HW 8, Answers

1. First step is to verify that the problem is feasible

$$\sum \frac{d_j}{P_j} = \frac{5600}{20000} + \frac{5600}{30000} + \frac{5600}{20000} = 0.7521 < 1, \quad \text{so there is a feasible solution}$$

$$\begin{aligned}
& & h_j^1 &= (1 - \tfrac{d_j}{P_j}) \cdot h_j \\
\Rightarrow & & h_1^{'} &= (1 - \tfrac{5600}{20000}) \times 3000 \times 0.2 = 432 \\
& & h_2^{'} &= (1 - \tfrac{5600}{30000}) \times 2500 \times 0.2 = 406.7 \\
& & h_3^{'} &= (1 - \tfrac{5600}{20000}) \times 4000 \times 0.2 = 576
\end{aligned}$$

Setup costs:

$$\begin{aligned}
& & K_1 &= 4000 \quad K_2 = 2000 \quad K_3 = 3000 \\
\Rightarrow & & T^* &= \sqrt{\frac{2 \cdot \sum K_j}{\sum h_j' d_j}} = \sqrt{\frac{2 \times (4000+2000+3000)}{(432+406.7+576) \times 5600}} \\
& & T^* &= 0.0477 \text{ years} \\
& & &\text{check whether} \quad T^* > T_{\min} \\
& & T_{\min} &= \frac{(2/(5\times50)+0.5/(5\times50)+1/(5\times50)}{1-0.75} = 0.056 \\
& & &\text{since } T^* < T_{\min} \Rightarrow \text{ choose } T_{\min} = 0.056 \\
\Rightarrow & & Q_1^* &= d_1 \cdot T_{\min} = 3136 \quad Q_2^* = Q_3^* = 3136.
\end{aligned}$$

2. Cost of in-house production $= \sum_{j=1}^{3} (\frac{K_j}{T_{\min}} + h_j^{'} \cdot \frac{Q_j}{2} + c_j \cdot d_j)$

$$\begin{aligned}
&= \tfrac{4000}{0.056} + 432 \times \tfrac{3136}{2} + 3000 \times 5600 \\
&+ \tfrac{2000}{0.056} + 406.7 \times \tfrac{3136}{2} + 2500 \times 5600 \\
&+ \tfrac{3000}{0.056} + 576 \times \tfrac{3136}{2} + 4000 \times 5600 \\
&= 55,578,963.89 \cong 55,578,964
\end{aligned}$$

Outsourcing cost was $68,266,017.42 \Rightarrow$ since cost of producing in-house is lower $\Rightarrow$ shift to in house.

To recover $\Rightarrow \frac{10000000}{12687054} = 0.788$ years required

However, in general, other issues should also be considered in choosing between in-house production and outsourcing. Such as quality differences, possibility of a significant (downward or upward) shift in demand, potential development of new technology (which could require an investment in a new machine often), lead times, and necessity/possibility of using the same machine for other products of the company.

Solution to problem 2 In HW #3

a) $C_o = 22 - 10 = 12$ \$/unit

C_u = profit = $50 - 22 = 28$ \$/unit

b) $F(Q^*) = \frac{C_u}{C_u + C_o} = 0.7$

$\Rightarrow \Phi\left(\frac{Q^* - \mu}{\sigma}\right) = 0.7$

$\Rightarrow$ From table A.1 $\frac{Q^* - \mu}{\sigma} = 0.53$

$\sigma = 2000 \quad \mu = 6000$

$\Rightarrow Q^* = \underline{7060}$

c) $F(Q^*) = 0.7$

$\Rightarrow \frac{Q^* - 4000}{6000 - 4000} = 0.7 \Rightarrow Q^* = \underline{5400}$

d)

Q	2000	3000	4000	5000	6000	7000	8000	9000	10000
F(Q)	0.02	0.05	0.15	0.3	0.6	0.75	0.90	0.95	1

$F(Q^*) = \boxed{0.7}$ in here

$\Rightarrow$ so by interpolation

$\frac{Q - 6000}{7000 - 6000} = \frac{0.7 - 0.6}{0.75 - 0.6} \Rightarrow Q^* = \underline{6667}$