Order Selection and Scheduling with Leadtime Flexibility
Kasarin Charnsirisaksul, Paul M. Griffin, Pınar Keskinocak
{kasarin, pgriffin, pinar} @ isye.gatech.edu
School of Industrial and Systems Engineering,
Georgia Institute of Technology
Atlanta, GA, 30332-0205, USA
Phone: 404-894-2431, Fax: 404-894-2301

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In this paper, we study integrated order selection and scheduling decisions, where the manufacturer has the flexibility to choose leadtimes. Our main goal is to provide a mechanism for coordinating order selection, leadtime, and scheduling decisions and to determine under what conditions leadtime flexibility is most useful for increasing the manufacturer’s profits. Through numerical analyses, we compare and contrast the benefits of leadtime flexibility and the flexibility to partially fulfill orders in different demand environments defined by the congestion level (or demand load), seasonality of the demand, and order size. We consider both the cases where the manufacturer has and does not have the flexibility to produce orders early before they are committed.

Key words: capacity management, scheduling, order selection, leadtime, flexibility.

1 Introduction

As business emphasis has moved from a cost focus to a value focus, companies can no longer solely compete on price, but must also provide value to their customers through other “quality features.” Short and reliable leadtimes provide value by helping customers reduce uncertainty in their businesses, leading to lower inventory and more accurate production/distribution plans (Rodin, 2001; Sheridan, 1999; Teresko, 2000).

Guaranteeing leadtimes by promising to deliver products by agreed due-dates and giving discounts to the customers for missed due dates has become a competitive leverage for companies to attract customers. Examples are Triple X composites, Tracewell Systems Inc., and Austin Traumanns Steel, who offer 15%, 5%, and 10% discounts, respectively, to their customers as a percentage of the value of the products or materials not delivered by the due-dates. The discounts given to the customers can be thought of as tardiness penalties. For example, in the aerospace industry, a tardiness penalty as high as $1 million per day is imposed on subcontractors of aircraft components (Slotnick and Sobel, 2001). Similarly, in the Finnish forest industry, a vendor pays the buyer a delay penalty amounting to 1% of the purchase price for each week, up to a maximum of 10%, plus additional damages arising from the delay.

Leadtime/due-date decisions depend on several factors, such as the manufacturer’s capacity and the customers’ demands and due-date preferences. In fulfilling customer orders, the manufac-
turer can benefit from various types of flexibility, including the flexibility to (i) complete orders after the specified due-dates (*leadtime flexibility*), (ii) produce early before orders arrive or are committed (*inventory holding flexibility*), and (iii) fulfill only part of the ordered quantity (*partial fulfillment flexibility*). Leadtime decisions may also depend on the demand environment, such as demand seasonality and system congestion.

Intuitively, higher flexibility leads to higher profits. For example, leadtime flexibility (potentially) enables the manufacturer to increase profits by accepting more orders or choosing more profitable orders that could not be completed otherwise. Similarly, the flexibility to produce early and hold inventory provides the manufacturer with the ability to better utilize her capacity and provide customers with shorter leadtimes. With leadtime or inventory holding flexibility, the manufacturer must tradeoff the benefits of completing orders late or early and the associated costs, such as tardiness and holding costs.

In this paper, we model simultaneous order selection, leadtime, and scheduling decisions when each customer has a preferred and latest acceptable due-date. The manufacturer incurs tardiness penalties for orders that cannot be completed before the preferred due-dates. In addition, we consider the cases where the manufacturer has the flexibility to produce early and hold inventory. Through a numerical study, we develop useful insights regarding the benefit of leadtime flexibility in different demand environments. We show that the magnitude of the benefit of leadtime flexibility (in terms of increasing profits) depends on various factors, such as demand load and seasonality, order size, and the flexibility to hold inventory. Understanding the benefit of leadtime flexibility in different demand and production environments is important for a manufacturer in designing effective due-date setting/negotiation policies.

The organization of this paper is as follows. We discuss the related literature in leadtime and scheduling decisions in Section 2. We define the problem and present a model and a solution approach in Section 3. In Section 4, we present a numerical study and discuss insights drawn from the study. Finally, Section 5 provides conclusions and future research directions.
2 Literature Review

Previous research most closely related to ours falls into four categories: (i) scheduling, (ii) due-date management, and (iii) manufacturing flexibility.

Most research in scheduling either ignores due-dates or assumes that due-dates are set \textit{a-priori} and are an input to the problem. For example, marketing often quotes due-dates at the time of order acceptance, and manufacturing attempts to satisfy those due-dates according to certain performance measures such as minimizing makespan or tardiness. Our research is related to the scheduling literature that considers time-windows (release times and due-dates). Examples include Abdul-Razaq et al. (1990), Crauwels et al. (1998), Holsenback and Russell (1992), Potts and Van Wassenhove (1985;1991), and Russell and Holsenback (1997), who study the case where all jobs have common release dates; and Akturk and Ozdemir (2000), Chu (1992), Hariri and Potts (1983), and Rinnooy Kan (1976), who study the case where jobs may have different release dates. A survey of the scheduling research can be found in Cheng and Gupta (1989) and Koulamas (1994).

In contrast to the traditional scheduling literature, due-date management literature studies situations where due-dates are set endogenously. In this case, the goal is to find a due-date management policy, which is a combination of due-date setting and scheduling policies. The objective is usually to minimize the total or average cost, which may be a function of earliness, tardiness, or the number of early/tardy orders (Bertrand, 1983; Conway, 1981; Cheng, 1984; Eilon and Chowdhury, 1976; Seidmann and Smith, 1981; Seidmann et al., 1981; Weeks, 1979), or to minimize the average weighted quoted due-date (or leadtime) subject to a service level constraint on the proportion of jobs completed on time or on the average job tardiness (Baker and Betrand, 1981, 1982; Bookbinder and Noor, 1985; Wein, 1991; Spearman and Zhang, 1999). See Keskinocak and Tayur (2003) for a review of due-date management literature.

