Inventory Control - Demand

Variability

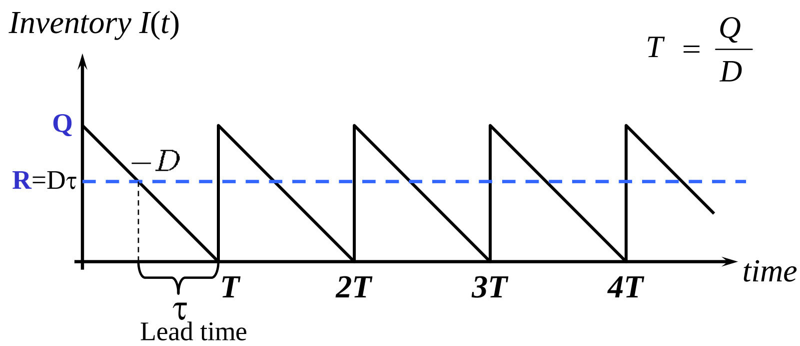
Uncertainty	Constant/Stationary	Variable/Non-Stationary
Deterministic	**Economic Order Quantity** (EOQ) – Tradeoff between fixed cost and holding cost	**Aggregate Planning** – Planning for capacity levels given a forecast **Materials Requirements Planning** (MRP)
Stochastic	**Lot size/Reorder level (Q,R) or (s,S) models** – Tradeoff between fixed cost, holding cost, and shortage cost	Very difficult problem!

→ **Newsvendor** – single period

Lot size/Reorder level (Q,R) Models

ISYE 3104 – Fall 2008

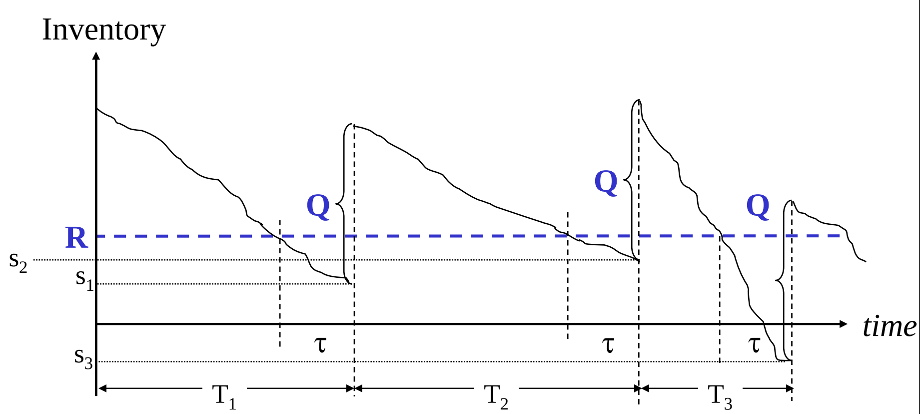
Recap: Basic EOQ



- Place an order when the inventory level is R. The order arrives after τ time periods
  - Q was the only decision variable
  - R could be computed easily because D was deterministic

Uncertain demand

- Both Q and R are decision variables
- Cycle time is no longer constant!



