Because of historically high fuel prices, the trucking industry’s operating expenses are higher than ever, and thus profit margins lower than ever. To cut costs, the trucking industry is searching for and exploring new ideas. We investigate the potential of collaborative opportunities in truckload transportation. When carriers serve transportation requests from many shippers, they may be able to reduce their repositioning costs by exchanging one or more of them. We develop optimization models to determine the maximum benefit that can be derived from collaborating, and we develop various exchange mechanisms, differing in terms of information sharing requirements and side payment options, which allow carriers to realize some or all of the costs savings opportunities.

**Key words:** carrier collaboration, truckload shipping, lane covering problem, lane exchanges.

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1. **Introduction**

Trucking is the backbone of U.S. freight movement. According to the American Trucking Association (ATA), the trucking industry’s share of the total volume of freight transported in the United States was 68.9% in 2005. The US trucking industry produced an annual revenue of $623 billion by hauling 10.7 billion tons in 2005, which was 84.3% of the nation’s freight bill. Mostly because of historically high fuel prices, the trucking industry’s operating expenses are higher than ever. “Each penny increase in diesel costs the trucking industry $381 million over a full year” (ATA 2007). Eventually, these increased operating costs are reflected in the prices charged to the shippers.

We focus on the full truckload segment of the trucking industry. Razor-thin profit margins have forced full truckload carriers to search for ways to cut costs. One way to do so is through mergers and acquisitions. There are economies of scale, in the form of increased buying power and reduced marketing and administrative expenses, and economies of scope, in the form of reduced repositioning costs. When shipment density increases, due to a merger or acquisition, the likelihood of geographical synergies between shipments increases, which in turn will reduce costly empty
travel between consecutive shipments. However, mergers and acquisitions may be beyond reach in many situations. In that case, companies may consider collaboration as a means to achieve some economies of scale and scope.

Collaboration among full truckload carriers may result in significant cost savings. However, realizing the full potential of such a collaboration is challenging. Ideally, a centralized decision maker with complete information about all participating carriers determines the optimal assignment of shipment requests to the carriers and identifies the minimum cost routes for the carriers. Besides the fact that a complex optimization problem needs to be solved, companies may not be willing to share the necessary information. Trust is a central issue for selfish individuals in collaborative relationships, especially in horizontal collaborations where competing entities collaborate.

In case centralized decision making is not viable, alternative collaboration mechanisms need to be explored. That is the focus of our paper. We design and analyze relatively simple mechanisms that allow full truckload carriers to initiate and manage a collaboration. Because the carriers are guided by their own self-interests, any proposed mechanism to manage the collaboration’s activities has to yield collectively and individually desirable solutions. Also, because of the trust issues associated with horizontal collaborations, the proposed mechanisms cannot rely on the availability of full information. The challenge is to design mechanisms that are simple, implementable, yet effective in terms of benefits to all participants. We will show that bilateral lane exchanges, where a lane represents a full truckload shipment request from an origin to a destination, satisfy all these requirements. We will also show that with more information sharing and with side payments, almost all the potential cost savings can be realized.

The remainder of the paper is organized as follows. In Section 2, we illustrate economies of scope in truckload transportation and discuss how collaboration may exploit economies of scope. In Section 3, we briefly review related work. In Section 4, we introduce and analyze the multi-carrier lane covering problem, which is the optimization problem that needs to be solved to determine the maximum benefits of collaborating. In Section 5, we present and investigate lane exchange mechanisms for different collaborative environments in terms of information sharing and side payment options.

2. Economies of Scope - An Example

We demonstrate the potential benefits of carrier collaboration with a simple example. Consider a network with four cities and two carriers A and B. We assume that the cost of traveling between two cities is the same for both carriers and, for simplicity, that there is no difference in cost between
traveling loaded or empty. We further assume that Carrier A has a contract in place to serve lane (3,2) and that Carrier B has a contract in place to serve lane (1,2). Figure 1 shows the relevant information.

![Figure 1 Cost information and existing contracts.](image)

Suppose that new shipment requests for lanes (2,4), (4,1), and (2,3) arrive and that they are assigned to the carriers by some procurement mechanism so that Carrier A gets lane (2,4) and Carrier B gets lane (4,1) and lane (2,3). We assume that these are not long term contractual agreements so can potentially be exchanged between the carriers. The minimum cost routes covering the lanes that have to be served for both carriers are presented in Figure 2, where a dashed line represents repositioning (or empty travel). The corresponding transportation costs are 3 for Carrier A and 4.73 for Carrier B.

If Carrier A and Carrier B collaborate and exchange lanes (2,4) and (2,3), they significantly reduce their repositioning costs and the transportation costs become 2 for Carrier A and 3.73 for Carrier B. In fact, this assignment of lanes to carriers and the accompanying routes is the best possible and would have been determined by a central decision maker.

3. Literature Review

In this section, we briefly review related work in the literature. The literature on truckload transportation procurement can be roughly divided into two streams: centralized and auction-based approaches.
Much of the research on centralized approaches considers only a single-carrier setting and focuses on optimizing the routing decisions for that carrier, i.e., finding continuous-move paths and tours covering the lanes and minimizing repositioning costs. Ergun et al. (2007b) introduce the Lane Covering Problem (LCP), i.e., the problem of finding a minimum-cost set of cycles covering a given set of lanes, and show that LCP is polynomially solvable. Ergun et al. (2007a) consider variants of LCP that include practically relevant side constraints, such as dispatch windows and driver restrictions, and show that these variants are NP-hard. Their proposed heuristics appear to be effective. Moore et al. (1991) develop a mixed integer programming model and a simulation tool to facilitate the centralized management of interstate truckload shipments, which involves selecting carriers and dispatching shipments.

The research on truckload transportation procurement using auction-based methods mostly focuses on the assignment of lanes from a single shipper to a set of carriers so as to minimize the total transportation cost. Combinatorial auctions are most suitable for transportation procurement auctions, because they allow the capturing of synergies between shipment requests through bundle bids. In this context, the shipper has to solve a winner determination problem to assign lanes to the carriers. Sheffi (2004) provides a survey on combinatorial auctions in procurement of truckload transportation services. He observes that solving large instances is quite challenging since real problems include thousands of lanes, dozens or hundreds of carriers, and millions of combination bids. See also De Vries and Vohra (2003), Caplice and Sheffi (2003), and Song and Regan (2003).
Elmaghraby and Keskinocak (2004) give an overview of the challenges in designing and implementing combinatorial auctions and presents a case study on how Home Depot utilizes combinatorial auctions in the procurement of transportation services. An et al. (2005) investigate how the participants in a combinatorial auction should select their bids as evaluating the bids for all possible bundles is not practical for both the bidders and the auctioneer. Ledyard et al. (2002) discuss the implementation of “combined-value auctions” for the purchasing of transportation services by Sears Logistics Services.

Our perspective in this paper is quite different, we are not interested in the transportation procurement process per se, i.e., in how shippers select carriers and assign lanes to carriers, but instead on how carriers can reduce cost by collaborating given the set of lanes that they have to serve. The goal is to design simple lane exchange processes to facilitate collaboration among carriers. We will measure the effectiveness of our simple exchange mechanisms by comparing the solutions they produce to a fully centralized setting. Furthermore, we will quantify the effects of more advanced mechanisms that require exchange of information and side payments.

4. The Multi-Carrier Lane Covering Problem
To assess the potential benefit of collaborating, the optimal assignment of lanes to carriers has to be determined and the optimal set of cycles covering the lanes assigned to each carrier needs to be found. We refer to this optimization problem as the Multi-Carrier Lane Covering Problem (MCLCP).

The total transportation costs break down into two components: lane covering costs and repositioning costs. The cost of covering a lane depends only on the carrier assigned to execute the associated shipment. Therefore, if the carriers have identical cost structures, the lane covering costs are independent of the assignment and routing decisions. On the other hand, the repositioning costs depend on the synergy among (or complementarity of) the set of lanes assigned to a carrier (irrespective of any differences in carrier cost structures), and thus on the assignment and routing decisions.

