HW #10 Solution

1) $K = \$40$ | $\sigma = 6$

$\lambda = 30/\text{year}$ | 95% service objective

$h = 0.12 \times 100 = \$12/\text{year}$ | Demand during lead time $\sim N(2.5, \sqrt{3})$

a) Type I Service

$F(R) = 0.95 \rightarrow z = 1.65$ (from Table A-4)

$R = \sigma z + \mu = 1.65(\sqrt{3}) + 2.5 = 5.36 \cong 6$

$Q = \sqrt{2K\lambda/h} = \sqrt{2(40)30/12} = 14.14 \cong 14$ $\quad (Q^*, R^*) = (14, 6)$

b) Type 2 service level for $(Q = 14, R = 6)$

$$\frac{n(R)}{Q} = 1-\beta \qquad n(R) = \sigma L(z)$$
$$= \sqrt{3}\, L(1.65) = \sqrt{3}\,(.0206)$$
$$= 0.036$$

$$\frac{0.036}{14} = 1-\beta, \quad \beta = 99.74\%$$

$$p = \frac{Qh}{(1-F(R))\lambda} \qquad p = \frac{14(12)}{0.05(30)} = \$112$$ (imputed shortage cost)

c) $\beta = 0.95$, $Q_0 = \sqrt{2K\lambda/h} \cong 14$

$n(R) = (1-\beta)Q \Rightarrow L(z) = Q(1-\beta)/\sigma$

$L(z) = 14(0.05)/\sqrt{3}$

$R_0 = \sigma z + \mu$ $\qquad L(z) = 0.404$

$= \sqrt{3}(-0.01) + 2.5 = 2.48 \cong 3$ $\qquad z = -0.01$ (From Table A-4)

$$Q_1 = \frac{n(R)}{1-F(R)} + \sqrt{\frac{2K\lambda}{h} + \left(\frac{n(R)}{1-F(R)}\right)^2} = \frac{0.05(14)}{0.504} + \sqrt{\frac{2(40)30}{12} + \left(\frac{0.05(14)}{0.504}\right)^2}$$
$$= 1.39 + 14.21 = 15.60 \cong 16$$

$n(R_1) = (1-\beta)Q \Rightarrow L(z) = Q(1-\beta)/\sigma$

$1 - F(R_1) = 0.5478$ $\qquad L(z) = 16(0.05)/\sqrt{3}$ $\qquad Q_2 = \frac{0.05(16)}{0.5478} + \sqrt{\frac{2(40)30}{12} + \left(\frac{(0.05)16}{0.5478}\right)^2}$

$R_1 = \sigma z + \mu$ $\qquad L(z) = 0.462$

$= \sqrt{3}(-0.12) + 2.5$ $\qquad z = -0.12$ $\qquad = 1.46 + 14.22 = 15.68 \cong 16$

$= 2.29 \cong 3$

$R_2 = R_1 \quad Q_2 = Q_1$ STOP $\quad Q^*, R^* = (16, 3)$

d) Type 1 service level of $(Q=16, R=3)$

$$P\{D \leq R\} = F(R) = F\left(\frac{R-\mu}{\sigma}\right) = F\left(\frac{3-2.5}{\sqrt{3}}\right) = F(0.289)$$
$$= 0.6141 = 61.41\%$$

$$p = \frac{Qh}{(1-F(R))\lambda} \qquad p = \frac{16(12)}{(0.386)(30)} = \$16.58 \text{ (imputed shortage cost)}$$

2)

Periods	1	2	3	4
From B	100	100	100	100
From A	150	0	180	90
Spare part	75	75	75	75
On Hand	(10)	(5)	(5)	
Net Req.	315	170	350	265