ISyE 3104 Fall 2005
HW10 – Answers

Using the hint to modify the requirements schedule:
R=(10,3,0,26,23)
h=1, K=30.

Computing Cij's where 1<=i<=5 and i+1<=j<=6

C12=30
C13=30+3=33
C14=33
C15=30+3+26*3=111
C16=30+3+26*3+23*4=203

C23=30
C24=30+0=30
C25=30+26*2=82
C26=30+26*2+23*3=151

C34=30
C35=30+26=56
C36=30+26+2*23=102

C45=30
C46=30+23=53

C56=30

Summarizing these cost in a matrix:

j i	1	2	3	4	5	6
1		30	33	33	111	203
2			30	30	82	151
3				30	56	102
4					30	53
5						30

Solution Technique 1: Complete enumeration

We can enumerate all the following 16 different paths to select the minimum cost path:

Path	Cost
1-2-3-4-5-6	?
1-2-3-4-6	?
1-2-3-6	?
1-2-6	?
1-6	?
1-2-3-5-6	?
1-2-4-5-6	?
1-3-4-5-6	?
1-3-6	?
1-4-6	86*
1-5-6	?
1-2-4-6	?
1-2-5-6	?
1-4-5-6	?
1-3-5-6	?
1-3-4-6	?

The number of different paths is 16. This is found from the Binomial Theorem's Pascal Triangle. We have 5 periods to produce. We have to produce in period 1 to satisfy first period's demand. Therefore we can choose to produce or not to produce in any other "4" periods from 2 to5. In the network sense, we have 6 vertices. Vertex 1 is the source and vertex 6 is the sink. We will start in 1 and end up in 6. If we visit any middle vertices (2 to 5), that means we select that period for production. These "4" vertices may or may not be selected for production. The total number of different paths from 1 to 6 is found by summing the coefficients of the following Pascal Triangle's 4th row.

```
0:                 1
1:               1   1
2:             1   2   1
3:           1   3   3   1
4:         1   4   6   4   1
5:       1   5  10  10   5   1
6:     1   6  15  20  15   6   1
7: 1   7  21  35  35  21   7   1
```

Here each row starts and ends with 1. These 1's are the options of not producing in the remaining 4 (in our case) periods or producing in each one of the 4 periods for that periods demand.

The coefficients of 1, 4, 6, 4, 1 is therefore showing the total "combinations" of each selection. Combination operation C(4,0)=1, shows selecting none (0) of the 4 periods to produce. Whereas, C(4,1)=4 shows selecting 1 period out of 4 to produce (in that case we have 4 different combinations). Going in this fashion C(4,2)=6, C(4,3)=4 and C(4,4)=1.

With the help of above explanation and using Pascal Triangle, one can calculate the total number of different production schedules, without trying to write them all. Because, as you see, it gets very complicated to enumerate the all different paths once we have more than 5 periods for the scheduling problem.

Therefore, we will use the second solution technique, which avoids this major calculation.

Solution Technique 2: Dynamic Programming

See Nahmias p.397 for the explanation of the technique:

$F5= 30$ at $j=6$

$F4= \min \{30+30, 53+0\}=53$ at $j=6$

$F3= \min \{30+53,56+30,102+0\}=83$ at $j=4$

$F2= \min \{30+83,30+53,82+30,151+0\}=83$ at $j=4$

$F1= \min \{30+83,33+83,33+53,111+30,203+0\}=86$ at $j=4$

Hence $y1= r1+r2+r3=13$. Production takes place next in period 4.
Since f4 is minimized at $j=6$, $y4=r4+r5=49$.
The optimal cost is $86.
The optimal path for technique 1 is then: 1-4-6