Most of the research in scheduling and due-date management ignores the impact of completion times or quoted due-dates on the customers’ decisions to place the orders. In particular, most of the scheduling literature assumes that all orders must be processed and the due-date management literature assumes that the customers accept the quoted due-dates however late they are. Among the literature in scheduling and due-date management, our research is most closely associated with
the work that considers order acceptance decisions. Scheduling literature that considers order acceptance decisions includes Arkin and Silverberg (1987), Chuzhoy and Ostrovsky (2002), Hall and Magazine (1994), Keskinocak et al. (2001), Liao (1992), Snoek (2002), Wester et al. (1992), Woeginger (1994). In these papers, the objective is to maximize the total profit from accepted orders subject to satisfying customer specified deadlines (latest acceptable due-dates). Most of these papers (except Keskinocak et al., 2001) assume that customers are indifferent as to when an order is completed (i.e., due-date indifferent) as long as it is within the specified deadline.

Papers in the due-date management literature that consider order acceptance decisions include Duenyas (1995) and Duenyas and Hopp (1995), where the probability of a customer placing an order is a decreasing function of the quoted due-date. The objective is to maximize the expected long-term average profit.

Most of the due-date management and scheduling research assumes that there is no limit on how late an order can be, while most of the research on order acceptance assumes “tight” time windows (i.e., an order is lost if it is not processed immediately at its arrival). The reality is somewhere in between these two extreme cases of unlimited versus no tolerance for waiting. In many manufacturing environments an increase in the waiting time usually decreases satisfaction, resulting in a decline in the revenue or an increase in cost (e.g., tardiness penalty). Our models capture this practical situation by differentiating between preferred and acceptable due-dates. We define the preferred due-date as the time period after which the customer’s satisfaction declines and the latest acceptable due-date as the date after which the customer does not accept the shipment. A tardiness penalty is incurred if an order is completed after the preferred due-date. Additionally, all of the papers discussed above assume that the release time and arrival time of each order are the same, i.e., orders cannot be produced early and held in inventory. In this research, we also consider the case where early production is allowed.

The focus of most due-date related research is on finding good due-date management policies. In contrast, our focus is on understanding when the flexibility to complete the orders after the customer specified due-dates is useful. We use an off-line model to study the impact of this leadtime flexibility. Clearly, using an online model would be more realistic. However, since an online model assumes that the information about an order is not available until the time of its arrival, the
performance of different systems would depend upon the heuristic due-date management policies. In the absence of the optimal policies, we would not be able to have a fair comparison of those systems.

Previous research on flexibility has focused on studying the flexibility of a manufacturer to adjust to the dynamic market environment, including volume, variety, process, and materials handling flexibility (Duclos et al., 2002; Sabri and Beamon, 2000; Goetschalckx et al., 2001; Voudouris, 1996). The objective is usually to maximize system flexibility or profit, or minimize costs while retaining a certain flexibility level. Another area of research on flexibility focuses on optimal investment in a flexible production resource that is capable of adjustments to multiple system states (e.g., Bish and Wang, 2002; Fine and Freund, 1990; Van Mieghem, 1998). While a considerable amount of research has been done on designing flexible manufacturing systems (i.e., supply-side flexibility), our focus is on studying flexibility based on demand-side factors such as customers’ preferences on leadtimes.

3 The Model

3.1 Problem definition

We consider a single-machine production system (e.g., a production line with a bottleneck and no buffers between stations) capable of producing multiple products, where setup times are negligible. The manufacturer is a price taker, who accepts the selling prices as given by the market. Customer orders differ in their arrival (or commitment) times, demanded quantities (expressed in the amount of production resources required), preferred and latest acceptable due-dates, unit production, holding, and tardiness costs, and the potential revenues generated per unit resource required.

The manufacturer has the option to accept or reject orders. She decides which orders to accept and when to produce the orders, which in turn affects the due-dates. The customer does not place an order if the manufacturer cannot complete the order by the latest acceptable due-date. Thus, an accepted order must be completed and shipped to the customer between the commitment time and the latest acceptable due-date specified by the customer. If an order is shipped after the preferred
due-date, it is considered late and incurs a tardiness penalty proportional to the number of periods and the quantity delayed. The tardiness penalty may include a discount offered to the customer, a penalty due to loss of goodwill, and the potential loss of future business.

We allow order preemption (i.e., different parts of an order can be produced at different times), however, we require the entire order be shipped at the same time. Hence, partially finished orders (work-in-process inventory) incur holding costs until they are completed. The restriction on shipping an order only after it is entirely completed is reasonable, for example, when the customer is responsible for the delivery cost and prefers to pay for a single truckload shipment rather than multiple less-than-truckload shipments.

The manufacturer’s objective is to maximize the total profit subject to capacity and delivery time constraints. The total profit is the sum of revenues received from all accepted orders minus the production, holding, and tardiness costs.