We assume that our starting point is a set of carriers each with two sets of lanes: lanes that have to be executed by the carrier and lanes that can be executed by any carrier. (The set of lanes that have to be executed by the carrier may be empty.) Of course only the lanes that can be executed by any carrier can be re-assigned during the optimization. We further assume that the cost of traversing an arc with a full truckload is independent of the direction along the arc and that the cost of repositioning along an arc is a percentage of the full truckload cost. These costs can be
different for the different carriers. There are no capacity restrictions; each carrier is able to handle all the assigned lanes. Finally, we observe that we ignore some relevant practical considerations, such as pickup and delivery windows, driver restrictions, etc.

Thus, MCLCP is defined on a complete graph $G = (N,A)$, where $N$ is the set of nodes $\{1, \ldots, n\}$ and $A$ is the set of arcs. Let $K$ be the set of carriers offering transportation services and let $c_{ij}^k$ denote the cost of traversing arc $(i,j)$ with a full truckload for Carrier $k$ ($k \in K$). The repositioning cost coefficient of Carrier $k$ is denoted by $\theta^k$ ($K \in K$), where $0 < \theta^k \leq 1$, and hence the repositioning cost along an arc $(i,j) \in A$ is equal to $\theta^k c_{ij}^k$ if traversed by Carrier $k$. The set of lanes that can be covered by any carrier is denoted by $L \subseteq A$ whereas the set of lanes that can only be covered by Carrier $k$ is denoted by $I_k \subseteq A$. The objective is to cover the lanes at minimum cost.

MCLCP can be formulated as an integer linear program as follows:

$$z_{LP} = \min \sum_{k \in K} \sum_{(i,j) \in L} c_{ij}^k x_{ij}^k + \sum_{k \in K} \theta^k \sum_{(i,j) \in A} c_{ij}^k z_{ij}^k$$

s.t. \[\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k + \sum_{j \in N} z_{ij}^k - \sum_{j \in N} z_{ji}^k = 0 \quad \forall i \in N, \forall k \in K\] (2)

\[\sum_{k \in K} x_{ij}^k \geq r_{ij} + \sum_{k \in K} I_{ij}^k \quad \forall (i,j) \in L\] (3)

\[x_{ij}^k \geq I_{ij}^k \quad \forall (i,j) \in I_k, \forall k \in K\] (4)

\[z_{ij}^k \geq 0 \quad \forall (i,j) \in A, \forall k \in K\] (5)

\[x_{ij}^k, z_{ij}^k \in Z.\] (6)

Variable $x_{ij}^k$ represents the number of times lane $(i,j) \in L$ is traversed with a full truckload by Carrier $k$ and variable $z_{ij}^k$ represents the number of times arc $(i,j) \in A$ is traversed empty (i.e., to reposition) by Carrier $k$. Furthermore, $I_{ij}^k$ represents the number of times lane $(i,j) \in L$ has to be traversed with a full truckload by Carrier $k$ and $r_{ij}$ represents the number of times lane $(i,j) \in L$ has to be traversed with a full truckload by any of the carriers. The objective is to minimize the sum of the lane covering costs of all the carriers plus the sum of the repositioning costs of all the carriers. Constraints (2) are flow balance constraints for all the nodes in the network and all the carriers offering service. Constraints (3) ensure that the transportation requirement of each lane is satisfied. Constraints (4) ensure that carrier specific requirements are satisfied. A solution to MCLCP gives the optimal assignment of the lanes to the carriers and the optimal routes for covering the lanes assigned to each carrier. The objective function represents the minimum cost for serving all shipment requests.

We have presented MCLCP as the optimization problem that the carriers need to solve to determine the most cost-effective way to serve their lane requests. However, it is important to note
that solving MCLCP is also equivalent to solving the shipper’s winner determination problem in the context of a fully expanded combinatorial transportation procurement auction. Hence all the results we present below regarding MCLCP are relevant in this combinatorial auction context as well.

Next, we examine the complexity of MCLCP. There are a few characteristics that impact the complexity: the number of carriers, whether or not carriers have sets of lanes that they have to serve, and whether or not the carriers have identical cost structures. We have the following set of results.

**Lemma 1.** When carriers have no sets of lanes that only they can serve and when their cost structures are identical, then MCLCP can be solved in polynomial time for any number of carriers.

**Proof** Follows from the fact that the unconstrained version of the single-carrier lane covering problem is polynomially solvable (Ergun et al. 2007b). □

**Theorem 1.** When there are three or more carriers with non-identical cost structures, then MCLCP is NP-hard.

**Proof** See the appendix. □

As MCLCP is NP-hard, the LP-relaxation for the formulation 2 - 6 may not yield an integral solution. Next, we present an observation concerning the integrality gap.

**Lemma 2.** The integrality gap for MCLCP is less than or equal to 100%.

**Proof** Let $z^*_{LP}$ be the objective function value of the LP-relaxation, then we have

$$z^*_{LP} \geq \sum_{k \in K} \min_{(i,j) \in L} c^k_{ij} r_{ij} + \sum_{k \in K} \sum_{(i,j) \in L} c^k_{ij} I^k_{ij},$$

because the right hand side represents the cost of traversing each lane in $L$ using a the minimum cost carrier with no repositioning. Furthermore, traversing each lane in $L$ with its lowest cost carrier option and returning back empty along that lane is an integral feasible solution to MCLCP. Because $\theta^k \leq 1$ for all $k \in K$, the objective function value of this feasible solution, say $z_{IP}$, satisfies

$$z_{IP} \leq 2 \{ \sum_{k \in K} \min_{(i,j) \in L} c^k_{ij} r_{ij} + \sum_{k \in K} \sum_{(i,j) \in L} c^k_{ij} I^k_{ij} \}.$$

Hence, the integrality gap is less than or equal to 100%. □

Because the size of instances of MCLCP encountered in different contexts may differ considerably and may sometimes be large, we have developed a few heuristic approaches for its solution and compared their performance to simply solving the integer programming (IP) formulation using a
commercial solver. The first heuristic approach relaxes the precision of the IP solver by specifying a relative optimality stopping criterion. The second heuristic exploits the structure of a solution to the LP-relaxation of 2 - 6 by fixing integral flows along cycles in the solution of the LP-relaxation at the root node (thus reducing the feasible solution space), before starting the branch-and-bound process. We refer the reader to Özener (2008) for the details of these approaches. The computational study to evaluate the performance of these heuristics uses randomly generated instances. The results demonstrate that integer programming with relaxed precision as well as the cycle fixing approach are effective heuristics for solving MCLCP because the maximum optimality gap over all instances (of various sizes) is less than 0.04% and 0.6%, respectively. The results also show that the computational advantages of these heuristic approaches becomes apparent only for relatively large instances (with at least 300 nodes). Again, we refer the reader to Özener (2008) for the details on the computational study.

5. Carrier Collaboration

We consider settings in which several carriers collaborate by means of bilateral lane exchanges, i.e., by two carriers exchanging shipment requests, sometimes accompanied by side payments. The goal is to reduce any geographic imbalances in the carriers’ networks so as to reduce repositioning costs. The carriers are selfish in the sense that their sole objective is to minimize their own costs, although they may be willing to share a portion of the benefits of a lane exchange if they believe this is required for the exchange to happen. The carriers have agreed up front on the rules and regulations governing the lane exchange process, primarily the level of information sharing and whether or not side payments can be offered. We have chosen to focus on bilateral lane exchanges because they represent the simplest setting for exchanges and thus facilitate the analysis. However, the ideas discussed can be relatively easily extended to lane exchanges involving more than two carriers. Lane exchanges with multiple carriers do significantly increase the computational challenges.

