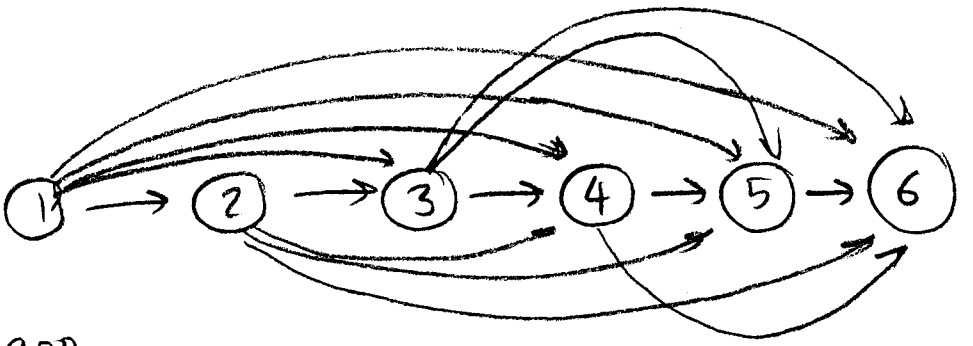
Week	1	2	3	4	5
req.	335	200	140	440	300

$K = \$200$ $\quad h = \$0.30/\text{unit/week}$

a)



$c_{12} = 200$

$c_{13} = 200 + 0.3 \times 200 = 260$

$c_{14} = 260 + 0.3 \times 2 \times 140 = 344$

$c_{15} = 344 + 0.3 \times 3 \times 440 = 740$

$c_{16} = 740 + 0.3 \times 4 \times 300 = 1100$

$c_{23} = 200$

$c_{24} = 200 + 0.3 \times 140 = 242$

$c_{25} = 242 + 0.3 \times 2 \times 440 = 506$

$c_{26} = 506 + 0.3 \times 3 \times 300 = 776$

$c_{34} = 200$

$c_{35} = 200 + 0.3 \times 440 = 332$

$c_{36} = 332 + 0.3 \times 2 \times 300 = 512$

$c_{45} = 200$

$c_{46} = 200 + 0.3 \times 300 = 290$

$c_{56} = 200$

$f_6 = 0$

$f_5 = \min_{j>5} (c_{5j} + f_j) = c_{56} + f_6 = 200 + 0 = 200 \quad @\ j = 5$

$f_4 = \min_{j>4} (c_{4j} + f_j) = \min \begin{Bmatrix} c_{45} + f_5 \\ c_{46} + f_6 \end{Bmatrix} = \min \begin{Bmatrix} 200 + 200 \\ 290 + 0 \end{Bmatrix} = 290$

$f_3 = \min_{j>3} (c_{3j} + f_j) = \min \begin{Bmatrix} c_{34} + f_4 \\ c_{35} + f_5 \\ c_{36} + f_6 \end{Bmatrix} = \min \begin{Bmatrix} 200 + 290 \\ 332 + 200 \\ 512 + 0 \end{Bmatrix} = 490$

$f_2 = \min_{j>2} (c_{2j} + f_j) = \min \begin{Bmatrix} c_{23} + f_3 \\ c_{24} + f_4 \\ c_{25} + f_5 \\ c_{26} + f_6 \end{Bmatrix} = \min \begin{Bmatrix} 200 + 490 \\ 242 + 290 \\ 506 + 200 \\ 776 + 0 \end{Bmatrix} = 532$

$f_1 = \min_{j>1} (c_{1j} + f_j) = \min \begin{Bmatrix} c_{12} + f_2 \\ c_{13} + f_3 \\ c_{14} + f_4 \\ c_{15} + f_5 \\ c_{16} + f_6 \end{Bmatrix} = \min \begin{Bmatrix} 200 + 532 \\ 260 + 490 \\ 344 + 290 \\ 740 + 200 \\ 1100 + 0 \end{Bmatrix} = 634$

$y_1 = r_1 + r_2 + r_3 = 675$

$y_2 = r_4 + r_5 = 730$

b)

week	1	2	3	4	5
req	335	200	140	440	300
cap	600	600	600	400	200

$r_1 = 335$	$c_1 = 600$
$r_1 + r_2 = 535$	$c_1 + c_2 = 1200$
$r_1 + r_2 + r_3 = 675$	$c_1 + c_2 + c_3 = 1800$
$r_1 + r_2 + r_3 + r_4 = 1115$	$c_1 + c_2 + c_3 + c_4 = 2200$
$r_1 + r_2 + r_3 + r_4 + r_5 = 1415$	$c_1 + c_2 + c_3 + c_4 + c_5 = 2400$

Feasible

c) First period demand exceeds capacity is week 4. Replace r_4 with c_4. Difference is added to r_3.

$r = (335 \quad 200 \quad 180 \quad 400 \quad 300)$

$c = (600 \quad 600 \quad 600 \quad 400 \quad 200)$

Next period with $r > c$ is week 5. Shift demand back to week 3, replacing r_5 by c_5 and adding the difference to r_3

$r' = (335 \quad 200 \quad 280 \quad 400 \quad 200)$

$c = (600 \quad 600 \quad 600 \quad 400 \quad 200)$

Setting $y = r'$ gives a feasible solution.

Cost = $5(200) + 100 \times 0.3 \times 2 + 40 \times 0.3 = \$1072$

d)

	1	2	3	4	5
r'	335	200	280	400	200
c	600	600	600	400	200
y	335	200	280	400	200
excess	265	400	320	0	0

We can shift $y_5 = 200$ to y_3, and save $200 - 2 \times 0.3 \times 200 = \$80$.

Shifting $y_4 = 400$ to y_3 and y_2, holding cost is $120 \times 0.3 + 280 \times 0.3 \times 2 = 204$.

Since this exceeds the setup cost, do not shift y_4.

$y' = (335 \quad 200 \quad 480 \quad 400 \quad 0)$

excess $= (265 \quad 400 \quad 120 \quad 0 \quad 200)$

Shifting y_2 to y_1 saves $200 - 200 \times 0.3 = \$140$.

$y' = (535 \quad 0 \quad 480 \quad 400 \quad 0)$

excess $(65 \quad 600 \quad 120 \quad 0 \quad 200)$

(In the previous step instead of shifting y_2 to y_1, shifting y_3 to y_2 and y_1 could have saved $200 - (400 \times 0.3 + 80 \times 0.3 \times 2) = \$32$, since shifting y_2 saves more, we did that instead)

r	335	200	140	440	300
y	535	0	480	400	0
endinv.	200	0	340	300	0

Total Cost $= 3 \times 200 + (200 + 340 + 300) 0.3$

$= 600 + 252 = 852$

Savings $= 1072 - 852 = \$220$

improvement $= \frac{\$220}{\$1072}$

$= \underline{\%20.52}$