Order information represents a forecast of the future arrivals of orders, which may include products with known characteristics or products whose characteristics are not known in advance but whose prices and capacity requirements are predictable. An order cannot be shipped to the customer before it is committed. In the case where product characteristics are predictable, the manufacturer has the flexibility to produce an order in advance and hold inventory until the time the customer commits to the order. In this case, production of an order can start as early as at the beginning of the planning horizon. The earliest possible shipping time of the order is equal to the commitment time. In the case where product characteristics are not predictable, the manufacturer does not have the flexibility to produce early, i.e., she does not have the flexibility to hold inventory. In this case, the earliest start time for the order is at its commitment time. An example of this type of production is in a printing business. The manufacturer can forecast the demand and production capacity requirements for different types of jobs, such as brochures, catalogs, or calendars. However, the manufacturer does not know the product details, i.e., job specifications, until they are actually committed. We consider both of these cases.

We model the problem of order acceptance, due-date setting, and scheduling, in the presence of preferred and latest acceptable due-dates, order preemption, and aggregated shipments as a mixed integer linear program. This problem is NP-hard; the proof of the problem’s complexity and special
cases that are solvable in polynomial time are presented in Charnsirisakskul (2003).

3.2 Notation

Sets:

$T$: set of time periods in the planning horizon

$O$: set of customers/orders

$O(t)$: set of orders with latest acceptable due-date in or after period $t$

Parameters:

$d_i$: order size, defined in units of capacity required for order $i$

$p_i$: unit price (or revenue per unit) of order $i$

$a_i$: unit tardiness penalty per period of order $i$

$e_i, f_i, l_i$: commitment time, preferred due-date, and latest acceptable due-date of order $i$

$h^i_t$: unit holding cost of order $i$ from the end of period $t$ to the beginning of period $t + 1$

$h^i_{t,k}$: cumulative holding cost per unit of order $i$ that is produced in period $t$ and delivered in period $k = \sum_{j=1}^{k-1} h^i_j$.

$c^i_t$: unit production cost of order $i$ in period $t$

$K_t$: units of production capacity available in period $t$

Unit costs and quantity are measured per unit of production capacity.

Order $i$ has commitment time at the beginning of period $e_i$, requires $d_i$ units of capacity, and yields a revenue of $p_i$ dollars per unit capacity. An accepted order must be shipped between the order commitment time and latest acceptable due-date. An order shipped within the preferred time window $[e_i, f_i]$ is considered on-time, while an order shipped within the leadtime window $[f_i+1, l_i]$ is considered late and incurs a tardiness penalty of $a_i$ per unit per period delay.

3.3 Mixed integer programming formulation

Decision Variables:

$x^i_{tk} = $ units of capacity in period $t$ used to produce order $i$ that is delivered to the customer at the end of period $k$. ($t = 1, .., k$, $k = e_i, .., l_i$, $i \in O$)

$I^i_k = 1$, if order $i$ is delivered at the end of period $k$; 0, otherwise. ($k = e_i, .., l_i$ $i \in O$)
Objectives Function:

Maximize Total profit

\[
= \sum_{i \in O} p_i \sum_{k = e_i}^{l_i} \sum_{t = 1}^{k} x_{ik}^t - \sum_{i \in O} \sum_{k = e_i}^{l_i} \sum_{t = 1}^{k} e_i^t x_{ik}^t - \sum_{i \in O} a_i \sum_{k = f_i + 1}^{l_i} (k - f_i) \sum_{t = 1}^{k} x_{ik}^t - \sum_{i \in O} \sum_{k = e_i}^{l_i} \sum_{t = 1}^{k-1} h_{t,k}^i x_{ik}^t
\]  

Constraints:

Demand Constraints:  
\[\sum_{k = e_i}^{l_i} L_i^k \leq 1 \quad \forall i \in O \]  
\[\sum_{t = 1}^{k} x_{ik}^t = d_i I_i^k \quad \forall i \in O, k \in [e_i, l_i] \]  

Capacity Constraints: 
\[\sum_{i \in O(t) k = \text{max}(e_i, t)}^{l_i} x_{ik}^t \leq K_t \quad \forall t \in T \]  

Variable Constraints: 
\[x_{ik}^t \geq 0 \quad \forall t \in T, \quad k \in [e_i, l_i], i \in O \]  
\[I_i^k \in \{0, 1\} \quad \forall k \in [e_i, l_i], i \in O \]

The four components of the objective function correspond to the total revenue, production cost, tardiness penalty, and holding cost, respectively. Constraints (2) ensure that if an order is accepted, it is aggregated into a single shipment within the time window specified by the customer. Constraints (3) ensure that an accepted order is fulfilled entirely. Constraints (4) ensure that the capacity in each period is not exceeded. Constraints (5) are non-negativity constraints and constraints (6) specify the binary variables.

The model can be modified to incorporate the release time of each order by setting the holding costs \(h_i^t\) to infinity, or equivalently the production variables \(x_{ik}^t\) to zero, for all periods \(t\) before the release time. Note that when early production is not allowed, as in the case with no inventory flexibility, the release time of each order is equal to the commitment time. To incorporate the case with partial fulfillment flexibility, the model can be easily modified by changing the equality in the demand constraints (3) to inequality (\(\leq\)).