Given a lane exchange mechanism, i.e., a set of rules and regulations governing the lane exchange process, a carrier determines his own strategy. That is, a carrier decides himself on the lane to offer to the other carrier and on the side payment, if allowed by the exchange mechanism. Depending, again, on the exchange mechanism, a carrier may have only his own individual cost and network information or may have information about the other carriers’ costs and networks. In order for a lane exchange to happen, both carriers must agree to the exchange, hence the result of the exchange must be acceptable to both carriers. This property ensures that the carriers are always better off participating in the collaboration, which is in line with the selfishness of the carriers.
The level of information sharing between carriers and the decision to allow or disallow side payments are the two primary factors defining a lane exchange mechanism. Carriers may be hesitant to share information with their competitors and therefore may decide not to do so. This, of course, forces them to select the lane to offer and to decide on the side payment based solely on their own cost and network information, so somewhat blindfolded; some beneficial lane exchange opportunities will not be recognized or identified. Limited information sharing not only restricts the number of possible exchanges considered but also decreases the possibility of an exchange being accepted, because a carrier is more likely to offer a lane or a side payment that is unacceptable to the other carrier. With information sharing, carriers can estimate the counter strategies of their collaborators, which makes it easier to identify the best overall strategy. Hence, increasing the level of information sharing usually increases the value of the collaboration. However, as will be shown later, this is not always the case.

Side payments allow carriers to share some of their benefits from an exchange with the other carrier. Because an exchange will only take place if both carriers agree to it, a carrier has to make his offer attractive to the other carrier so as to make it acceptable. Suppose that Carrier A offers a lane with low synergies with the other lanes in his network. Due to the low synergies, the operating cost for this lane is high and if the offer is accepted, it will likely result in considerable cost savings. Although Carrier B may be able to handle this lane at a lower cost, because of possible synergies with the other lanes in his network, the exchange may be unacceptable for Carrier B as his costs may still increase. In this case, Carrier A can share some of his cost savings with Carrier B via a side payment and make the exchange offer acceptable to Carrier B. Hence, allowing side payments increases the number of acceptable exchanges.

Designing an effective lane exchange mechanism presents several challenges. The mechanism should identify exchanges that are acceptable to the individual carriers while trying to reach a system optimal solution (i.e., an optimal solution to the MCLCP). Furthermore, the mechanism should be computationally viable and be able to determine a candidate exchange without having to enumerate all possible exchanges.

Different levels of information sharing and whether or not side payments are allowed present different challenges, all of which a mechanism should accommodate. For example, without information sharing and side payments, the carriers are myopic when deciding on a lane to offer and have no means to make this offer acceptable to the other carrier. If side payments are allowed, determining a side payment for the offered lane is non-trivial because of the lack of information about the other carrier. With full information sharing, the number of alternative strategies is quite
large, which makes the exchange process complicated, especially if side payments are allowed. A carrier has to evaluate all lanes and simultaneously evaluate all possible side payments, since the benefits of the a lane exchange depend on both the lane and the associated side payment.

5.1. Lane Exchanges - Illustrative Examples

We demonstrate the challenges associated with designing lane exchange mechanisms by means of examples. We consider a network with five cities and three carriers, namely A, B and C. The cost of covering a lane is assumed to be the same for all carriers; the costs are shown in Figure 3(a).

For simplicity, we also assume that the cost of traversing an arc with an empty truck is equal to the cost of traversing an arc with a loaded truck. The lanes of each carrier and the optimal way to cover them are shown in Figure 3(b), (c), and (d), respectively, where a dashed line represents repositioning. The corresponding transportation costs are 3, 4.73, and 5.46 respectively.

Consider a lane exchange setting in which there is no information sharing and there are no side payments. In this setting, the only task of a carrier is to select the lane to offer to the other carrier. Suppose that Carrier A and B try to exchange lanes. Because a carrier has no information about the other carrier, he is likely to choose a lane that is costly to handle, such as the highest marginal cost lane or the highest per mile marginal cost lane, hoping that the lane has a greater synergy within the other carrier’s network. Hence, Carrier A may offer either lane (3,2) or lane (2,4); they are equivalent from Carrier A’s point of view. Carrier B may offer lane (2,3). As a consequence, if Carrier A decides to offer lane (3,2), the exchange will not be acceptable, but if Carrier A decides to offer lane (2,4) instead, both carriers will agree on the exchange as it reduces their costs to 2 and 3.73, respectively.

Allowing side payments results in more acceptable lane exchanges. Suppose that Carrier B and C try to exchange lanes. Furthermore, let Carrier B offer lane (2,3) as it is the highest marginal

![Figure 3](attachment:image.png)
cost lane and, similarly, let Carrier C offer lane (1,4). If the carriers exchange these lanes, then their costs become 5.46 and 3, respectively. Hence, Carrier B will not agree to the exchange. However, if Carrier C is willing to share some of his benefits of 5.46 - 3 = 2.46 and offer a side payment of, say one unit, then Carrier B will accept the improved offer as it results in a cost of 5.46 - 1 = 4.46, which is less than 4.73. At the same time, Carrier C still benefits too as his costs are 3 + 1 = 4, which is still less than 5.46. This shows that carriers can benefit from allowing side payments. However, it is not obvious how a carrier should determine the amount of the side payment, especially without information about the other carrier’s network.

Information sharing allows carriers to calculate the value and outcome of any possible exchange. Therefore, carriers can initiate lane exchanges with higher benefits and higher likelihood of acceptance. Suppose that Carrier A and B try to exchange lanes and share network and cost information. Carrier B realizes that if he offers any lane but (2,3) Carrier A will reject the exchange, regardless of the lane offer by Carrier A itself. Therefore, Carrier B offers lane (2,3). Similarly, Carrier A realizes that Carrier B will offer lane (2,3) and so he chooses to offer lane (2,4). The exchange will be accepted as both carriers reduce their costs, to 2 and 3.73 respectively.

Not surprisingly, with information sharing determining a strategy becomes considerably more involved. Evaluating each possible lane exchange is not only challenging and time consuming, but may not yield a dominant solution, i.e., a unique lane exchange that results in the largest cost savings for both carriers. Consider a network with eight cities and two carriers A and B. The cost of covering a lane is assumed to be the same for all carriers; the costs are shown in Figure 4(a). Furthermore, assume that the cost of traversing an arc with an empty truck is equal to the cost of traversing an arc with a loaded truck. The networks of Carrier A and B are shown in Figure 4(b). The carriers share network and cost information. If Carrier A offers lane (2,3), then his cost savings will be 0 in case Carrier B offers (3,2) or 2 if Carrier B offers (7,6). On the other hand, if Carrier A offers lane (6,7), then his cost savings will be 2 in case Carrier B offers (3,2) or 0 if Carrier B offers (7,6). The situation is identical for Carrier B. That is, there is no dominant solution and the carriers are still uncertain about their strategies even though they have perfect information. The reason for the uncertainty is that the carriers do not know the strategy of the other carrier.

Finally, although information sharing is generally helpful, it is easy to show that this is not always the case. Suppose the networks of Carrier A and B are as shown in Figure 5. Suppose that Carrier A can offer only lanes (4,1) and (8,6) and Carrier B can offer only lanes (5,3) and (10,7). Without information sharing, the carriers will likely offer lanes with the highest marginal costs, i.e., (4,1) and (10,7) respectively, resulting in cost savings of 1 for both carriers. With information
sharing, if Carrier A offers (4,1), then his cost savings will be 1 in case Carrier B offers (10,7) or 0 if Carrier B offers (5,3). On the other hand, if Carrier A offers lane (8,6), then his cost savings will be 2 in case Carrier B offers (10,7) or 0 if Carrier B offers (5,3). As the potential payoffs from offering lane (8,6) dominate the alternative, Carrier A chooses to offer lane (8,6). Similarly, Carrier B chooses to offer lane (5,3). Thus their cost savings will be 0 whereas without information sharing their cost savings would have been 1.