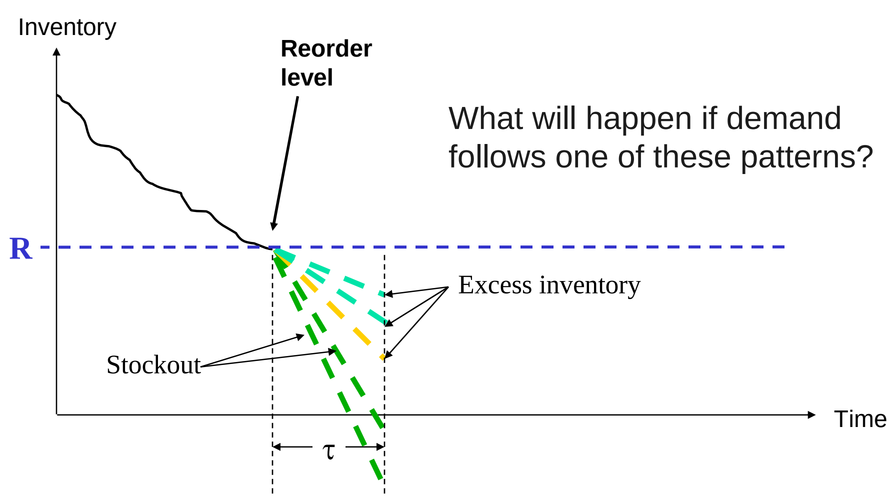
(Q,R) Decisions

- We choose R to meet the demand during lead time
  - Service levels: Protect against uncertainties in demand (or lead time)
  - Balance the costs: stock-outs and inventory
- Tradeoff in Q: Fixed cost versus holding cost

Objective:

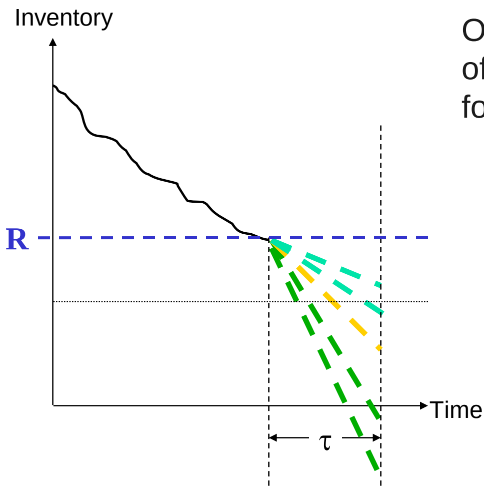
- Minimize
  - fixed cost + holding cost + stockout (backorder) cost

Demand during lead time

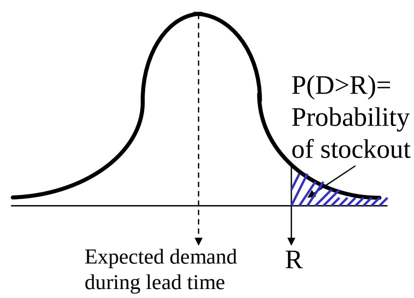


What will happen if demand follows one of these patterns?

Demand during lead time



Often the probability distribution of demand during lead time follows a Normal pattern



(Q,R) Model Assumptions

- Continuous review
- Demand is random and stationary. Expected demand is d per unit time.
- Lead time is τ
- Costs
  - K: Setup cost per order
  - h: Holding cost per unit per unit time
  - c: Purchase price (cost) per unit
  - p: Stockout cost per unit
- Demand during lead time is a continuous random variable D with
  - pdf (density function) $f(x)$ and cdf (distribution function) $F(x)$
  - Mean=μ and standard deviation=σ

(Q,R) Model – Expected total cost per unit time

$$C(Q) = h\overbrace{\left(s+\frac{Q}{2}\right)}^{\text{Holding cost}} + \overbrace{\frac{K}{T}}^{\text{Fixed cost}} + \overbrace{p\frac{n(R)}{T}}^{\text{Shortage cost}} \qquad \text{Recap}: T = \frac{Q}{d}$$

s = Average inventory level before an order arrives

= (Reorder level) – (expected demand during leadtime) = R-μ

$n(R)$ = Expected shortage per cycle

$D > R \Rightarrow$ shortage = D-R

$D < R \Rightarrow$ shortage = 0

Standard loss function

$$n(R) = \int_0^R 0\, f(x)dx + \int_R^\infty (x-R)\, f(x)dx = \int_R^\infty (x-R)\, f(x)dx = \sigma\, L(z)$$

(Q,R) Model – Expected total cost per unit time

$$C(Q) = \text{Holding cost} + \text{Fixed cost} + \text{Shortage cost}$$

$$= h\left(\frac{Q}{2} + R - d\tau\right) + K\frac{d}{Q} + p\frac{d\, n(R)}{Q}$$

Optimal solution:

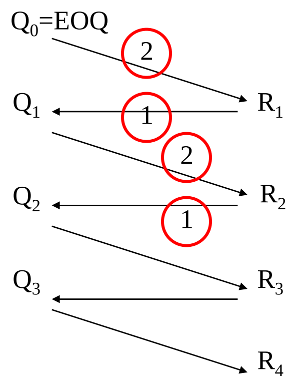
(1) $$Q = \sqrt{\frac{2d[K + p\, n(R)]}{h}}$$

$$F(R) = 1 - \frac{Qh}{pd}$$ (2)

How do we pull Q and R from these equations? → Solve iteratively!!