Although in the numerical study we require an entire order to be shipped at the same time, the above model can be easily modified to accommodate the case where disaggregation is allowed as follows. The variables \(x_{ik}^t\) and \(I_i^k\) are replaced by \(x_i^t\) (nonnegative) and \(I_i\) (binary), where \(x_i^t\) is the
amount of capacity used to produce order $i$ in period $t$ and $I_i$ is the binary order acceptance variable, equal to 1 if order $i$ is accepted and 0 otherwise. Assuming that the quantity produced (in units of capacity consumed) before the commitment time $e_i$ is delivered in $e_i$ and the quantity produced in $[e_i,l_i]$ is delivered immediately after its completion in each period, the objective function (1) is replaced by

$$
\sum_{i \in O} p_i \sum_{t=1}^{l_i} x^i_t - \sum_{i \in O} \sum_{t=1}^{l_i} c^i_t x^i_t - \sum_{i \in O} a_i \sum_{t=f_i+1}^{l_i} (t-f_i)x^i_t - \sum_{i \in O} \sum_{t=1}^{e_i-1} h^i_{t,e_i-1}x^i_t.
$$

Constraints (2) are removed. Constraints (3) are replaced by $\sum_{t=1}^{l_i} x^i_t = d_i I_i$, $\forall i \in O$. The term $\sum_{k=\max\{e_i,t\}}^{l_i} x^i_{tk}$ in constraints (4) is replaced by $x^i_t$.

4 A Numerical Study

A manufacturer’s order acceptance, scheduling, and due-date decisions and profitability depend on the flexibility she has in fulfilling orders, which comes partly from negotiations with the customers. In a typical manufacturing environment, the negotiation process usually takes place before orders are accepted and the production plan is executed. In such cases, analyzing the benefit of different leadtime flexibility levels based on currently available demand information or forecasts can provide insights to the manufacturer regarding the level of flexibility to negotiate for future orders. In reality, offering longer leadtimes (through higher leadtime flexibility) may result in additional costs to the manufacturer, such as the loss of goodwill or a decrease in the probability of the customer placing an order. Such costs can be incorporated into the analysis by considering a fixed cost of leadtime flexibility proportional to $R$.

In order to develop an efficient negotiation policy, the manufacturer must have a thorough understanding of the benefits of different types of flexibility that she could negotiate for. The objective of this numerical study is to develop insights that serve as guidelines for manufacturers in due-date negotiations. We compare the benefits of leadtime flexibility and partial fulfillment flexibility in different environments to answer questions such as: In which environments is the flexibility important? How much or what type of flexibility is required to significantly benefit the manufacturer?
4.1 Experimental design

The manufacturing environment can be described by several factors such as demand load, order size, and seasonality of the demand. In order to test how the benefit of flexibility depends on these conditions, we simulate different demand environments defined by the factors above. We use a 90-period forecast horizon. Note that the production period can assume any time unit. In our numerical study, a production period represents 4 days and the 90-period forecast horizon is approximately one year. For simplicity, we assume that the manufacturer has one unit of capacity per period (since the planning period can be discretized and other parameters scaled accordingly, there is no loss of generality).

Demand load dictates how congested the system is and is defined as the expected ratio of the total requested amount of production capacity over the total available capacity during the forecast horizon. Four levels of demand load are included in the study: low (0.75), moderate (1.0), high (1.5), and extremely high (2.0). The total number of order arrivals generated during the forecast horizon is equal to: \( \text{demand load} \times \frac{\text{the length of the forecast horizon}}{\text{the average order size}} \).

To test the impact of order size on the benefit of flexibility, we used two distributions to model small \( (d_i \sim \text{uniform}[2,6]) \) and large order sizes \( (d_i \sim \text{uniform}[8,12]) \). Four different distributions of commitment times are tested: uniform \( (U: e_i \sim \text{uniform}[1,90]) \), high number of order commitments near the beginning \( (LT: e_i \sim \text{triangular}[1,20,90]) \), the middle \( (MT: e_i \sim \text{triangular}[1,45,90]) \), and the end \( (RT: e_i \sim \text{triangular}[1,70,90]) \) of the forecast horizon. The processing time requirement for each order \( (pt_i) \) is equal to the order size (in units of capacity required) divided by the production capacity per period. Since we assume unit capacity per period, the processing time requirement for each order is equal to the order size (i.e., \( pt_i = d_i \)).

The length of the planning horizon is set to the maximum value of the latest acceptable due-date (i.e., \( |T| = \max_i \{ l_i \} \)). To illustrate the benefit of leadtime flexibility excluding the effect of leadtime preferences among different orders/customers, we assume that the preferred due-date of each order is equal to the commitment time plus the processing time \( (f_i = e_i + pt_i) \). The latest acceptable due-date \( (l_i = f_i + R \times pt_i) \) is equal to the preferred due-date plus the leadtime window, which is proportional to leadtime flexibility factor \( (R) \) and the processing time \( (pt_i) \). Leadtime flexibility factor \( R \) is the ratio of the length of the leadtime window over the processing time of an
order. In each experiment, we assume that $R$ is the same for all orders.

Unit price (in dollars) of each order is generated from a uniform[1000,2000] distribution. Unit holding and tardiness costs are 0.5% and 2% of the selling price per period, respectively. The unit production cost is zero and prices, unit costs, and order sizes have integer values.

We simulated a complete combination of all factor levels (32 environments), each with nine levels of leadtime flexibility: $R \in \{0, 0.3, 0.6, 1, 2, 3, 4, 5, 6\}$. For each scenario (environment and $R$), we generated five replications of problem instances and solved the corresponding mixed integer programs using CPLEX7.0. The time spent to solve an instance (Pentium III, 500 MHz) of the mixed integer program in the experiments varied from less than 1 second to more than 15,000 seconds, with an average of 2500 seconds. The time used to solve the instances tends to be shorter when order commitment times are uniformly distributed than when the commitment times exhibit seasonality, and tends to increase as the order size, the demand load, and leadtime flexibility increase.