Also with information sharing, allowing side payments is generally beneficial as it increases the number of acceptable lane exchanges. However, determining an appropriate side payment with an offered lane is even more difficult in this setting. The cost savings for a carrier depend on the lane offered by the other carrier, so even if a carrier is willing to share the benefits, he is uncertain about the actual cost savings of a lane exchange until the moment of the exchange.
lane to offer calculating an appropriate side payment for all possible counter offers may be too time consuming. Determining an appropriate side payment is further complicated by the fact that the other carrier may also offer a side payment.

5.2. Lane Exchange Mechanisms

We propose and analyze lane exchange mechanism for four different carrier collaboration settings: no information sharing and no side payments (NINS), no information sharing with side payments (NIWS), information sharing without side payments (WINS), and information sharing with side payments (WIWS). We first discuss some common properties of these lane exchange mechanisms and then describe them in detail and discuss their advantages and disadvantages.

Let Carrier A and B be the two carriers in a lane exchange process. Let $L^A$ and $L^B$ be the sets of lanes the carriers can offer for exchange. (Recall that some lanes may have to be served by the carrier itself.) Let $c^A_{ij}$ represent the cost of covering arc $(i, j)$ for Carrier A, i.e., the cost of traversing the lane with a full truckload, and let $\theta c^A_{ij}$ be the repositioning cost for Carrier A along that same arc. Let $z^*_A(L^A)$ be the minimum cost of covering all the lanes in the lane set $L^A$ by Carrier A, i.e., the optimal objective function value of the lane covering problem (LCP) for Carrier A. The marginal cost $MC^A_{ij}(L^A)$ of lane $(i, j) \in L^A$ for Carrier A is $z^*_A(L^A) - z^*_A(L^A\setminus(i, j))$. Furthermore, let $p^A_{ij}$ denote the side payment for lane $(i, j) \in L^A$ by Carrier A, i.e., the amount that Carrier A will pay to Carrier B if the lane exchange is accepted. The “payoff” of an exchange to a carrier refers to the cost savings for the carrier if the lane exchange is accepted and performed. The payoff to Carrier A is denoted by $\pi^A_{ij,uv}$, where $(i, j)$ represents the lane offered by Carrier A and $(u, v)$ the lane offered by Carrier B, and similarly the payoff to Carrier B is denoted by $\pi^B_{uv,ij}$. The payoffs are equal to

$$\pi^A_{ij,uv} = \begin{cases} MC^A_{ij}(L^A) - MC^A_{uv}(L^A\setminus(i, j) \cup (u, v)) - p^A_{ij} + p^B_{uv} & \text{if exchange occurs}, \\ 0 & \text{otherwise}, \end{cases}$$

and

$$\pi^B_{uv,ij} = \begin{cases} MC^B_{uv}(L^B) - MC^B_{ij}(L^B\setminus(u, v) \cup (i, j)) - p^B_{uv} + p^A_{ij} & \text{if exchange occurs}, \\ 0 & \text{otherwise}. \end{cases}$$

Note that the payoffs depend on both lanes. That is, the marginal cost of an offered lane depends on the lane that the other carrier offers. Therefore, strategies that ignore the other carrier’s actions are less likely to be effective.

5.2.1. No Information Sharing - No Side Payments

In this most basic lane exchange setting, a carrier only needs to select the lane to offer to the other carrier. In the following discussion on the lane exchange mechanism we propose, we assume that the carriers offer the lanes with the
highest marginal costs. That is, Carrier A offers $\text{argmin}_{(i,j) \in L_A} \{MC_A^{ij}(L^A)\}$ and Carrier B offers $\text{argmin}_{(u,v) \in L_B} \{MC_B^{uv}(L^B)\}$. The selection strategy is completely based on a carrier’s individual network and ignores the other carrier’s network. The motivation for this selection strategy is that it maximizes the known benefit from a potential exchange as it discards the lane with highest cost.

An alternative selection strategy is to offer the lane with the highest marginal cost per mile, which is an indicator for the repositioning requirement of the lane. The motivation for this selection strategy is that the highest marginal cost lane may be a long lane rather than a lane with low synergies. On the other hand, the lane with the highest marginal cost per mile has limited or no synergies with the existing lanes of the carrier.

The computational requirements of both selection strategies are small as the exchange mechanism is simple and only requires the ranking of the lanes according to their marginal costs (or per mile marginal costs).

The following algorithm evaluates the benefits of this exchange mechanism for two carriers A and B, where we assume that the carriers continue to exchange lanes as long as beneficial lane exchanges are identified:

**NINS:**

**Step 1:** Rank the lanes that can be offered according to marginal costs (or per mile marginal costs) for both carriers and select the best one from each to form the lane exchange pair.

**Step 2:** Let $(i,j), (u,v)$ be the selected pair of lanes. Compute the payoffs for each carrier:

$$\hat{\pi}_A^{ij,uv} = MC_A^{ij}(L^A) - MC_A^{uv}(L^A \setminus (i,j) \cup (u,v)),$$

$$\hat{\pi}_B^{uv,ij} = MC_B^{uv}(L^B) - MC_B^{ij}(L^B \setminus (u,v) \cup (i,j)).$$

**Step 3:** If the individual payoffs are nonnegative, i.e., $\hat{\pi}^A \geq 0$ and $\hat{\pi}^B \geq 0$, and the combined payoff is strictly positive, i.e., $\hat{\pi}^A + \hat{\pi}^B > \epsilon$, execute the exchange, update the lane sets of the carriers, and return to Step 1. Otherwise, discard the exchange and stop.

This algorithm terminates after a finite number of iterations because the combined payoff is greater than $\epsilon$ at each iteration and the minimum total cost for the carriers is positive. The only computationally intensive step in the algorithm is the ranking of the lanes according to their marginal costs. To compute the marginal cost of a lane, we have to solve two LCPs. Fortunately, LCP can be solved efficiently as a min-cost flow problem.
5.2.2. No Information Sharing With Side Payments

In this lane exchange setting, a carrier needs to select a lane to offer and needs to determine an appropriate side payment for the offered lane. Note that we assume the side payments for lanes are set beforehand. In the lane mechanism we propose, the carriers still offer the lanes with the highest marginal costs (or highest marginal costs per mile). However, as a lane exchange may result in cost savings, the carriers have an incentive to make the offer attractive in order to realize the cost savings. Side payments provide a mechanism for making an offered lane more attractive. We propose a method for determining side payments that is based on the synergies of a lane within the network of the carrier. We can assess the synergies of a lane by comparing the marginal cost to the lane cost. For instance, when the marginal cost of lane \((i,j)\) for Carrier \(A\) is equal to \((1 + \theta)c_{ij}^A\), the lane has no synergies within the network of Carrier \(A\), since the marginal cost value implies that Carrier \(A\) has to reposition from \(j\) to \(i\). On the other hand, when the marginal cost of lane \((i,j)\) for Carrier \(A\) is equal to \((1 - \theta)c_{ij}^A\), the lane has perfect synergies within the network of Carrier \(A\), since it implies that covering the lane also serves as a repositioning move for other lanes in the network of Carrier \(A\).

As we increase the side payment the likelihood of acceptance increases. However, at the same time, the benefits from the lane exchange decrease. As such, side payments determine how the cost savings that result from exchanging lanes are shared among the carriers. As a carrier does not know these cost savings in advance, since they depend on the other carrier’s action, an estimate has to be used. One option is to use \(MC_{ij}^A(L^A) - c_{ij}^A\) as an estimate of the cost savings. When the marginal cost of a lane is high compared to the lane cost, the synergies within the network are low. Consequently, the other carrier may be able to cover this lane more cheaply due to potential synergies within his network. Sharing the estimated cost savings equally provides a reasonable balance between reducing the risk of a rejection by the other carrier and reducing the size of the benefit of the lane exchange. Thus, the carrier may put forth a side payment equal to half the difference between the marginal cost and original lane cost: \(\frac{MC_{ij}^A(L^A) - c_{ij}^A}{2}\).

