-250	0	5750
c9:	-750	0	1484.85
c10:	-infinity	0	750
C11:	1400	3000	3500
c12:	3400	5000	5500
c13:	3750	4000	+infinity
c14:	1250	2000	3484.85
c15:	-infinity	1000	1750
c16:	0	500	2100
c17:	-infinity	250	1000
c18:	0	60	69.697
c19:	-359.375	0	1600
c20:	-359.375	0	1600
c21:	-359.375	0	250
c22:	-200	0	+infinity
c23:	-200	0	+infinity

Month	Starting Workforce	Hired	Fired	Ending Workforce	Starting Inventory	Produced	Overtime Produced	Demand	Ending Inventory
December	60	10	0	70	500	3500	300	3000	1300
January	70	0	0	70	1300	3500	350	5000	150
February	70	0	0	70	150	3500	350	4000	0
March	70	0	30	40	0	2000	0	2000	0
April	40	0	0	40	0	2000	0	1000	1000
TOTAL		10	30	290			1000		2450
unit cost		\$1,600	2000	\$1,500			41.6		3
		\$16,000	60000	\$435,000			41600		\$7,350

\$559,950

e Additional Decision Variable for part e:

U_i : number of units backordered at the end of month i

Formulation: (Changes from part d are in bold font)

Minimize
$$\sum_{i=1}^5 (1600 H_i + 2000 F_i + 3 I_i + \mathbf{6} U_i + 1500 W_i + 41.6 O_i)$$

Subject To:

$$W_{i-1} + H_i - F_i - W_i = 0 \quad \text{for } i: 1..5$$

$$50 W_i + O_i - P_i = 0 \quad \text{for } i: 1..5$$

$$I_0 + P_1 - I_1 + U_1 = D_1$$

$$I_{i-1} - U_{i-1} + P_i - I_i + U_i = D_i \quad \text{for } i: 2..5$$

$$O_i - 5 W_i \leq 0 \quad \text{for } i: 1..5$$

$$I_0 = 500$$

$$I_5 - U_5 \geq 250$$

$$W_0 = 60$$

$$W_i \geq 0, P_i \geq 0, H_i \geq 0, F_i \geq 0, U_i \geq 0 \quad \text{for } i: 1..5$$

Minimize

$$\begin{aligned} \text{obj: } & 1600 H_1 + 1600 H_2 + 1600 H_3 + 1600 H_4 + 1600 H_5 \\ & + 2000 F_1 + 2000 F_2 + 2000 F_3 + 2000 F_4 + 2000 F_5 \\ & + 3 I_1 + 3 I_2 + 3 I_3 + 3 I_4 + 3 I_5 \\ & + 6 U_1 + 6 U_2 + 6 U_3 + 6 U_4 + 6 U_5 \\ & + 1500 W_1 + 1500 W_2 + 1500 W_3 + 1500 W_4 + 1500 W_5 \\ & + 41.6 O_1 + 41.6 O_2 + 41.6 O_3 + 41.6 O_4 + 41.6 O_5 \end{aligned}$$

Subject To:

$$c1: W_0 + H_1 - F_1 - W_1 = 0$$

$$c2: W_1 + H_2 - F_2 - W_2 = 0$$

$$c3: W_2 + H_3 - F_3 - W_3 = 0$$

$$c4: W_3 + H_4 - F_4 - W_4 = 0$$

$$c5: W_4 + H_5 - F_5 - W_5 = 0$$

$$c6: 50 W_1 + O_1 - P_1 = 0$$

$$c7: 50 W_2 + O_2 - P_2 = 0$$

$$c8: 50 W_3 + O_3 - P_3 = 0$$

$$c9: 50 W_4 + O_4 - P_4 = 0$$

$$c10: 50 W_5 + O_5 - P_5 = 0$$

$$C11: I_0 + P_1 - I_1 + U_1 = 3000$$

$$c12: I_1 - U_1 + P_2 - I_2 + U_2 = 5000$$

$$c13: I_2 - U_2 + P_3 - I_3 + U_3 = 4000$$

$$c14: I_3 - U_3 + P_4 - I_4 + U_4 = 2000$$

$$c15: I_4 - U_4 + P_5 - I_5 + U_5 = 1000$$

$$c16: I_0 = 500$$

$$c17: I_5 - U_5 \geq 250$$

$$c18: W_0 = 60$$

$$c19: O_1 - 5 W_1 \leq 0$$

$$c20: O_2 - 5 W_2 \leq 0$$

$$c21: O_3 - 5 W_3 \leq 0$$

$$c22: O_4 - 5 W_4 \leq 0$$

$$c23: O_5 - 5 W_5 \leq 0$$

Bounds

$$0 \leq W_1$$

$$0 \leq W_2$$

$$0 \leq W_3$$

$$0 \leq W_4$$

$$0 \leq W_5$$

$$0 \leq U_1$$

$$0 \leq U_2$$

$$0 \leq U_3$$

$$0 \leq U_4$$

$$0 \leq U_5$$

$$0 \leq P_1$$

$$0 \leq P_2$$

$$0 \leq P_3$$

$$0 \leq P_4$$

$$0 \leq P_5$$

End

Solver Output:

LP problem filename: /tmp/cpxweb.8582.lp

min +1600 H1 +1600 H2 +1600 H3 +1600 H4 +1600 H5 +2000 F1 +2000 F2 +2000 F3 +2000 F4 +2000 F5 +3 I1 +3 I2
+3 I3 +3 I4 +3 I5 +6 U1 +6 U2 +6 U3 +6 U4 +6 U5 +1500 W1 +1500 W2 +1500 W3 +1500 W4 +1500 W5
+41.6 O1 +41.6 O2 +41.6 O3 +41.6 O4 +41.6 O5

subject to

(c1) +1 H1 -1 F1 -1 W1 +1 W0 = 0
(c2) +1 H2 -1 F2 +1 W1 -1 W2 = 0
(c3) +1 H3 -1 F3 +1 W2 -1 W3 = 0
(c4) +1 H4 -1 F4 +1 W3 -1 W4 = 0
(c5) +1 H5 -1 F5 +1 W4 -1 W5 = 0
(c6) +50 W1 +1 O1 -1 P1 = 0
(c7) +50 W2 +1 O2 -1 P2 = 0
(c8) +50 W3 +1 O3 -1 P3 = 0
(c9) +50 W4 +1 O4 -1 P4 = 0
(c10) +50 W5 +1 O5 -1 P5 = 0
(C11) -1 I1 +1 U1 +1 P1 +1 I0 = 3000
(c12) +1 I1 -1 I2 -1 U1 +1 U2 +1 P2 = 5000
(c13) +1 I2 -1 I3 -1 U2 +1 U3 +1 P3 = 4000
(c14) +1 I3 -1 I4 -1 U3 +1 U4 +1 P4 = 2000
(c15) +1 I4 -1 I5 -1 U4 +1 U5 +1 P5 = 1000
(c16) +1 I0 = 500
(c17) +1 I5 -1 U5 >= 250
(c18) +1 W0 = 60
(c19) -5 W1 + O1 <= 0
(c20) -5 W2 + O2 <= 0
(c21) -5 W3 + O3 <= 0
(c22) -5 W4 + O4 <= 0
(c23) -5 W5 + O5 <= 0
0 <= H1 <= +infinity
0 <= H2 <= +infinity
0 <= H3 <= +infinity
0 <= H4 <= +infinity
0 <= H5 <= +infinity
0 <= F1 <= +infinity
0 <= F2 <= +infinity
0 <= F3 <= +infinity
0 <= F4 <= +infinity
0 <= F5 <= +infinity
0 <= I1 <= +infinity
0 <= I2 <= +infinity
0 <= I3 <= +infinity
0 <= I4 <= +infinity
0 <= I5 <= +infinity
0 <= U1 <= +infinity
0 <= U2 <= +infinity
0 <= U3 <= +infinity
0 <= U4 <= +infinity
0 <= U5 <= +infinity
0 <= W1 <= +infinity
0 <= W2 <= +infinity
0 <= W3 <= +infinity
0 <= W4 <= +infinity
0 <= W5 <= +infinity
0 <= O1 <= +infinity
0 <= O2 <= +infinity
0 <= O3 <= +infinity
0 <= O4 <= +infinity
0 <= W0 <= +infinity
0 <= P1 <= +infinity
0 <= P2 <= +infinity
0 <= P3 <= +infinity
0 <= P4 <= +infinity
0 <= P5 <= +infinity
0 <= I0 <= +infinity

No dictionary printed (LP representation is not canonical).