Solving for optimal Q and R

- Start with a Q_0 value and iterate until the Q values converge



Remember: To find *Q*, you need $n(R) = \sigma L(z)$
Lookup for *z* in the Normal tables

Example – Rainbow Colors

- Rainbow Colors paint store uses a (Q,R) inventory system to control its stock levels. For a popular eggshell latex paint, historical data show that the distribution of monthly demand is approximately Normal, with mean 28 and standard deviation 8. Replenishment lead time for this paint is about 14 weeks. Each can of paint costs the store \$6. Although excess demands are backordered, each unit of stockout costs about \$10 due to bookkeeping and loss-of-goodwill. Fixed cost of replenishment is \$15 per order and holding costs are based on a 30% annual interest rate.
  - What is the optimal lot size (order quantity) and reorder level?
  - What is the expected inventory level (safety stock) just before an order arrives?

Example – Rainbow Colors

- Input
  - Monthly demand Normal mean=28 std.dev.=8
  - τ=14 weeks
  - c=\$6, p=\$10, K=\$15
  - h=ic=(0.3)(6)=\$1.8/unit/year
- Computed input
  - d=(28)(12)=336 units/year (Expected annual demand)
  - Expected demand during lead time

$$\boxed{\mu} = \frac{(28)(12)\text{ units/year}}{52 \text{ weeks/year}} \times (14 \text{ weeks}) = \boxed{90} \text{ units}$$

  - Variance of demand during lead time

$$\text{Annual variance} = (12)(8^2) = 768$$

$$\text{Variance of lead time demand} = 768 \times \frac{14}{52} = 206.77 \Rightarrow \boxed{\sigma = 14.38}$$

Example – Rainbow Colors

- Input
  - Monthly demand Normal mean=28 std.dev.=8

As the lead time increases, so does the mean and variance of demand during lead time

Shorter lead times ⇔ Less variability of demand during lead time

$$\boxed{\mu} = \frac{(28)(12)\text{ units/year}}{52 \text{ weeks/year}} \times (14 \text{ weeks}) = \boxed{90} \text{ units}$$

  - Variance of demand during lead time

$$\text{Annual variance} = (12)(8^2) = 768$$

$$\text{Variance of lead time demand} = 768 \times \frac{14}{52} = 206.77 \Rightarrow \boxed{\sigma = 14.38}$$

Example – Rainbow Colors

- Iteration 0: Compute EOQ

$$\boxed{Q_0} = \sqrt{\frac{2Kd}{h}} = \sqrt{\frac{(2)(15)(336)}{1.8}} = \boxed{75}$$

Example – Rainbow Colors

- Iteration 1: Compute R_1 (given Q_0) and then compute Q_1 (given R_1)

$$F(R_1) \overset{②}{=} 1 - \frac{Q_0 h}{pd} = 1 - \frac{(75)(1.8)}{(10)(336)} = 0.96 = \Phi(z) \;\Rightarrow\; z = 1.75$$

← From standard Normal table

$$\text{Recap: } F(R) = P(D \le R) = P\left(\underbrace{\frac{D-\mu}{\sigma}}_{Z} \le \underbrace{\frac{R-\mu}{\sigma}}_{z}\right) = P(Z \le z) = \Phi(z)$$

↑ Standard Normal

$$z = \frac{R-\mu}{\sigma} \Rightarrow R = \sigma z + \mu \Rightarrow \boxed{R_1} = (14.38)(1.75) + 90 \approx \boxed{115}$$

Safety Stock (σz)

Expected Demand during Lead time (μ)