When the marginal cost of a lane is low compared to the lane cost (but still greater than the marginal cost), there are synergies within the network. Consequently, it is less likely that the other carrier can cover this lane more cheaply as a result of potential synergies within his network. Therefore, the carrier’s priority is to make the exchange attractive to the other carrier so as to avoid a rejection of the lane exchange. This means that a higher portion of the estimated cost savings has to be “transferred” to the other carrier to increase the likelihood of acceptance. Thus, the carrier may opt to offer a side payment that is more than half the difference between the
marginal cost and the original cost. We have chosen to use the adjustment factor \( \frac{(1+\theta)c^A_{ij}}{MC^A_{ij}(L^A)} \), which results in the following rule for determining the side payment:

\[
p^A_{ij} = \begin{cases} 
(1+\theta)c^A_{ij} \times \frac{MC^A_{ij}(L^A) - c^A_{ij}}{2} & \text{if } MC^A_{ij}(L^A) \geq c^A_{ij}, \\
0 & \text{otherwise.}
\end{cases}
\]

Note that the adjustment factor is a value between 1 and 2 (since \( \theta \leq 1 \)) and that it gets larger when the marginal cost gets smaller.

The following algorithm evaluates the benefits of this exchange mechanism for two carriers \( A \) and \( B \), where we assume that the carriers continue to exchange lanes as long as beneficial lane exchanges are identified:

**NIWS:**

**Step 1:** Rank the lanes that can be offered according to marginal costs (or per mile marginal costs) for both carriers and select the best one from each to form the lane exchange pair.

**Step 2:** Let \( ((i,j),(u,v)) \) be the selected pair of lanes. Compute the side payments on the lanes:

\[
p^A_{ij} = \begin{cases} 
\frac{(1+\theta)c^A_{ij}}{2 \times MC^A_{ij}(L^A)} \times (MC^A_{ij}(L^A) - c^A_{ij}) & \text{if } MC^A_{ij}(L^A) > c^A_{ij}, \\
0 & \text{otherwise.}
\end{cases}
\]

\[
p^B_{uv} = \begin{cases} 
\frac{(1+\theta)c^B_{uv}}{2 \times MC^B_{uv}(L^B)} \times (MC^B_{uv}(L^B) - c^B_{uv}) & \text{if } MC^B_{uv}(L^B) > c^B_{uv}, \\
0 & \text{otherwise.}
\end{cases}
\]

Compute the payoffs for each carrier:

\[
\hat{\pi}^A_{ij,uv} = MC^A_{ij}(L^A) - MC^A_{uv}(L^A \setminus (i,j) \cup (u,v)) - p^A_{ij} + p^B_{uv}
\]

\[
\hat{\pi}^B_{uv,ij} = MC^B_{uv}(L^B) - MC^B_{ij}(L^B \setminus (u,v) \cup (i,j)) - p^B_{uv} + p^A_{ij}.
\]

**Step 3:** If the individual payoffs are nonnegative, i.e., \( \hat{\pi}^A \geq 0 \) and \( \hat{\pi}^B \geq 0 \), and the combined payoff is strictly positive, i.e., \( \hat{\pi}^A + \hat{\pi}^B > \epsilon \), execute the exchange, update the lane sets of the carriers, and return to Step 1. Otherwise, discard the exchange and stop.

As before, the algorithm terminates after a finite number of iterations and the only computationally intensive step is computing the marginal cost of the lanes.

**5.2.3. With Information Sharing - No Side Payments** In this lane exchange setting, the carriers share information to better be able to identify lane exchange opportunities. More specifically, they share their cost structure as well as the lanes they need to serve. With information about the other carrier, a carrier can consider and analyze potential counter offers and estimate the outcome of possible lane exchanges, and choose a strategy accordingly to increase the value of the benefits as well as the probability of acceptance of a lane exchange.
A carrier only needs to select a lane to offer to the other carrier. However, this selection should be based not only on information about their own network, but also on information about the other carrier’s network and thus expectations about the other carrier’s strategy. We propose to model the selection problem as a two-person non-zero sum game where the rows correspond to lanes from Carrier A and columns correspond to lanes from Carrier B. As before, the payoff for a carrier for a given a lane exchange pair is the value of the cost savings resulting from the exchange. Both carriers can enumerate all possible lane exchange pairs and compute the corresponding payoffs (thus constructing the payoff matrix for the game). Then, based on this information, the carriers can choose the strategy that maximizes the minimum payoff to themselves, which corresponds to finding a Nash equilibrium of the two-person non-zero sum game.

We have the following results:

**Lemma 3.** A mixed-strategy Nash equilibrium point always exists for the lane exchange game.

**Proof** Directly follows from the well-known Nash’s theorem: “Every finite strategic game has a mixed strategy Nash equilibrium” (Osborne and Rubinstein 1994).

**□**

**Lemma 4.** A pure-strategy Nash equilibrium point may not exist for the lane exchange game.

**Proof** Consider the example in Figure 6. Two carriers, namely I and II, have four lanes each, \{A, B, E, F\} and \{C, D, G, H\} respectively and they have identical cost structures on this network (cost figures are shown in Figure 6). Suppose that Carrier I can offer either lane A or B for an exchange, and Carrier II can offer either lane C or D. Hence, the resulting payoff matrix is as follows:

\[
\begin{bmatrix}
2.5 & 10.7 \\
10.7 & 2.5 \\
\end{bmatrix}
\]

Consequently, this instance has no pure strategy Nash equilibrium point.  

**□**

Based on these results, we first check whether there exists a pure-strategy Nash equilibrium point for the lane exchange game. If one or more pure-strategy Nash equilibria exist, then we select the one with the highest combined benefit to both carriers. When a pure-strategy Nash equilibrium point does not exist, we find the set of mixed-strategy equilibria and select the one with the highest combined benefit to both carriers. For that mixed-equilibrium point, we select the row and column with the highest probabilities as the strategies for the carriers. Note that regardless of the type of the equilibrium, this method may not return the best lane exchange pair from a system-wide perspective. This is a result of the selfishness of the carriers. Intuitively, though, this method should
be quite effective in terms of the resulting benefits. With information sharing, the carriers can evaluate the response of the other carrier and thus choose a strategy that maximizes the expected cost savings.

The following algorithm evaluates the benefits of this exchange mechanism for two carriers $A$ and $B$, where we assume that the carriers continue to exchange lanes as long as beneficial lane exchanges are identified:

WINS:

Step 1: For every possible pair of lanes $((i,j),(u,v))$ compute the payoffs for each carrier:

$$
\hat{\pi}^A_{ij,uv} = MC^A_{ij}(L^A) - MC^A_{uv}(L^A \setminus (i,j) \cup (u,v)),
$$

$$
\hat{\pi}^B_{uv,ij} = MC^B_{uv}(L^B) - MC^B_{ij}(L^B \setminus (u,v) \cup (i,j)),
$$

$$
\pi^A_{ij,uv} = \begin{cases} 
\hat{\pi}^A_{ij,uv} & \hat{\pi}^A_{ij,uv}, \hat{\pi}^B_{uv,ij} \geq 0, \hat{\pi}^A_{ij,uv} + \hat{\pi}^B_{uv,ij} > 0, \\
0 & \text{otherwise},
\end{cases}
$$

$$
\pi^B_{uv,ij} = \begin{cases} 
\hat{\pi}^B_{uv,ij} & \hat{\pi}^A_{ij,uv}, \hat{\pi}^B_{uv,ij} \geq 0, \hat{\pi}^A_{ij,uv} + \hat{\pi}^B_{uv,ij} > 0, \\
0 & \text{otherwise}.
\end{cases}
$$

Step 2: Construct the payoff matrix using $\pi^A$'s and $\pi^B$'s.