Optimal solution found after 20 iterations.
Objective Value: 482161

Solutions:

	Primal solution	Dual solution
Variable	Value	Reduced costs
H1:	0	3600
H2:	0	3439.53
H3:	0	3429.06
H4:	0	3118.59
H5:	0	2508.11
F1:	1.04716	0
F2:	0	160.472
F3:	0	170.943
F4:	0	481.415
F5:	0	1091.89
I1:	447.642	0
I2:	0	9
I3:	0	9
I4:	0	9
I5:	250	0
U1:	0	9
U2:	1604.72	0
U3:	2657.07	0
U4:	1709.43	0
U5:	0	9
W1:	58.9528	0
W2:	58.9528	0
W3:	58.9528	0
W4:	58.9528	0
W5:	58.9528	0
O1:	0	14.8094
O2:	0	11.8094
O3:	0	17.8094
O4:	0	23.8094
W0:	60	0
P1:	2947.64	0
P2:	2947.64	0
P3:	2947.64	0
P4:	2947.64	0
O5:	11.7906	0
P5:	2959.43	0
I0:	500	0
Row	Slack	Dual prices
c1:	0	-2000
c2:	0	-1839.53
c3:	0	-1829.06
c4:	0	-1518.59
c5:	0	-908.114
c6:	0	26.7906
c7:	0	29.7906
c8:	0	23.7906
c9:	0	17.7906
c10:	0	11.7906
C11:	0	26.7906
c12:	0	29.7906
c13:	0	23.7906
c14:	0	17.7906
c15:	0	11.7906
c16:	0	-26.7906
c17:	0	14.7906
c18:	0	2000

c19:	58.9528	0
c20:	58.9528	0
c21:	58.9528	0
c22:	58.9528	0
c23:	0	-2.35811

Sensitivity:

Objective Function

Variable	Lower Bound	Current	Upper Bound
H1:	-2000	1600	+infinity
H2:	-1839.53	1600	+infinity
H3:	-1829.06	1600	+infinity
H4:	-1518.59	1600	+infinity
H5:	-908.114	1600	+infinity
F1:	-954.72	2000	2200.55
F2:	1839.53	2000	+infinity
F3:	1829.06	2000	+infinity
F4:	1518.59	2000	+infinity
F5:	908.114	2000	+infinity
I1:	-1.01099	3	62.0944
I2:	-6	3	+infinity
I3:	-6	3	+infinity
I4:	-6	3	+infinity
I5:	-6	3	+infinity
U1:	-3	6	+infinity
U2:	-3	6	8.84754
U3:	-3	6	10.2685
U4:	-3	6	14.5199
U5:	-3	6	+infinity
W1:	1299.45	1500	4454.72
W2:	1215.25	1500	2303
W3:	297.904	1500	1927.7
W4:	-1450	1500	1927.7
W5:	-1450	1500	1927.7
O1:	26.7906	41.6	+infinity
O2:	29.7906	41.6	+infinity
O3:	23.7906	41.6	+infinity
O4:	17.7906	41.6	+infinity
W0:	-infinity	0	+infinity
P1:	-4.01099	0	59.0944
P2:	-5.69507	0	16.06
P3:	-22.2573	0	8.554
P4:	-29.7558	0	8.554
O5:	-15688.2	0	11.8
P5:	-62.503	0	8.51992
I0:	-infinity	0	+infinity

Right Hand Side

Row	Lower Bound	Current	Upper Bound
c1:	-infinity	0	1.04716
c2:	-11.1888	0	1.30869
c3:	-14.9134	0	1.74434
c4:	-22.3553	0	2.61477
c5:	-44.6215	0	5.21912
c6:	-2673.1	0	262
c7:	-2240	0	262
c8:	-2240	0	262
c9:	-2240	0	262
c10:	-2240	0	262
C11:	326.897	3000	3262
c12:	2760	5000	5262
c13:	1760	4000	4262
c14:	-240	2000	2262
c15:	-1240	1000	1262
c16:	238	500	3173.1
c17:	0	250	512