4.2 Analysis of numerical results

In this section we discuss the numerical results obtained from the simulations.\(^1\)

4.2.1 Preliminary analysis

We consider four different combinations of inventory and partial fulfillment flexibility: no inventory and no partial fulfillment (NOINV-NOPF), inventory and no partial fulfillment (INV-NOPF), no inventory but with partial fulfillment (NOINV-PF), and both inventory and partial fulfillment (INV-PF). For each of these combinations, Figure 1 shows the total average percentage profit increase due to different levels of leadtime flexibility, compared to the base case of NOINV-NOPF with $R = 0$. From Figure 1, for all combinations we observe that leadtime flexibility is useful, however, it exhibits diminishing marginal returns. The benefit of partial fulfillment flexibility is relatively high under NOINV for low values of $R$, however, the benefits are smaller under INV or higher values of $R$.

An interesting question is which of these three types of flexibility, namely leadtime, partial

\(^1\)For detailed numerical and statistical test results, please contact any of the authors.
fulfillment, and inventory flexibility, is most useful. For any level of leadtime flexibility, the profits under INV-NOPF are higher than the profits under NOINV-PF. This suggests that if the manufacturer has to choose between inventory and partial fulfillment flexibilities, she would prefer the former. In comparing the benefits of partial fulfillment and leadtime flexibilities, the profit increase under NOINV-NOPF, $R \geq 0.6$ is higher than the profit increase under NOINV-PF, $R = 0$, suggesting that a leadtime flexibility of $R \geq 0.6$ is more useful than partial fulfillment flexibility under NOINV (Figure 1). Similarly, the profit increase under INV-NOPF, $R \geq 0.3$ is higher than the profit increase under INV-PF, $R = 0$, suggesting that a leadtime flexibility of $R \geq 0.3$ is more useful than partial fulfillment flexibility under INV. These observations suggest that even a small amount of leadtime flexibility is more useful than partial fulfillment flexibility. In comparing the benefits of inventory and leadtime flexibility, we see that the percentage profit increase due to inventory flexibility only (INV-NOPF, $R = 0$) is higher than the profit increase due to the maximum tested leadtime flexibility (NOINV-NOPF, $R = 6$), suggesting that inventory flexibility is more useful than leadtime flexibility. In summary, the three types of flexibility are ranked as inventory, leadtime, and partial fulfillment in decreasing order of their usefulness.

To better understand the impact of leadtime flexibility on profit, costs, and accepted orders, in Table 1 we summarize the (average) decomposition of the potential revenue (the total revenue that would be achieved if all orders were accepted and completed by the preferred due-dates) into profit, holding cost, tardiness cost, and lost revenue at different flexibility levels under NOPF. Lost revenue is the potential revenue minus the actual revenue received. From Table 1 we see that as leadtime flexibility increases, the percentages of holding cost, lost revenue, and rejected orders decrease, while the percentages of the profit and the tardiness cost increase. Intuitively, the increase in profits due to higher leadtime flexibility is primarily due to the manufacturer’s ability to accept more profitable orders. The gradual decrease in lost revenue, which parallels the decrease in the percentage of rejected orders, as leadtime flexibility increases supports this hypothesis. Although the decrease in lost revenue (equivalently the increase in the revenue received) is accompanied by an increase in tardiness cost, the increase in tardiness cost is smaller than the corresponding increase in revenue.

Next, we analyze the marginal benefit of (i) leadtime flexibility under each of the four combi-
nations of inventory and partial fulfillment flexibility (Table 2) and (ii) partial fulfillment flexibility under INV and NOINV for various levels of leadtime flexibility (Table 3).

**Leadtime flexibility** Table 2 shows the (average) percentage profit increase for different $R$ values compared to $R = 0$ for each of the four combinations of inventory and partial fulfillment flexibilities. For instance, the value of 4.04 under INV-NOPF means that there was a 4.04% increase in profits by having a leadtime flexibility of $R = 1$ compared to $R = 0$ in INV-NOPF. Since this value is a marginal benefit, it is over and above the percentage profit increase due to inventory flexibility alone.

The analysis of the results in Table 2 show that the additional benefit due to leadtime flexibility is statistically significant\(^2\) even when both inventory and partial fulfillment flexibilities are available. The additional benefit of leadtime flexibility is even more pronounced when either inventory or partial fulfillment flexibility or both are not available. Pairwise *t*-tests also show that the (percentage) profit difference between INV and NOINV decreases significantly as leadtime flexibility ($R$) increases. These results suggest that leadtime flexibility and inventory holding flexibility are partial substitutes.

**Partial fulfillment flexibility** Table 3 shows the marginal benefit of partial fulfillment flexibility. In this Table, the average percentage profit increase of NOINV-PF over NOINV-NOPF and INV-PF over NOINV-NOPF is given for all values of $R$. The additional benefit due to partial fulfillment flexibility is statistically significant for all cases though there are diminishing marginal returns as $R$ increases. Partial fulfillment flexibility allows a manufacturer to satisfy a large fraction of an otherwise rejected order. Since inventory flexibility allows the manufacturer to produce orders before they are committed and, thus, accept more orders (entirely), we see a higher benefit of partial fulfillment flexibility under NOINV-PF. As can be seen from Table 3 even the smallest increase of 1.40% for NOINV-PF over NOINV-NOPF ($R = 6$) is greater than the largest increase of 1.24% for INV-PF over INV-NOPF($R = 0$).

In the following sections, we discuss how the benefits of leadtime and partial fulfillment flexibility depend on different environmental factors. Note that in order to distinguish between the

\(^2\)For detailed *t*-test results, please contact any of the authors.
effects of the factors on the benefits of leadtime flexibility and partial fulfillment flexibility, we do not consider the cases where both types of flexibility are available simultaneously.