Step 3: Solve the two-person non-zero sum game to find the pure-strategy Nash equilibrium points. If multiple pure-strategy Nash equilibria exist, then select the one with the highest combined benefits ($\pi^A + \pi^B$). If no pure-strategy Nash equilibrium point exists, solve the game to find the mixed-strategy Nash equilibrium points. If multiple mixed Nash equilibria exist, then select the one...
with the highest combined benefits ($\pi^A + \pi^B$). For the selected mixed-strategy Nash equilibrium point, select the row and column strategies with highest probabilities and take corresponding payoffs as the payoffs of the carriers.

**Step 4:** If the individual payoffs are nonnegative, i.e., $\pi^A \geq 0$ and $\pi^B \geq 0$, and the combined payoff is strictly positive, i.e., $\pi^A + \pi^B > \epsilon$, execute the exchange, update the lane sets of the carriers, and return to Step 1. Otherwise, discard the exchange and stop.

The algorithm terminates after a finite number of iterations. This algorithm is much more computationally intensive. To construct the payoff matrix, we have to compute the payoffs for all possible lane exchange pairs. For a given lane exchange pair, this involves solving four LCPs (two for each marginal cost calculation). After constructing the payoff matrix, we have to find the Nash equilibria of the game.

5.2.4. With Information and With Side Payments As mentioned before, by allowing side payments the number of lane exchange opportunities increases. Observe that with information sharing, but without side payoffs, if for a given lane exchange pair one of the payoffs is negative, the exchange will not be acceptable. On the other hand, the payoff for the offering carrier may be quite high due to the fact that he is giving away a high cost lane. A side payment may result in nonnegative payoffs for both carriers.

In this lane exchange setting, a carrier needs to select a lane to offer and needs to determine an appropriate side payment for the offered lane. Again, we propose to model the decision process as a two-person non-zero sum game where the rows correspond to lanes from Carrier $A$ and columns correspond to lanes from Carrier $B$. The only difference is that the payoff values have to include any side payments. Determining appropriate side payments, however, is quite complicated. The basic idea of a side payment is to make an offer attractive to the other carrier. However, when a carrier includes a side payment with a lane, it affects the payoffs of all the lane exchange pairs that involve the lane. Furthermore, the other carrier may include a side payment with the offered lane as well, in effect returning a portion of the side payment. We propose to use a “conservative approach” for determining side payments. If Carrier $A$ offers lane $(i, j)$ and Carrier $B$ would choose $(u, v)$ as the counter-offer to maximize his payoff, i.e., $(u, v) = \text{argmax}_{(u,v) \in L^B} \hat{\pi}^B_{uv,ij}$, but $\hat{\pi}^B_{uv,ij} < 0$, then Carrier $A$ will offer side payment $\hat{p}^A_{ij} = \epsilon - \hat{\pi}^B_{uv,ij}$ resulting in a positive payoff for Carrier $B$. Of course, Carrier $A$ will do so only if $\hat{\pi}^A_{ij,uv} + \hat{\pi}^B_{uv,ij} > \epsilon$ otherwise his own payoff would become negative.

The following algorithm evaluates the benefits of this exchange mechanism for two carriers $A$ and $B$, where we assume that the carriers continue to exchange lanes as long as beneficial lane
exchanges are identified:

**WIWS:**

Step 1: For every possible lane exchange pair \(((i, j), (u, v))\) compute the tentative payoffs for each carrier:

\[
\pi_{ij, uv}^A = MC_{ij}^A(L^A) - MC_{uv}^A(L^A \setminus \{i, j\} \cup \{u, v\})
\]

\[
\pi_{uv, ij}^B = MC_{uv}^B(L^B) - MC_{ij}^B(L^B \setminus \{u, v\} \cup \{i, j\})
\]

Step 2: For every lane \((i, j)\) of Carrier \(A\), determine \((u, v) = \arg\max_{(u, v) \in L^B} \pi_{uv, ij}^A\) and

\[
p_{ij}^A = \begin{cases} 
\epsilon - \pi_{uv, ij}^A & \pi_{uv, ij}^A < 0, \\
0 & \text{otherwise.}
\end{cases}
\]

For every lane \((u, v)\) of Carrier \(B\), determine \((i, j) = \arg\max_{(i, j) \in L^A} \pi_{ij, uv}^A\) and

\[
p_{uv}^B = \begin{cases} 
\epsilon - \pi_{ij, uv}^A & \pi_{ij, uv}^A < 0, \\
0 & \text{otherwise.}
\end{cases}
\]

Update the tentative payoff values as

\[
\hat{\pi}_{ij, uv}^A = \pi_{ij, uv}^A - p_{ij}^A + p_{uv}^B
\]

\[
\hat{\pi}_{uv, ij}^B = \pi_{uv, ij}^B + p_{ij}^A - p_{uv}^B
\]

For every possible lane exchange pair \(((i, j), (u, v))\) compute the actual payoffs

\[
\pi_{ij, uv}^A = \begin{cases} 
\hat{\pi}_{ij, uv}^A & \hat{\pi}_{ij, uv}^A \geq 0, \\
0 & \text{otherwise,}
\end{cases}
\]

\[
\pi_{uv, ij}^B = \begin{cases} 
\hat{\pi}_{uv, ij}^B & \hat{\pi}_{uv, ij}^B \geq 0, \\
0 & \text{otherwise.}
\end{cases}
\]

Step 3: Construct the payoff matrix using \(\pi^A\)’s and \(\pi^B\)’s.

Step 4: Solve the two-person non-zero sum game to find the pure-strategy Nash equilibrium points. If multiple pure-strategy Nash equilibria exist, then select the one with the highest combined benefits \((\pi^A + \pi^B)\). If no pure-strategy Nash equilibria exists, solve the game to find the mixed-strategy Nash equilibrium points. If multiple mixed-strategy Nash equilibria exist, then select the one with the highest benefits \((\pi^A + \pi^B)\). For the select mixed-strategy Nash equilibria, select the row and column strategies with highest probabilities and take corresponding payoffs as the payoffs of the carriers.

Step 5: If the individual payoffs are nonnegative, i.e., \(\pi^A \geq 0\) and \(\pi^B \geq 0\), and the combined payoff is strictly positive, i.e., \(\pi^A + \pi^B > \epsilon\), execute the exchange, update the lane sets of the carriers, and return to Step 1. Otherwise, discard the exchange and stop.
This algorithm terminates after a finite number of iterations and has the same computational challenges as the algorithm for WINS.

**Computational Efficiency.** All proposed algorithms require the solution of a large number of LCPs to compute the marginal costs of the lanes of the carriers. The following property concerning the marginal cost of a lane allows us to significantly reduce the computational time of our algorithms.

**Theorem 2.** The marginal cost of a lane is less than or equal to the value of the corresponding dual variable in an optimal solution to the dual of the lane covering problem.

**Proof** Let $L^k$ be the lane set of carrier $k$ and $r^k_{ij}$ be the number of times lane $(i, j) \in L^k$ is required to be covered. Hence, for a carrier $k$, the dual of the lane covering problem is as follows:

$$DLCP^k \quad z(L^k) = \max \sum_{(i,j) \in L^k} r^k_{ij} a^k_{ij}$$

$$\text{s.t. } a^k_{ij} + y^k_i - y^k_j = c^k_{ij} \quad \forall (i,j) \in L^k$$

$$y^k_i - y^k_j \leq \theta c^k_{ij} \quad \forall (i,j) \in A$$

$$a^k_{ij} \geq 0 \quad \forall (i,j) \in L^k.$$ (1)

