

Homework #3

Supply Chain Models: Manufacturing & Warehousing (ISyE 3104) - Fall 2001 Due September 20, 2001

Show all your steps to get full credit. (Total 45 points)

Reading assignment: Read Supplement 1, Chapter 3 and Chapter 4.

The Yeasty Brewing Company produces a popular local beer known as Iron Stomach. Beer sales are somewhat seasonal, and Yeasty is planning its production and manpower levels on March 31 for the next six months. The demand forecasts are:

Month	Production Days	Forecasted Demand
April	11	8,500
May	22	9,300
June	20	12,200
July	23	17,600
August	16	14,000
September	20	6,300

As of March 31, Yeasty had 86 workers on the payroll. Over a period of 26 working days, when there were 100 workers on the payroll, Yeasty produced 12,000 cases of beer. The cost to hire each worker is \$125 and the cost of laying off each worker is \$300. The regulation imposes that the number of hired or laid off workers should not exceed 10 per month. Regular worker cost is \$10/hr/worker (assume 8-hour days) whereas overtime worker cost is \$20/hr/worker. The overtime working is a 4-hr shift after a regular working day. (Overtime workers are also hired on a monthly basis.) Subcontracting is available at a cost of \$25 per case.

As of March 31, Yeasty expects to have 4,500 cases of beer in stock, and it wants to maintain a minimum buffer inventory of 1,000 cases each month. It plans to start October with 3,000 cases on hand.

Attached are an LP formulation and the corresponding Lindo output for this problem.

- a) (7 points) Explain how this problem can be modeled by the following LP. Associate appropriate definitions for the variables and explain the constraints.

$$1) \text{ Min } 125H1 + 125H2 + 125H3 + 125H4 + 125H5 + 125H6 + 300F1 + 300F2 + 300F3 + 300F4 + 300F5 + 300F6 + 0.75I1 + 0.75I2 + 0.75I3 + 0.75I4 + 0.75I5 + 0.75I6 + 880W1 + 1760W2 + 1600W3 + 1840W4 + 1280W5 + 1600W6 + 880O1 + 1760O2 + 1600O3 + 1840O4 + 1280O5 + 1600O6 + 25S1 + 25S2 + 25S3 + 25S4 + 25S5 + 25S6$$

Subject to

$$2) W0 + H1 - F1 - W1 = 0$$

$$3) W1 + H2 - F2 - W2 = 0$$

$$4) W2 + H3 - F3 - W3 = 0$$

$$5) W3 + H4 - F4 - W4 = 0$$

$$6) W4 + H5 - F5 - W5 = 0$$

$$7) W5 + H6 - F6 - W6 = 0$$

$$8) H1 \leq 10$$

$$9) H2 \leq 10$$

$$10) H3 \leq 10$$

$$11) H4 \leq 10$$

$$12) H5 \leq 10$$

$$13) H6 \leq 10$$

$$14) F1 \leq 10$$

$$15) F2 \leq 10$$

$$16) F3 \leq 10$$

$$17) F4 \leq 10$$

$$18) F5 \leq 10$$

$$19) F6 \leq 10$$

$$20) 50.7W1 + 25.35O1 - U1 - P1 = 0$$

$$21) 101.4W2 + 50.70O2 - U2 - P2 = 0$$

$$22) 92.2W3 + 46.10O3 - U3 - P3 = 0$$

$$23) 106.0W4 + 53.00O4 - U4 - P4 = 0$$

$$24) 73.7W5 + 36.88O5 - U5 - P5 = 0$$

$$25) 92.2W6 + 46.10O6 - U6 - P6 = 0$$

$$26) I0 + P1 + S1 - I1 = 8500$$

$$27) I1 + P2 + S2 - I2 = 9300$$

$$28) I2 + P3 + S3 - I3 = 12100$$

$$29) I3 + P4 + S4 - I4 = 17600$$

$$30) I4 + P5 + S5 - I5 = 14000$$

$$31) I5 + P6 + S6 - I6 = 6300$$

$$32) I0 = 4500$$

$$33) I1 \geq 1000$$

$$34) I2 \geq 1000$$

$$35) I3 \geq 1000$$

$$36) I4 \geq 1000$$

$$37) I5 \geq 1000$$

$$38) I6 \geq 3000$$

$$39) W0 = 86$$

$$40) O1 - W1 \leq 0$$

$$41) O2 - W2 \leq 0$$

$$42) O3 - W3 \leq 0$$

- 43) $O4 - W4 \leq 0$
- 44) $O5 - W5 \leq 0$
- 45) $O6 - W6 \leq 0$

Answer the following questions using sensitivity analysis. Remember that the “solution” to the problem has two components: objective function value and the values of the variables.