4.2.2 The impact of environmental factors on the benefit of leadtime flexibility

In this section we consider leadtime and partial fulfillment flexibilities separately to analyze the impact of the environmental factors. To test statistically the significance of the effect of each factor, we performed an Analysis Of Variance (ANOVA) using a 10% significance level. We refer to the effects of order commitment time distribution, demand load, and order size as the main factor effects and refer to the combined effects between two or more factors as the interaction effects. The percentage profit increase due to leadtime flexibility under NOPF is summarized in Table 4 for various demand environments. ANOVA results show that all main factors have significant impact on the benefit of leadtime flexibility at all flexibility levels ($R$) in INV. For NOINV, the significance of different factors depends on the level of leadtime flexibility. At low flexibility level ($R = 0.3$), no main effects are significant, but the combined (interaction) effect between order size and demand load is significant. At $R = 0.6$ and 1, order size is the only significant factor. At $R = 2$, order size and demand load are significant. At $R \geq 3$, order size and commitment time distribution are significant.

The impact of demand load   When the demand load is high, a manufacturer with insufficient production capacity to accept and complete all orders on time must reject several orders. As leadtime flexibility increases, the manufacturer has more flexibility to adjust the due-dates and accept more orders. Thus, we expect leadtime flexibility to be more useful when demand load is high. Our observations in the case of INV confirm this intuition. From Table 4, we observe that the benefit of leadtime flexibility (at any flexibility level) increases as demand load increases. The highest profit increase is achieved at extremely high load (2.0) except when $R = 0.6$, where the benefit is highest at high load (1.5). We also observe that under INV the diminishing marginal returns of leadtime flexibility are more pronounced at lower demand loads.

Contrary to our intuition, for sufficiently high leadtime flexibility ($R \geq 1$), the highest benefit under NOINV is achieved when the demand load is either low or moderate. (However, the effect
of the demand load on the benefit of leadtime flexibility is not statistically significant except at $R = 2$.) One possible explanation for this observation is that under INV, when the demand load is low to moderate, the manufacturer can complete most of the orders by holding inventory, i.e., the additional benefit due to leadtime flexibility is relatively small. However, under NOINV, the manufacturer can significantly benefit from leadtime flexibility even for low to moderate demand load. For high demand load, since the manufacturer already uses a significant portion of its capacity and captures most of the demand, the benefit of leadtime flexibility may not be as high as in the case of low or moderate demand load.

The impact of commitment time distribution (demand seasonality) We observe from Table 4 that the benefit of leadtime flexibility is smallest when order commitment times are uniformly distributed throughout the forecast horizon both in the cases of INV and NOINV (at any flexibility level, except for $R=0.6$ in NOINV). Intuitively, when order commitment times are evenly distributed over the horizon, the manufacturer can complete an order that is being processed before a new attractive order is committed. Thus, having higher leadtime flexibility does not add a significant benefit in terms of accepting more profitable orders. On the other hand, in a highly seasonal demand situation most of the profitable orders are committed during the same periods, making it harder for the manufacturer to accept and complete many orders without leadtime flexibility. In such cases, having higher leadtime flexibility allows the manufacturer to satisfy more orders and achieve higher profits.

Under INV, the benefit of leadtime flexibility (at any flexibility level) is highest when commitment times of most orders are near the beginning of the forecast horizon (LT), followed by high number of commitments near the middle (MT), and the end of the horizon (RT). This observation can be explained as follows. Under RT, the manufacturer can produce several orders during earlier periods where the congestion is low and hold inventory until the orders are committed. Therefore, the additional benefit of leadtime flexibility is relatively small. Under LT, however, since the congestion is towards the beginning of the planning horizon, the flexibility to produce early is not very beneficial and therefore the additional benefit of leadtime flexibility is relatively high. Under NOINV, for $R \geq 1$, the pattern is the reverse of what we observe under INV, namely, the benefit
of leadtime flexibility is highest under RT, followed by MT, and LT.

To understand the impact of the commitment time distribution on the benefit of leadtime flexibility under INV and NOINV, we compute for each $R$ the percentage difference between the largest and the smallest profit increase due to leadtime flexibility across all commitment time patterns. For example, under INV, $R = 0$, the percentage difference is $\frac{2.54 - 1.24}{1.24} \times 100 = 104.8\%$. These percentage differences are 104.8, 82.4, 54.0, 64.2, 64.9, 65.2, 64.2, 65.9 under INV and 22.2, 1.1, 9.7, 22.4, 24.2, 25.4, 25.9 under NOINV, for $R = 0, 0.3, 0.6, 1, 2, 3, 4, 5, 6$, respectively. A pairwise comparison of these percentages for each $R$ value shows that the difference between the maximum and the minimum benefit is higher under INV than NOINV. Hence, our results suggest that the impact of the commitment time distribution on the benefit of leadtime flexibility is higher under INV than NOINV.

**The impact of order size** Intuitively, we would expect the number of rejected orders to be higher when orders are large, since large orders require more time periods to complete (provided that the demand load is adequately high such that some orders are rejected). Accordingly, we would expect leadtime flexibility to be more useful in the environments with large orders. From the numerical results (Table 4), we observe that this intuition holds under both INV and NOINV, except for the low flexibility level ($R = 0.3$) in NOINV. In the $R = 0.3$ case, however, the result is not statistically significant.

**4.2.3 The impact of environmental factors on the benefit of partial fulfillment flexibility**

In this section, we discuss how the benefit of partial fulfillment depends on the environmental factors for the cases of INV and NOINV. Table 5 summarizes the percentage profit increase due to partial fulfillment flexibility under INV and NOINV when $R = 0$.