Let $(\hat{a}^k, \hat{y}^k)$ represent the optimal solution to $DLCP^k$. As the optimal objective function value of the dual LCP is equal to the total cost of covering all the lanes by Carrier $k$, the marginal cost of lane $(u, v)$ to Carrier $k$’s network is equal to:

$$MC_{uv}^k(L^k) = z^*(L^k \cup (u, v)) - z^*(L^k)$$

where $z^*(.)$ represents the optimal objective function value of the associated dual LP. After adding lane $(u, v)$ to the network, the resulting cost of Carrier $k$ can be determined by solving the modified dual problem:

$$DLCP^k \quad z(L^k \cup (u, v)) = \max \sum_{(i,j) \in L^k} r^k_{ij} a^k_{ij} + a^k_{uv}$$

$$\text{s.t. } a^k_{ij} + y^k_i - y^k_j = c^k_{ij} \quad \forall (i,j) \in L^k$$

$$a^k_{uv} + y^k_u - y^k_v = c^k_{uv}$$

$$y^k_i - y^k_j \leq \theta c^k_{ij} \quad \forall (i,j) \in A$$

$$a^k_{ij} \geq 0 \quad \forall (i,j) \in L^k \cup (u, v).$$ (15)
Let \((\bar{a}^k, \bar{y}^k)\) represent the optimal solution to \(\text{DLCP}^k\). As \((\bar{a}^k, \bar{y}^k)\) is feasible for the original dual problem \(\text{DLCP}^k\), we have
\[
\sum_{(i,j) \in L^k} r^k_{ij} \bar{a}_{ij} \leq \sum_{(i,j) \in L^k} r^k_{ij} \hat{a}_{ij}
\]
since \(\text{DLCP}^k\) is a maximization problem. Combining these two results, we obtain the following property for the marginal costs:
\[
MC^k_{uv}(L^k) = z^*(L^k \cup (u,v)) - z^*(L^k) = \sum_{(i,j) \in L^k} r^k_{ij} \bar{a}_{ij} + \bar{a}^k_{ij} - \sum_{(i,j) \in L^k} r^k_{ij} \hat{a}_{ij} \leq \bar{a}^k_{ij}. \quad \square
\]

The optimal values of the dual variables can easily be found using linear programming technology. Consequently, we have an efficient mechanism for obtaining upper bounds on the marginal costs of the lanes, which we will use instead of the marginal costs unless these values are insufficient. For example, the lane exchange mechanism without information sharing and without side payments requires the identification of the lane with the highest marginal cost. We sort the lanes based on the upper bounds on the marginal cost and compute the actual marginal cost for the lane with the largest upper bound, which provides a lower bound on the value of the highest marginal cost. For the remaining lanes, if the upper bound is less than or equal to the lower bound on the value of the highest marginal cost, we simply skip the computation of the true marginal cost. Otherwise, we compute the true marginal cost of the lane and update the lower bound if necessary. Computational experience shows that this simple procedure significantly reduces computation times. The lane exchange mechanism with information sharing but without side payments can take advantage of Theorem 2. The computation of the tentative payoffs (\(\tilde{\pi}'s\)) involves two marginal costs. One of these can be replaced with its upper bound to get an upper bound on the payoff. Only if the upper bound on the payoff is positive, do we compute the actual marginal cost of the lane for which we have used the upper bound. Let \(\bar{a}^A_{ij}\) and \(\bar{a}^B_{uv}\) represent the relevant upper bounds, then we examine
\[
\tilde{\pi}^A_{ij,uv} = \bar{a}^A_{ij} - MC^A_{uv}(L^A \cup (u,v))
\]
and
\[
\tilde{\pi}^B_{uv,ij} = \bar{a}^B_{uv} - MC^B_{ij}(L^B \cup (i,j)).
\]
to see if true payoff values need to be computed. The same idea can be applied in case there is information sharing and there are side payments. Computational experience shows significant reductions in computation times.
5.3. Computational Study

We have carried out a computational study to evaluate the performance of the proposed lane exchange mechanisms.

To do so, we generated a large set of instances in a 1,000 × 1,000 mile square region by varying the number of locations, the average number of lanes incident to a location, the number of clusters, and the ratio of cluster locations to total locations. Lanes are created by randomly picking an origin and a destination, where we ensure that no lanes are generated between two locations in the same cluster. The cost of traveling between locations with a full truckload is equal to the Euclidean distance between the locations and the repositioning cost coefficient for all carriers is equal to 0.75. We generated 270 instances with 100, 200, and 300 locations, with 2, 4, and 6 as the average number of lanes incident to a location, with 4 clusters, and with 45% of the locations within clusters. To be able to investigate the impact of lanes that have to be served by the carrier, we created three additional instances for every instance by adding lanes that have to be served by a carrier. (Thus, the number of lanes that can be exchanged is the same for the four instances.) The three additional instances are created by adding 50%, 100%, and 200% of the number of lanes that can be exchanged. That means that if the original instance has 120 lanes, the variant with 50% of lanes that have to be served by a carrier has a total of 180 lanes, i.e., 120 that can be exchanged (assigned randomly to the two carriers) and 60 that cannot be exchanged (assigned randomly and evenly among the carriers). All the experiments assume two carriers with identical cost structures, which is the setting with the least potential for cost savings opportunities.

Our goal is to show that bilateral lane exchange mechanisms are capable of identifying and exploiting the synergies that exist among the lanes of the two carriers. For a given random assignment of lanes to Carrier A and Carrier B, we determine the maximum possible savings by computing the minimum cost for Carrier A (obtained by solving LCP over the lanes assigned to Carrier A) plus the minimum cost for Carrier B (obtained by solving LCP over the lanes assigned to Carrier B) minus the cost of a centralized solution (obtained by solving MCLCP over all lanes). We use the term optimality gap to refer to these maximum possible savings as a percentage of the cost of the perfect collaboration.

We evaluate each of the four lane exchange mechanisms (NINS, NIWS, WINS, and WIWS) by computing the optimality gap after the lane exchange mechanism has been applied. We solve the two-person non-zero sum games in mechanisms WINS and WIWS using the Gambit software package (Ruchira 2003). Table 1 summarizes the performance of the exchange methods. We report the optimality gap for the random assignment (Random), the average optimality gap for mechanism
NINS (NINS), the average computation time for mechanism NINS in CPU seconds (cpu NINS), the average optimality gap for mechanism NIWS (NIWS), the average computation time for mechanism NIWS (cpu NIWS), the average optimality gap for mechanism WINS (WINS), the average computation time for mechanism WINS (cpu WINS), the average optimality gap for mechanism WIWS (WIWS), and the average computation time for mechanism WIWS (cpu WIWS). For each instance size, we present summary statistics for the different levels of lanes that have to be served by the carrier itself.

### Table 1 Optimality gaps for the proposed lane exchange mechanisms

<table>
<thead>
<tr>
<th></th>
<th>Random</th>
<th>NINS</th>
<th>cpu NINS</th>
<th>NIWS</th>
<th>cpu NIWS</th>
<th>WINS</th>
<th>cpu WINS</th>
<th>WIWS</th>
<th>cpu WIWS</th>
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<td>40 lanes</td>
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<td></td>
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<tr>
<td>0%</td>
<td>5.06</td>
<td>3.53</td>
<td>3.20</td>
<td>3.14</td>
<td>2.00</td>
<td>0.37</td>
<td>74.60</td>
<td>0.16</td>
<td>268.00</td>
</tr>
<tr>
<td>50%</td>
<td>8.07</td>
<td>4.76</td>
<td>3.00</td>
<td>3.81</td>
<td>2.40</td>
<td>0.92</td>
<td>69.40</td>
<td>0.36</td>
<td>302.20</td>
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<tr>
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<td>2.60</td>
<td>7.48</td>
<td>1.80</td>
<td>1.97</td>
<td>82.00</td>
<td>0.73</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
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<td>3.80</td>
<td>2.29</td>
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<td>0.11</td>
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</tr>
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<td>3.20</td>
<td>4.07</td>
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<td>2.61</td>
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<td>697.40</td>
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<td>596.60</td>
<td>0.13</td>
<td>2747.40</td>
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<td>4.96</td>
<td>3.59</td>
<td>3.40</td>
<td>3.20</td>
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<td>200%</td>
<td>5.61</td>
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<td>3.20</td>
<td>4.54</td>
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<td>3.12</td>
<td>3.88</td>
<td>1.88</td>
<td>0.55</td>
<td>356.55</td>
<td>0.27</td>
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The computational experiments reveal that the proposed lane exchange mechanisms perform quite well and are capable of identifying and exploiting the synergies among the lanes of the two carriers. The average optimality gap for NINS is 4.12% and the average optimality gap for of NIWS is 3.88%, which implies that 26% and 30% of the possible cost savings has been realized. Although NIWS outperforms NINS on average, for some instances NINS has lower optimality gap than NIWS. As we mentioned earlier, allowing side payments is not always beneficial and here we observe this phenomenon in our experiments.