- b) (3 points) Rather than keeping 1000 units in inventory at the end of July, the firm is considering to keep only 500 units. Would this change improve the objective function or make it worse? How much would the objective function change per unit decrease in the buffer inventory?
- c) (2 points) If the demand in April increased from 8500 to 9000, what would be the new objective function value?
- d) (2 points) Suppose the demand in April decreases to 8000. By looking at the sensitivity output from Lindo, can you tell how the solution would change?
- e) (3 points) If the number of available workers to hire increased from 10 to 15 in May, would Yeasty be interested in hiring any additional workers (in addition to the 10 workers they will hire according to the current solution)? If yes, how much would they be willing to pay each additional worker?
- f) (2 points) If the cost of hiring a worker in June increased from \$125 to \$250 due to a shortage in the labor market, how would this change impact the current solution?
- g) (2 points) If the cost of subcontracting in April increased from \$25 to \$30, how would this change impact the current solution?
- h) (3 points) If the cost of subcontracting reduced from \$25 to \$24 in May, would Yeasty be interested in subcontracting part of the production in May? What if the cost of subcontracting reduces to \$20 in May?

LP OPTIMUM FOUND AT STEP 37

OBJECTIVE FUNCTION VALUE

1) 1265754.

VARIABLE	VALUE	REDUCED COST
H1	10.000000	.000000
H2	10.000000	.000000
H3	10.000000	.000000
H4	.000000	352.500100
H5	.000000	1162.500000
H6	.000000	1725.000000
F1	.000000	1718.750000
F2	.000000	1331.250000
F3	.000000	708.349900
F4	.000000	72.499920
F5	10.000000	.000000
F6	10.000000	.000000
I1	1000.000000	.000000
I2	2448.400000	.000000
I3	1043.600000	.000000
I4	1000.000000	.000000
I5	1000.000000	.000000
I6	3000.000000	.000000
W1	96.000000	.000000
W2	106.000000	.000000
W3	116.000000	.000000
W4	116.000000	.000000
W5	106.000000	.000000
W6	96.000000	.000000
O1	.000000	246.250000
O2	.000000	568.550000
O3	.000000	482.075000
O4	.000000	515.000000
O5	.000000	358.000000
O6	.000000	1600.000000
S1	132.799900	.000000
S2	.000000	1.500000
S3	.000000	.750000
S4	5260.400000	.000000
S5	6187.800000	.000000
S6	.000000	25.000000
W0	86.000000	.000000
U1	.000000	25.000000
P1	4867.200000	.000000

U2	.000000	23.500000
P2	10748.400000	.000000
U3	.000000	24.250000
P3	10695.200000	.000000
U4	.000000	25.000000
P4	12296.000000	.000000
U5	.000000	25.000000
P5	7812.200000	.000000
U6	551.199700	.000000
P6	8300.000000	.000000
I0	4500.000000	.000000

ROW SLACK OR SURPLUS DUAL PRICES

2)	.000000	-1418.750000
3)	.000000	-1031.250000
4)	.000000	-408.349900
5)	.000000	227.500100
6)	.000000	1037.500000
7)	.000000	1600.000000
8)	.000000	1293.750000
9)	.000000	906.249900
10)	.000000	283.349900
11)	10.000000	.000000
12)	10.000000	.000000
13)	10.000000	.000000
14)	10.000000	.000000
15)	10.000000	.000000
16)	10.000000	.000000
17)	10.000000	.000000
18)	.000000	737.500100
19)	.000000	1300.000000
20)	.000000	-25.000000
21)	.000000	-23.500000
22)	.000000	-24.250000
23)	.000000	-25.000000
24)	.000000	-25.000000
25)	.000000	.000000
26)	.000000	-25.000000
27)	.000000	-23.500000
28)	.000000	-24.250000
29)	.000000	-25.000000
30)	.000000	-25.000000
31)	.000000	.000000
32)	.000000	25.000000
33)	.000000	-2.250000

34)	1448.400000	.000000
35)	43.599810	.000000
36)	.000000	-.750000
37)	.000000	-25.750000
38)	.000000	-.750000
39)	.000000	1418.750000
40)	96.000000	.000000
41)	106.000000	.000000
42)	116.000000	.000000
43)	116.000000	.000000
44)	106.000000	.000000
45)	96.000000	.000000