**The impact of demand load** We observe that the benefit of partial fulfillment flexibility depends on the demand load as follows. For INV, the benefit of partial fulfillment is significant in all environments with moderate to extremely high load. When the demand load is low, partial fulfillment is only useful when orders are large and commitment time distribution is LT. For NOINV,
the benefit of partial fulfillment is significant at all load levels, though the benefit under low and moderate demand loads is significantly smaller than under high and extremely high demand loads. As expected, for both INV and NOINV, partial fulfillment flexibility is more useful for higher levels of demand load, though clearly as demand load gets high enough, the benefit of partial fulfillment will not increase further since all capacity will be utilized.

**The impact of commitment time distribution** Under INV, the benefit of partial fulfillment is highest on average when order commitment time distribution is LT, followed by MT, RT, and uniform. Note that this is very similar to what we observed in Section 4.2.2 for the benefit of lead time flexibility. Intuitively, when most orders have early commitment times, inventory flexibility is not very beneficial, and hence, the benefit of leadtime or partial fulfillment flexibility is more pronounced. Under NOINV, the benefit of partial fulfillment flexibility is highest when the commitment time distribution is RT, followed by LT, MT, and uniform.

**The impact of order size** On average, the benefit of partial fulfillment is highest when orders are large under both INV and NONV. In congested systems, it is harder to accept and fully complete large orders than to complete small orders fully. Thus, partial fulfillment flexibility is especially beneficial when orders are large.

## 5 Conclusions

In this paper, we consider simultaneous order selection, scheduling, and due-date decisions and provide a framework for analyzing the benefits of leadtime and partial fulfillment flexibility. Through numerical analyses, we develop insights on how leadtime flexibility benefits the manufacturer in various demand and production environments.

Our results suggest that leadtime flexibility is beneficial and more so than partial fulfillment flexibility. This is true both when the manufacturer has (INV) and does not have (NOINV) inventory holding flexibility. The benefit of higher leadtime flexibility is higher in NOINV though exhibits diminishing marginal returns under both cases. While the benefit in NOINV is purely attributed to the capability to accept more profitable orders, the benefit in INV may also be attributed to the
tradeoffs between the options to produce orders late or early.

The magnitude of the benefit of leadtime flexibility depends on several factors, including demand load, seasonality of demand, and order size. The impact of these factors becomes consistent when leadtime flexibility reaches a moderate level and can be summarized as follows. In INV, leadtime flexibility is more useful when the demand load is high. On the contrary, in NOINV the benefit is higher when the load is low or moderate. In both cases, leadtime flexibility is more useful when order commitment times exhibit some seasonality. In INV, the benefit increases as the periods of high order commitments are closer to the beginning of the forecast horizon. The converse is true for NOINV. For both cases, leadtime flexibility is more useful when orders are large.

In our model, demand is deterministic, setup costs and setup times are negligible, and order preemption is allowed. Future research directions include alternate models that relax these assumptions, such as considering probabilistic demand, order setup times, and no order preemptions. While this paper, like most of the research in the literature, assumes that prices are exogenous (i.e., the manufacturer takes prices as given by the market), the case where the manufacturer has some power to set prices and incorporate price decisions into production decisions is an interesting area to explore. We have taken a first attempt in this direction in Charnsirisakskul et al. (2003).
Figure 1: Average percentage profit increase over the base case (NOINV-NOPF with $R = 0$) for the combinations of inventory and partial fulfillment flexibility at different levels of leadtime flexibility.
### Table 1: Profit and costs as percentages of potential revenue, and rejected orders as a percentage of total orders for different levels of leadtime flexibility, under NOPF.

<table>
<thead>
<tr>
<th></th>
<th>Leadtime Flexibility ($R$)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.3</td>
<td>0.6</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>NOINV</td>
<td>74.34</td>
<td>76.64</td>
<td>77.28</td>
<td>77.88</td>
<td>78.27</td>
<td>78.30</td>
<td>78.32</td>
<td>78.33</td>
</tr>
<tr>
<td></td>
<td>3.05</td>
<td>2.91</td>
<td>2.81</td>
<td>2.79</td>
<td>2.69</td>
<td>2.60</td>
<td>2.58</td>
<td>2.56</td>
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<tr>
<td>INV</td>
<td>0.00</td>
<td>0.42</td>
<td>0.90</td>
<td>1.48</td>
<td>2.92</td>
<td>4.18</td>
<td>5.12</td>
<td>5.59</td>
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<tr>
<td></td>
<td>22.61</td>
<td>20.03</td>
<td>19.01</td>
<td>17.85</td>
<td>16.12</td>
<td>14.92</td>
<td>13.98</td>
<td>13.52</td>
</tr>
<tr>
<td></td>
<td>25.81</td>
<td>24.27</td>
<td>22.98</td>
<td>21.75</td>
<td>19.04</td>
<td>17.25</td>
<td>16.08</td>
<td>15.44</td>
</tr>
</tbody>
</table>

### Table 2: Average percentage profit increase due to leadtime flexibility over the base case. For each row, the base case is when there is no leadtime flexibility ($R = 0$) for that combination.