It is also clear that the lane exchange mechanism with information sharing perform considerably better than those without information sharing. The average optimality gap for WINS is 0.55% and the average optimality gap for WIWS is only 0.27%, which implies that 90% and 95% of the possible cost savings have been realized. In fact, in most of the instances WIWS achieves the optimal solution (i.e., the centralized solution representing perfect collaboration). Information sharing allows carriers to select their strategies based on the anticipated counter offers and thus
allowing carriers to identify the best possible lane exchange. Allowing side payments helps too, but its effect is much less significant than sharing information.

Next, we generate 60 instances in which the sets of lanes that have to be served by a carrier are geographically clustered in different regions. This is accomplished by dividing the square region in an upper triangular area and the complementing lower triangular area and generating the lanes that have to be served by one carrier in the upper triangular region and the lanes that have to be served by the other carrier in the lower triangular region. The results can be found in Table 2. For these instances, the average optimality gap for NINS and NIWS is 4.43% and 3.92% respectively. These results imply that 40% and 47% of the possible cost savings have been realized, hence the performance is better than when the lanes that have to be served by the carriers are randomly assigned to carriers and have no special geographical properties. The average optimality gap for WINS and WIWS are 0.72% and 0.32%, respectively, and these results imply that 90% and 96% of the possible cost savings have been realized. Comparing these results to the previous results, it appears that the performance of the lane exchange process is better when the carriers have a "clear identity" in the form of geographically separated and clustered sets of lanes that have to be served by them. This is probably due to the fact that fewer synergies arise with such sets.

The computation times of the exchange mechanisms are within acceptable limits. Although the average computation times of the proposed mechanisms increase as the mechanisms becomes more complex (and more effective), the highest computation time is still less than 4500 seconds.

### Table 2  
Optimality gaps for the proposed lane exchange mechanisms when the carriers have geographically separated sets of lanes that they have to serve

<table>
<thead>
<tr>
<th></th>
<th>Random</th>
<th>NINS</th>
<th>cpu</th>
<th>NINS</th>
<th>NIWS</th>
<th>cpu</th>
<th>NIWS</th>
<th>WINS</th>
<th>cpu</th>
<th>WINS</th>
<th>WIWS</th>
<th>cpu</th>
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<td>40 lanes</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>5.23</td>
<td>4.10</td>
<td>3.20</td>
<td>4.10</td>
<td>1.20</td>
<td>0.70</td>
<td>64.00</td>
<td>0.34</td>
<td>277.40</td>
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<tr>
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<td>6.64</td>
<td>4.63</td>
<td>2.80</td>
<td>4.37</td>
<td>2.00</td>
<td>1.32</td>
<td>60.20</td>
<td>0.54</td>
<td>229.40</td>
<td></td>
<td></td>
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<tr>
<td>200%</td>
<td>10.15</td>
<td>6.91</td>
<td>3.00</td>
<td>5.70</td>
<td>2.00</td>
<td>2.29</td>
<td>63.40</td>
<td>0.59</td>
<td>236.60</td>
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<td>80 lanes</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>50%</td>
<td>10.55</td>
<td>4.47</td>
<td>4.60</td>
<td>4.12</td>
<td>3.00</td>
<td>0.24</td>
<td>543.40</td>
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<td>3.93</td>
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<td>120 lanes</td>
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<td>2.56</td>
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<td>100%</td>
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<tr>
<td>200%</td>
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<td>3.39</td>
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<td>3.26</td>
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<td>0.40</td>
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<td>3987.00</td>
<td></td>
<td></td>
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<tr>
<td>Average</td>
<td>7.32</td>
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<td>3.80</td>
<td>3.92</td>
<td>2.64</td>
<td>0.72</td>
<td>456.44</td>
<td>0.32</td>
<td>1897.69</td>
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</table>

Acknowledgments

Ozlem Ergun and Martin Savelsbergh were partially supported under NSF grant DMI-0427446.
References


Appendix. Complexity Proof

**Theorem 1.** When there are three or more carriers with non-identical cost structures, then MCLCP is NP-hard.
**Proof:** By a reduction from 3-SAT. In 3-SAT, we have a set of boolean variables \( B = \{B_1, \ldots, B_n\} \) and a set of clauses \( U = \{CLA_1, \ldots, CLA_m\} \). Each clause has exactly three literals, where a literal corresponds to a boolean variable \( B_i \) or its complement \( \overline{B}_i \). A clause is satisfied when it contains at least one true literal. The question is whether every clause can be satisfied by setting each boolean variable to either true or false.

We give a polynomial reduction from 3-SAT to \( MCLCP \) with three or more carriers and non-identical cost structures \( (3^+ - MCLCP) \).

Each clause is converted to an arc in such a way that these arcs form a path, the so-called clause path (see Figure 7). For each boolean variable \( B_i \), we construct a network with an upper path corresponding to the variable itself, and a lower path corresponding to its complement. An example is depicted in Figure 8, where the upper path is shown in solid lines and the lower path is shown in dashed lines. Each of these paths intersects the clause path whenever the variable appears in the clause. In the example, the upper path intersects with the clause path twice, in arcs (2,3) and (4,5), indicating that variable \( B_i \) is included in both \( CLA_2 \) and \( CLA_4 \). Similarly, the lower path intersects with the clause path once, in arc (3,4), indicating that \( \overline{B}_i \) is only included in \( CLA_3 \). The network is completed with two more arcs: a shared arc and a returning arc; also shown in Figure 8. The lane set \( L \) consists of the arcs in the clause path and the shared arcs in each of the networks associated with the boolean variables. We set the \( r_{ij} \) value for each clause arc and for each shared arc to one. The latter guarantees that there is at least a unit flow on either the upper or the lower path of every boolean variable.

![Figure 7](image1.png) **The clause path.**

![Figure 8](image2.png) **Network corresponding to a boolean variable.**
Next, we create a carrier for each boolean variable, in the sense that carrier can only cover arcs in the network associated with the boolean variable. This can be done by setting the cost of all other arcs to infinity for that carrier. This guarantees that a flow in the network associated with a boolean variable stays on the lower or upper path. For the arcs in the network associated with a boolean variable, we set the costs to zero except for the shared arc which gets a cost of one. Repositioning costs are assumed to be equal to the original lane costs for every carrier.

Note that each clause arc appears in exactly three boolean variable paths (either an upper or a lower path). A clause is satisfied if and only if the corresponding clause arc is covered by at least one of the three carriers associated with the boolean variables. Note that by setting $r_{ij}$ equal to one for all the clause arcs, we ensure that each clause in $U$ is satisfied. The constructed instance of $3^+ - MCLCP$ is always feasible since by sending a unit flow on all upper and lower paths corresponding to the boolean variables we will satisfy every clause. In that case, the cost for each carrier is equal to two, since the cost of the shared arc is equal to 1, and thus the total cost of the carriers is equal to $2 \times |n|$. Hence, an instance of 3-SAT is feasible if and only if the corresponding $3^+ - MCLCP$ instance has an optimal solution with a value of $|n|$. Any feasible solution with cost strictly greater than $|n|$ will have at least one boolean variable with a flow of one on both the upper and lower path, which implies that the 3-SAT instance is not feasible.

We conclude that $MCLCP$ with three or more carriers and non-identical cost structures is NP-Hard. □