

ISyE 3104 MIDTERM 1 SOLUTIONS

1.a. 2 years: cost when outsourced = $750 \times 1,000 \times 24 = 18,000,000$

cost when produced in house = $5,000,000 + 1,000 \times 24 \times 0.6 \times 750 = 15,800,000$

⇒ It is worth producing in house.

1.b. $5,000,000 + 1000 \times 24 \times 0.7 \times P^* = 750 \times 1,000 \times 24 \times 0.7$

⇒ $P^* = \$452.38$

1.c. $5,000,000 + \sum_{x=1}^n (500 - \frac{x}{1000}) = 750 \times n$

$$5,000,000 + 500n - \frac{1}{10,000} \times \frac{n(n+1)}{2} = 750n$$

$$\frac{n^2+n}{20,000} + 250n - 5,000,000 = 0$$

⇒ $n \cong 19,921$

Also OK, if understood cost of producing X units (not Xth unit) is $500 - x/1000$;

$$750x = 5,000,000 + (500 - x / 1000) x$$

$$x^2 / 1000 + 250x - 5,000,000 = 0$$

$$x = 18,615$$

2.a. $2^{-b} = 0.92 \Rightarrow b = 0.12$

$$Y(1) = 120 \text{ min.}$$

$$120 = a \times 1^{-b} \Rightarrow a = 120$$

$$Y(500) = 120 \times 500^{-b} = 56.82 \text{ minutes} = 0.947 \text{ hours}$$

2.b. $45 = 120 \times 1000^{-b} \Rightarrow b = 0.213 \Rightarrow$ it does not fly

$$\ln(Y(u)) = \ln(a) - b \cdot \ln(u)$$

$$y(1) = 120 \Rightarrow \ln(Y(1)) = 4.7875 \quad \ln(1) = 0$$

$$Y(1000) = 45 \Rightarrow \ln(Y(1000)) = 3.807 \quad \ln(1000) = 6.908$$

$$\frac{Y - Y_1}{X - X_1} = \frac{Y_2 - Y_1}{X_2 - X_1} \Rightarrow \text{finding equation of line}$$

$$\frac{\ln(Y(u)) - 4.4875}{\ln(u) - 0} = \frac{3.807 - 4.7875}{6.908 - 0}$$

$$6.908 \ln(Y(u)) - 33.702 = -0.9805 \times \ln(u)$$

$$\ln(Y(u)) = 4.7875 - 0.142 \times \ln(u)$$

$$2^{-b} = 2^{-0.142} = 0.906 = 90.6\%$$

Also OK if found new b by ;

$$Y(1000) = 2 \times (1000)^{-b} = 0.75 \text{ hour}$$

$$(1000)^{-b} = 0.75/2$$

$$b = 0.142$$

$$L^{\text{new}} = 2^{-b} = 2^{-0.142} = 90.6\%$$

3.a. annual demand = 75 x 52 = 3,900 units

$$Q^* = \sqrt{\frac{2 \times 20 \times 3900}{0.2 \times 1.5}} \cong 721$$

$$T = \frac{Q}{D} = \frac{721}{3900} = 0.185 \text{ years} = 9.61 \text{ weeks}$$

3.b. If 4 orders per year,

$$T = \frac{1}{4} \text{ years}$$

$$\frac{1}{4} = \frac{Q}{3900} \Rightarrow Q = 975$$

$$975 = \sqrt{\frac{2 \times K \times 3900}{0.2 \times 1.5}} \Rightarrow K = 36.56$$

4.a. K = \$100

$$h = \$2/\text{year}$$

$$P = 8,000 \text{ unit / year}$$

$$d = 2,000 \text{ unit / year}$$

$$Q^* = \sqrt{\frac{2 \times 100 \times 2,000}{2x \left(1 - \frac{2,000}{8,000}\right)}} \cong 517$$

$$4.b. H = 517 \times \left(1 - \frac{2,000}{8,000}\right) \cong 388$$

$$4.c. T_1 = \frac{Q}{P} = \frac{517}{8,000} = 0.064625 \text{ years} = 3.3605 \text{ weeks}$$

$$4.d. T = \frac{Q}{d} = \frac{517}{2,000} = 0.2585 \text{ years} = 13.442 \text{ weeks}$$

$$T_2 = T - T_1 = 10.0815 \text{ weeks}$$

$$5.a. \text{ A worker can produce } \frac{7}{2.5} \times 20 = 56 \frac{\text{units}}{\text{month}}$$

Cumulative production per worker	cumulative demand	# of workers required	cumulative prodn.	ending inventory
56	6750	121	6,776	26
112	10,750	96	13,552	2,802
168	17,750	106	20,328	2,578
224	21,750	97	27,104	5,354 + 500

Fire 200 - 121 = 79 workers at the beginning

$$\begin{aligned} \text{Total cost} &= \underline{79 \times 2,500} + \underline{30 \times 7 \times 20 \times 121 \times 4} + \underline{(26 \times 2,802 + 2,578 + 5,354 + 500) \times 15} \\ &\quad \text{Firing cost} \quad \text{worker cost} \quad \text{inventory holding cost} \\ &= 197,500 + 2,032,800 + 16,890 \\ &= 2,399,200 \end{aligned}$$

5.b.

Prodn per worker	Demand	# of workers required	Cumulative ending inventory	Hire	Fire

56	6,750	121	26		79
56	4,000	72	58		49
56	7,000	125	58	53	
56	4,000	72	90 + 500		53

Total cost = $(79 + 49 + 53) \times 2,500 + 53 \times 1,250 + 7 \times 30 \times 20 \times 121 + 7 \times 30 \times 20 \times 72 + 7 \times 30 \times 20 \times 125$
 $+ 7 \times 30 \times 20 \times 72 + (26 + 58 + 58 + 90 + 500) \times 15 = 2,167,730$

5.c. From part a, apply a constant workforce plan with 106 workers (2nd highest). Fire 94 workers at the beginning

Cumulative production	Ending inventory	Backorder
5,936	-	814
11,872	308	-
17,808	58	-
23,744	1,994 + 500	-

Total cost = $94 \times 2,500 + 30 \times 7 \times 20 \times 106 \times 4 + (308 + 58 + 1,994 + 500) \times 15 + 814 \times 50$
 $= 2,099,400$

6.a. No, because the EOQ model is for systems which have constant demand. However, in this case demand is seasonal.

6.b. Uncertainties, speculators, economies of scale

6.c. (b)

6.d. (c)

6.e. (c)