

ISyE 3104
HW #8 SOLUTIONS

① $K = \$200$
 $c = \$5/\text{gallon}$
 $h = ic = \$1/\text{gallon/year}$
 $d = 10,000 \text{ gallons/yr}$
 $D \sim N(400, 900)$
 $\mu = 400$
 $\sigma = 30$

(a) $\frac{10,000}{400} = 25 \text{ times/year} \rightarrow \approx 2 \text{ weeks}$

(b) $F(R) = \alpha = 0.9 \Rightarrow z = 1.285$
 $\Rightarrow R = \mu + \sigma z = 400 + 30(1.285) \approx 439 \text{ gallons}$
 $s = R - \mu = 439 - 400 = 39 \text{ gallons}$

(c) $Q = 1000 \text{ gallons}$
 $p = \$4$

$1 - F(R) = \frac{qh}{pd} = \frac{1000(1)}{4(10,000)} \Rightarrow F(R) = 0.975 \Rightarrow z = 1.96$
 $R = \mu + \sigma z = 400 + 30(1.96) \approx 459 \text{ gallons}$

② $p = \$4$

(a) $R = 400$ $n(R) = \sigma L\left(\frac{R - \mu}{\sigma}\right) = 30 L\left(\frac{400 - 400}{30}\right) = 30 L(0)$
 $= 30(0.3989) = 11.967$

\hookrightarrow expected # of stock-outs incurred in a cycle

$Q^* = \sqrt{\frac{2\lambda[K + pn(R)]}{h}} = \sqrt{\frac{2(10,000)[200 + 4(11.967)]}{1}}$

$\approx 2227 \text{ gallons}$

(b) Probability that lead-time demand exceeds the reorder point

$1 - F(R) = 1 - 0.5 = 0.5 \Rightarrow 50\%$

(c) $Q_0 = \text{EOQ} = \sqrt{\frac{2\lambda K}{h}} = \sqrt{\frac{2(200)(10,000)}{1}} = 2000$

$1 - F(R_0) = \frac{qh}{pd} = \frac{2000(1)}{4(10,000)} = 0.05$ $F(R_0) = 0.95 \Rightarrow z = 1.64$
 $R_0 = 400 + 30(1.64) = 449 \text{ gallons}$

$$L(1.64) = 0.0211 \Rightarrow n(R_0) = 30(0.0211) = 0.633$$

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$$Q_1 = \sqrt{\frac{2(10,000)[200 + 4(0.633)]}{1}} \approx 2013 \text{ gallons}$$

$$1 - F(R_1) = \frac{Q_1 h}{p\lambda} = \frac{2013(1)}{4(10,000)} = 0.05 \Rightarrow F(R_1) = 0.95 \Rightarrow z = 1.64$$

$$R_1 = 449 \text{ gallons}$$

We stop!

$$(Q^*, R^*) = (2013, 449)$$

$$\text{Holding Cost} = h \left[\frac{Q}{2} + R - \mu \right] = 1 \left[\frac{2013}{2} + 449 - 400 \right] = \$1055.5$$

$$\text{Setup Cost} = \frac{K\lambda}{Q} = \frac{200(10,000)}{2013} = \$993.5$$

$$\text{Stock-out Cost} = p\lambda \frac{n(R)}{Q} = \frac{4(10,000)(0.633)}{2013} = \$12.6$$

$$\text{Total Average Annual Cost} = \$2061.6$$

$$\textcircled{3} \quad \beta = 0.99 \quad \frac{n(R)}{Q} = (1 - \beta) \quad Q = \frac{n(R)}{1 - F(R)} + \sqrt{\frac{2K\lambda}{h} + \left(\frac{n(R)}{1 - F(R)} \right)^2}$$

$$\Rightarrow L(z) = (1 - \beta)Q/\sigma$$

$$Q_0 = \text{EOQ} = 2000 \text{ (from previous question)}$$

$$L(z) = 0.04(2000)/30 = 0.667 \Rightarrow z = -0.46$$

$$\Rightarrow R_0 = 400 + 30(-0.46) = 386 \text{ gallons}$$

$$n(R_0) = 2000(0.01) = 20$$

$$1 - F(R_0) = 0.6772$$

$$Q_1 = \frac{n(R)}{1 - F(R)} + \sqrt{\frac{2K\lambda}{h} + \left(\frac{n(R)}{1 - F(R)} \right)^2}$$

$$= \frac{20}{0.6772} + \sqrt{(2000)^2 + \left(\frac{20}{0.6772} \right)^2} \approx 2030 \text{ gallons}$$

n(R1) = (1 - beta) Q1 = 0.01(2030) = 20.3 L(z) = 20.3/30 = 0.677

Q2 = 20.3 / 0.6808 + sqrt((2000)^2 + (20.3 / 0.6808)^2) approx 2030 gallons => z = -0.47 => R1 = 400 + 30(-0.47) approx 386 gallons

p = (Qh) / [(1 - F(R)) lambda] = (2030(1)) / (0.6808(10,000)) approx \$0.298

1 - F(R1) = 0.6808

we stop!

↳ imputed shortage cost

(Q*, R*) = (2030, 386)

Question 4:

a. Fixed lead time. Since the lead time does not depend on how much work is in the plant, the implicit assumption is that the line will always have sufficient capacity regardless of the load. This can create problems when the production levels are at or near capacity. One way to address this problem is to make sure that the master production schedule is capacity feasible. We will see one way to do this when we cover Section 7.4. Another problem is that lead times may be stochastic in practice. But since the production planner needs to specify one deterministic lead time, he may tend to choose pessimistic (long) estimates for the planned lead times to make sure he feeds the next stage by the required date. Suppose the average lead time is three weeks with a standard deviation of one week. Suppose also that to maintain a good customer service, the planned lead time is set to five weeks. If production lead times follow a normal distribution, 95 items will be ready on time, but 50 two weeks or more. The result can be a large amount of inventory. One remedy is to try to reduce the variability in lead times by good process control.

b. BOM explosion based on forecasts. The BOM explosion is carried out as if the forecasts of future demands would not change. But forecasts are invariably updated when new information becomes available. This has two implications in the MRP system. One is that all of the lot sizing decisions that were determined in the last run could be incorrect, and even more problematic, former decisions that are currently being implemented in the production process may be inappropriate. The analysis we've done so far showed that an optimal policy includes safety stock to protect against the uncertainty in demand. The same logic can be applied to MRP systems by building suitable safety levels into the forecasts for the end item. These will be transmitted automatically down through the system to the lower levels through the explosion calculus. This method will avoid constant changes in the MRP schedule.