<table>
<thead>
<tr>
<th></th>
<th>Leadtime Flexibility ($R$)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.3</td>
<td>0.6</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>NOINV</td>
<td>12.76</td>
<td>19.87</td>
<td>24.24</td>
<td>30.33</td>
<td>33.11</td>
<td>33.88</td>
<td>34.24</td>
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<tr>
<td>INV</td>
<td>1.80</td>
<td>3.15</td>
<td>4.04</td>
<td>6.09</td>
<td>6.95</td>
<td>7.34</td>
<td>7.54</td>
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<tr>
<td>NOINV</td>
<td>5.94</td>
<td>9.07</td>
<td>11.33</td>
<td>15.25</td>
<td>16.81</td>
<td>17.36</td>
<td>17.68</td>
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<tr>
<td>INV</td>
<td>1.59</td>
<td>2.73</td>
<td>3.56</td>
<td>5.24</td>
<td>5.99</td>
<td>6.31</td>
<td>6.52</td>
</tr>
</tbody>
</table>

### Table 3: Average percentage profit increase due to partial fulfillment flexibility for different levels of leadtime flexibility under NOINV and INV (NOINV-PF compared to NOINV-NOPF and under INV-PF compared to INV-NOPF).

<table>
<thead>
<tr>
<th></th>
<th>Leadtime Flexibility ($R$)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.3</td>
<td>0.6</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>NOINV</td>
<td>15.78</td>
<td>8.78</td>
<td>5.35</td>
<td>3.75</td>
<td>2.38</td>
<td>1.60</td>
<td>1.49</td>
<td>1.50</td>
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<tr>
<td>INV</td>
<td>1.24</td>
<td>1.03</td>
<td>0.83</td>
<td>0.77</td>
<td>0.43</td>
<td>0.33</td>
<td>0.27</td>
<td>0.28</td>
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20
<table>
<thead>
<tr>
<th>Factor</th>
<th>Factor level</th>
<th>Leadtime Flexibility ($R$)</th>
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<tbody>
<tr>
<td>Low</td>
<td>0.40</td>
<td>1.00 1.03 1.11 1.11 1.11</td>
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<tr>
<td>Moderate</td>
<td>1.83</td>
<td>3.40 4.79 6.71 6.98 7.04</td>
</tr>
<tr>
<td>High</td>
<td>2.44</td>
<td>4.15 5.16 8.23 9.84 10.48</td>
</tr>
<tr>
<td>Ext. high</td>
<td>2.53</td>
<td>4.06 5.17 8.30 9.86 10.74</td>
</tr>
<tr>
<td>Commitment</td>
<td>U</td>
<td>1.24 2.22 3.24 4.61 5.16</td>
</tr>
<tr>
<td></td>
<td>LT</td>
<td>2.54 4.05 4.99 7.57 8.51</td>
</tr>
<tr>
<td></td>
<td>MT</td>
<td>1.82 3.38 4.00 6.50 7.43</td>
</tr>
<tr>
<td></td>
<td>RT</td>
<td>1.60 2.97 3.91 5.67 6.69</td>
</tr>
<tr>
<td>Order size</td>
<td>Small</td>
<td>1.29 2.34 3.02 4.78 5.76</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>2.31 3.96 5.05 7.39 8.13</td>
</tr>
</tbody>
</table>

Table 4: Percentage profit increase from the case with no leadtime flexibility ($R = 0$), due to different levels of leadtime flexibility under NOPF.
<table>
<thead>
<tr>
<th>Factor</th>
<th>Factor level</th>
<th>INV-PF over INV-NOPF</th>
<th>NOINV-PF over NOINV-NOPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand load</td>
<td>Low</td>
<td>0.37</td>
<td>14.93</td>
</tr>
<tr>
<td></td>
<td>Moderate</td>
<td>1.57</td>
<td>14.43</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>1.49</td>
<td>17.24</td>
</tr>
<tr>
<td></td>
<td>Ext. high</td>
<td>1.52</td>
<td>16.50</td>
</tr>
<tr>
<td>Commitment time</td>
<td>U</td>
<td>0.95</td>
<td>14.14</td>
</tr>
<tr>
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<td>LT</td>
<td>1.91</td>
<td>16.26</td>
</tr>
<tr>
<td></td>
<td>MT</td>
<td>1.12</td>
<td>15.74</td>
</tr>
<tr>
<td></td>
<td>RT</td>
<td>0.96</td>
<td>16.96</td>
</tr>
<tr>
<td>Order size</td>
<td>Small</td>
<td>0.34</td>
<td>11.64</td>
</tr>
<tr>
<td></td>
<td>Large</td>
<td>2.13</td>
<td>19.91</td>
</tr>
</tbody>
</table>

Table 5: Percentage profit increase for INV-PF over INV-NOPF and NOINV-PF over NOINV-NOPF when there is no leadtime flexibility \((R = 0)\).
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**Biographies**

Kasarin Charnsirisakskul received her undergraduate degree in Industrial Engineering from Sirindhorn Institute of Technology, Thammasat University, Thailand, in 1997. She received her M.S. and Ph.D. degrees in Industrial Engineering from the School of Industrial and Systems Engineering at Georgia Institute of Technology in 1998 and 2003, respectively.

Paul M. Griffin is an Associate Professor in the School of Industrial and Systems Engineering at the Georgia Institute of Technology. He received his Ph.D. in Industrial Engineering from Texas A&M University. His teaching and research interests are in manufacturing systems, logistics systems and economic decisions analysis.

Pınar Keskinocak is the Coca Cola Assistant Professor in the School of Industrial and Systems Engineering at Georgia Institute of Technology. Before joining Georgia Tech, she worked at IBM T.J. Watson Research Center, Yorktown Heights, New York. She holds a Ph.D. degree in Operations Research from Carnegie Mellon University. Her research focuses on supply chain management, with an emphasis on lead time and pricing decisions. She is also interested in the applications of optimization techniques in real world environments.