Q_0=75 —②→ R_1=115 —①→ Q_1

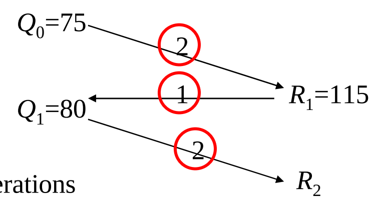
Example – Rainbow Colors

- Iteration 1 (continued): Compute Q_1 (given R_1)

$$Q = \sqrt{\frac{2d[K + p\,n(R)]}{h}} \quad (1)$$

$$n(R_1) = \sigma L(z) = (14.38)(0.0162) = 0.233$$

$$\boxed{Q_1} = \sqrt{\frac{(2)(336)[15 + (10)(0.233)]}{1.8}} \approx \boxed{80}$$



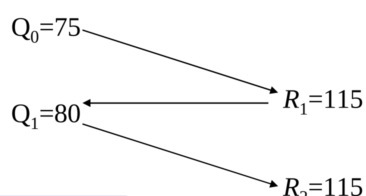
Q_0 and Q_1 not close, continue iterations

Example – Rainbow Colors

- Iteration 2: Compute R_2 (given Q_1) and then compute Q_2 (given R_2)

$$F(R_1) = 1 - \frac{Q_1 h}{pd} = 1 - \frac{(80)(1.8)}{(10)(336)} = 0.957 = \Phi(z) \;\Rightarrow\; z = 1.72$$

$$R = \sigma z + \mu \Rightarrow \boxed{R_2} = (14.38)(1.72) + 90 \approx \boxed{115}$$



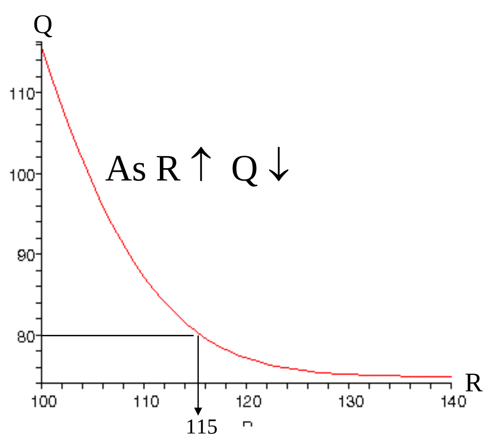
STOP! R values converged, optimal (*Q*,*R*)=(80,115)

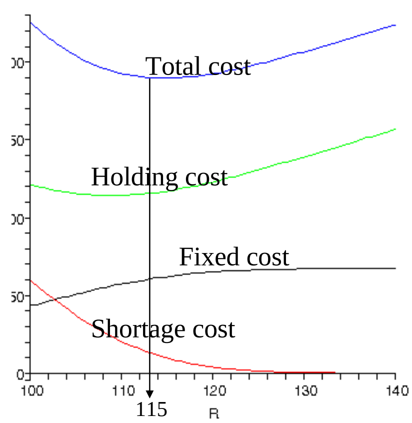
Example – Rainbow Colors

- (Q,R)=(80,115)
  - Optimal order quantity is larger than EOQ. Why?
  - Reorder level is larger than expected demand during lead time. Why?
- Safety stock
  - $s=R-\mu=115-90=25$ units

Impact of R on the costs and Q

R ↑ inventory ↑, therefore Q ↓





Optimal R as a function of Q

$$F(R) = 1 - \frac{Qh}{pd}$$

As the order quantity increases, the reorder level decreases

Q ↑ holding cost ↑ , therefore R ↓ so that we can bring the holding cost back down

The Impact of Holding Cost on the Optimal (Q,R)

As h goes up, both Q and R go down, but Q drops at a faster rate!

i	Q	R
0.2	97	116
0.3	81	115
0.4	71	114
0.5	64	113
0.6	59	112
0.7	55	111

The Impact of Stockout Cost on the Optimal (Q,R)

As p goes up, Q goes down and R goes up!

p	Q	R
2	84	101
6	81	111
10	81	115
14	81	117
18	80	118
22	80	120

Summary: (Q,R) Models

- Balance between holding cost, setup/fixed cost, and shortage cost
  - To save on the shortage cost, we want large R
  - To save on the holding cost, we want small Q and small R
  - To save on the fixed cost, we want large Q

Choose Q and R to strike a good balance among these three costs!!!