Solution to HW #9
ISYE 3104

a) $d \sim N(5600, 500^2) \underset{\text{During lead time}}{\Longrightarrow} N(215.385, 98.058^2)$

$c = 4000$

$\tau = \frac{2}{52} = \frac{1}{26}$ years $= 2$ weeks

$h = I \cdot C = 0.2 \times 4000 = 800$

$p = 1500$/unit

$$Q = \sqrt{\frac{2 \cdot d \cdot (K + P \cdot n(R))}{h}}$$

$$1 - F(R) = \frac{Q \cdot h}{p \cdot d} \quad \text{and} \quad n(R) = \sigma \, L\left(\frac{R - \mu}{\sigma}\right)$$

For the first iteration $Q_0 = \sqrt{\frac{2 \cdot K \cdot d}{h}} = 374$

$1 - F(R) = 0.106857 \Rightarrow F(R) = 0.893143$

$\Rightarrow R_0 = 337 \Rightarrow n(R_0) = 98.058 \times L(1.24) = 5.06$

$\Rightarrow$ By following the iterations according to the formula given above

$Q_1 = 419$ $F(R_1) = 0.8803$ $R_1 = 331$ $n(R_1) = 5.7727$

$Q_2 = 425$ $F(R_2) = 0.8786$, $R_2 = 330$ $n(R_2) = 5.8737$

$Q_3 = 426$ $F(R_3) = 0.8783$ $R_3 = 330$ $n(R_3) = 5.8906$

$Q_4 = 426$ $F(R_4) = 0.8783$ $R_4 = 330$ $n(R_4) = 5.8906$

$\Rightarrow$ so optimal $(Q, R) = (426, 330)$

$\Rightarrow$ Total cost (Annual) $= h\cdot\left(\frac{Q}{2} + R - \mu\right) + \frac{K\cdot d}{Q} + \frac{p\cdot d\cdot n(R)}{Q}$

$= 800 \times \left(\frac{426}{2} + 330 - \frac{5600}{26}\right) + \frac{10000\cdot 5600}{426}$

$+ \frac{500 \times 5600 \times 5.8906}{426} = 432265.3$

Additional cost due to uncertainty in demand

$432265.3 - 299333.3 = \underline{132932}$

b) i) Type 1 service: $\alpha = F(R) = \underline{0.8783}$

ii) Type 2 service $\Rightarrow n(R) = EOQ\cdot(1-\beta)$

$5.8906 = 374 \times (1-\beta)$

$\Rightarrow \beta = \underline{0.98425}$

c) $\alpha = 0.9 = F(R) \Rightarrow z = 1.28$ $\quad n(R) = 98 \times 0.0475 = 4.655$

$R = \sigma\cdot z + \mu = 98 \times 1.28 + 215 = 341$

$Q = EOQ = 374 \Rightarrow (Q,R) = (374, 341)$

Annual cost $= 800\left(\frac{374}{2} + 341 - 215\right) + \frac{10000 \times 5600}{374} + \frac{500 \times 5600 \times 4.655}{374}$

$= \underline{435783}$

Imputed shortage cost $= Q\cdot h / ((1-F(R))\cdot d) = 374 \times 800 / ((1-0.9) \times 5600) = \underline{534.3}$

d) 95% type II service level $\Rightarrow \beta = 0.95$

$n(R) = EOQ \cdot (1-\beta) \Rightarrow n(R) = 18.7$

$\Rightarrow L(z) = \frac{18.7}{98} = 0.1908 \Rightarrow z = 0.52$

$\Rightarrow R_1 = \sigma . z + \mu = 266 \Rightarrow F(R_1) = 0.6985$

$\Rightarrow Q_1 = \frac{n(R_1)}{1-F(R_1)} + \sqrt{\frac{2kd}{h} + \left(\frac{n(R_1)}{1-F(R_1)}\right)^2} = 441$

$\Rightarrow$ continue iterations as:

$n(R_2) = 22.2$, $R_2 = 257$, $Q_2 = 445$

$n(R_3) = 22.2$, $R_3 = 257$, $Q_3 = 445$

$\Rightarrow$ stop. So, $(Q, R) = (445, 257)$

$\Rightarrow$ Annual cost $= 300\left(\frac{445}{2} + 257 - 215\right) + \frac{10000 \times 5600}{445} + \frac{500 \times 5600 \times 22.2}{445}$

$= 477128$

$\Rightarrow$ Imputed shortage cost $= \frac{445 \times 300}{5600 \times (1 - 0.6591)} = 186.48$

e) $\alpha = 0.9 = F(R) \Rightarrow z = 1.28$

$R = \sigma . z + \mu = \frac{500}{\sqrt{52}} \times 1.28 + \frac{5600}{52} \cong 197$

$Q = EOQ = 374 \Rightarrow (Q, R) = (374, 197)$

Annual cost $= 391436.24$

Imputed shortage cost $= \frac{Q . h}{d . (1 - F(R))} = 534.3$

As $z \downarrow \Rightarrow R \downarrow \Rightarrow$ and overall cost $\downarrow$

Answer to HW #9
ISYE 3104

(2)

* A

week	1	2	3	4
Planned order Release	120	120	120	120

* B

week	1	2	3	4
POR	60	100	80	50

* Spare Part

week	1	2	3	4
Net Demand	50	50	50	50

* C

week	0	1	2	3	4
Gross Requir.	–	350	470	410	320
Scheduled receipts	–	15	10	0	0
On-hand inv.	10	0	0		
Net Req.	–	325	460	410	320