NO. ITERATIONS= 37

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	125.000000	1293.750000	INFINITY
H2	125.000000	906.249900	INFINITY
H3	125.000000	283.349900	INFINITY
H4	125.000000	INFINITY	352.500100
H5	125.000000	INFINITY	1162.500000
H6	125.000000	INFINITY	1725.000000
F1	300.000000	INFINITY	1718.750000
F2	300.000000	INFINITY	1331.250000
F3	300.000000	INFINITY	708.349900
F4	300.000000	INFINITY	72.499920
F5	300.000000	737.500100	INFINITY
F6	300.000000	1300.000000	INFINITY
I1	.750000	INFINITY	2.250000
I2	.750000	8.937375	1.500000
I3	.750000	3.073209	.750000
I4	.750000	INFINITY	.750000
I5	.750000	INFINITY	25.750000
I6	.750000	INFINITY	.750000
W1	880.000000	1293.750000	INFINITY
W2	1760.000000	906.249900	INFINITY
W3	1600.000000	283.349900	INFINITY
W4	1840.000000	72.499920	352.500100
W5	1280.000000	72.499920	352.500100
W6	1600.000000	72.499920	352.500100
O1	880.000000	INFINITY	246.250000
O2	1760.000000	INFINITY	568.550000

O3	1600.000000	INFINITY	482.075000
O4	1840.000000	INFINITY	515.000000
O5	1280.000000	INFINITY	358.000000
O6	1600.000000	INFINITY	1600.000000
S1	25.000000	9.714004	2.250000
S2	25.000000	INFINITY	1.500000
S3	25.000000	INFINITY	.750000
S4	25.000000	.750000	.683962
S5	25.000000	.750000	.983717
S6	25.000000	INFINITY	25.000000
W0	.000000	INFINITY	INFINITY
U1	.000000	INFINITY	25.000000
P1	.000000	25.000000	9.714004
U2	.000000	INFINITY	23.500000
P2	.000000	8.937375	11.214000
U3	.000000	INFINITY	24.250000
P3	.000000	3.073209	10.457160
U4	.000000	INFINITY	25.000000
P4	.000000	.683962	3.325472
U5	.000000	INFINITY	25.000000
P5	.000000	.983717	4.782905
U6	.000000	.750000	3.823211
P6	.000000	25.000000	.750000
I0	.000000	INFINITY	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	.000000	.225206	2.619328
3	.000000	.225206	17.558080
4	.000000	.472883	26.540870
5	.000000	5.978305	49.626420
6	.000000	5.978305	83.959300
7	.000000	5.978305	INFINITY
8	10.000000	2.619328	.225206
9	10.000000	17.558080	.225206
10	10.000000	26.540870	.472883
11	10.000000	INFINITY	10.000000
12	10.000000	INFINITY	10.000000
13	10.000000	INFINITY	10.000000
14	10.000000	INFINITY	10.000000
15	10.000000	INFINITY	10.000000
16	10.000000	INFINITY	10.000000
17	10.000000	INFINITY	10.000000
18	10.000000	5.978305	10.000000
19	10.000000	5.978305	10.000000

20	.000000	4867.200000	132.799900
21	.000000	43.599810	5260.400000
22	.000000	43.599810	5260.400000
23	.000000	12296.000000	5260.400000
24	.000000	7812.200000	6187.800000
25	.000000	551.199700	INFINITY
26	8500.000000	INFINITY	132.799900
27	9300.000000	43.599810	5260.400000
28	12100.000000	43.599810	5260.400000
29	17600.000000	INFINITY	5260.400000
30	14000.000000	INFINITY	6187.800000
31	6300.000000	551.199700	8300.000000
32	4500.000000	132.799900	4500.000000
33	1000.000000	5260.400000	43.599810
34	1000.000000	1448.400000	INFINITY
35	1000.000000	43.599810	INFINITY
36	1000.000000	6187.800000	1000.000000
37	1000.000000	8300.000000	551.199700
38	3000.000000	551.199700	3000.000000
39	86.000000	2.619328	.225206
40	.000000	INFINITY	96.000000
41	.000000	INFINITY	106.000000
42	.000000	INFINITY	116.000000
43	.000000	INFINITY	116.000000
44	.000000	INFINITY	106.000000
45	.000000	INFINITY	96.000000

2. (5 points) What is the opportunity cost of inventory?

3. (6 points) What are the major functions of inventory?

4. HAL Ltd. produces a line of high-capacity disk drives for mainframe computers. The housings for the drives are produced in Hamilton, Ontario, and shipped to the main plant in Toronto. HAL uses the drive housings at a fairly steady rate of 720 per year. Suppose that the housings are shipped in trucks that can hold 40 housings at one time. It is estimated that the fixed cost of loading the housings onto the truck and unloading them on the other end is \$300 for shipments of 120 or fewer housings (i.e., three or fewer truckloads). Each trip made by a single truck costs the company \$160 in driver time, gasoline, oil, insurance and wear and tear on the truck.

a) (9 points) Compute the annual costs of transportation and loading and unloading the housings for the following policies: (1) shipping one truck per week, (2) shipping one full truckload as often as needed, and (3) shipping three full truckloads as often as needed.

b) (3 points) For what reasons might the policy in (a) with the highest annual cost be more desirable from a systems point of view than the policy having the lowest annual cost?