Inventory Systems Overview

1. Introduction
2. Independent Demand Systems
3. Dependent Demand Systems

Inventory Systems Overview

- Introduction
- Independent Demand Systems
- Dependent Demand Systems

Inventory

- Material Being Held in Storage
- Buffer Between and Decoupling of Two Subsequent Processes
**Historical Perspective on Inventory**

- **Before 20th Century Desirable Wealth**
  - Maximized
- **Early 20th Century Wasteful**
  - Minimized
- **Currently an Expensive But Necessary Evil**
  - Tradeoff or Balance

**Inventory Objective and Cost**

- **Provide Some Specified Service Level at Minimum Cost**
- **Common Service Level Definition** = Proportion of Long Range Demand Delivered From Inventory (P2 or $\beta$)
- **Cost** = 25% of Value per Year and Rising

**Customer Service Level Definitions**

- **Cycle service level $\alpha$ (P1)** long range probability of no stock-out (count)
- **Fill rate $\beta$ (P2)** long range fraction of demand delivered from inventory (size)
- **$\gamma$ (P3)** ratio of cumulative unsatisfied demand over total average demand (size and time)
- **ready rate alternative definition**

**Customer Service Levels Example Demand Graph**

[Graph showing demand over time with peaks and valleys]
Customer Service Levels

Example Computations

\[ d = \frac{\sum_{i=1}^{T} d_i}{T} = \frac{160}{40} = 4 \]

\[ \alpha = 1 - \frac{2}{4} = 0.50 \]

\[ \beta = 1 - \frac{20 + 10}{160} = 0.81 \]

\[ \gamma = 1 - \frac{(5 + 10 + 15 + 20 + 5 + 10)}{40} = 0.59 \]

Inventory Performance

- **Stock Turnover (Turns per Year)**
  - Cost of Annual Sales Divided by Average Value of Inventory

- **Average Time in Inventory (Years)**
  - Inverse of Stock Turnover

Inventory Justification

- **Economies of Scale**
  - Production
  - Transportation

- **Response Time Constraints**
  - Emergency
  - Seasonal

- **Required Aging Processes**

- **Hedging Against Price Changes**

- **Amplified by Uncertainty**
  - Demand
  - Production yield and lead time
  - Transportation time
  - Sales price and exchange rates
Inventory Tradeoffs

- Tradeoff with Transportation Costs and Schedules
- Tradeoff with Production or Purchasing Costs and Schedules
- Tradeoff with Lost Profit and Lost Sales

Direct Shipping Cost Tradeoff

- Plant
- Customer
- Expensive Delivery
- Cheap Linehaul
- Warehouse

Cross Docking Material Handling Tradeoff

Inventory Types

- Pipeline Inventory
- Cycle Inventory
- Safety Inventory
- Seasonal Inventory
- Aging Inventory
- Speculative Inventory
Inventory Materials

- Raw Materials
  - Incoming to Organization
- Work in Process
  - Internal to Organization
- Finished Goods
  - Outgoing from Organization

Inventory Questions

- What to Store
- Where to Store
- When to Place an Order
- How Much to Order

Inventory Costs

- Unit Cost or Unit Value ($/Unit)
- Reorder Cost ($/Order)
- Holding Cost ($/Unit-Year)
  - Holding cost rate ($/$-Year)
- Shortage Cost ($/Unit)
  - Backorder cost
  - Lost sales cost

Inventory Policy Classes

- Fixed Order Quantity Systems
  - Adjust order time
  - Small irregular demand
- Fixed Order Frequency Systems
  - Adjust order quantity
  - High or regular demand
- Direct Demand Satisfaction
  - Adjust quantity and time
  - Push or Pull
**Pull Inventory Policies**
- Each Stocking Point Considered Independent
- Replenishment Based on Local Conditions
- Precise Local Inventory Control
- Ignores Impact on Other Elements of the Supply Chain
- Retail Systems

**Push Inventory Systems**
- Stocking Point Viewed as Component in the Supply Chain
- Replenishment Based on Forecasted Requirements
- Central Inventory Control
- Inventory Levels are Set Collectively
- Depends Strongly on Forecast Quality
- Production Systems

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**Independent Demand Systems**
- Deterministic or Known Demand
- Stochastic or Uncertain Demand
**Pipeline Inventory**

- In-Transit or Aging & Curing Inventory
- Notation
  - \( PI = \) Pipeline inventory (units)
  - \( D = \) Total demand (units / year)
  - \( TT = \) Transit time (years)
  - \( PI = D \cdot TT \)

**Cycle Inventory**

- Transition Between Regular Input and Output Quantities (Batch Sizes)
- Notation
  - \( CI = \) Cycle inventory (units)
  - \( d = \) Periodic demand (units / period)
  - \( CT = \) Cycle time (years)
  - \( ct = \) Cycle time (periods)
  - \( CI = \frac{D \cdot CT}{2} = \frac{d \cdot ct}{2} \)

**Cycle Inventory Versus Replenishment Cycle Time**

\[ CI = \frac{d \cdot ct}{2} \]

**Seasonal Inventory**

- Reduced production capacity by inventory accumulation in low demand periods
- Notation
  - \( h_k = \) holding cost rate for period \( k \) ($/$.t)
  - \( l_k = \) inventory at end of period \( k \)
  - \( v = \) product value
  - \( SIC = \) seasonal inventory cost
**Seasonal Inventory With Constant Rates**

- **Inventory level over periods**:
  - Period 1: \( I_1 \)
  - Period 2: \( I_2 \)
  - Period 3: \( I_3 \)
  - Period 4: \( I_4 \)

**Seasonal Inventory Equations**

- **Constant demand and production rate per period**
  \[
  SIC = v \sum_{i=1}^{N} h_i (I_{i-1} + I_i) / 2
  \]

- **Equal periods and cyclical system**
  \[
  SIC = v \cdot h \sum_{i=1}^{N} I_i
  \]

**Repetitive Order Quantities With Instantaneous Resupply**

- **Total Cost**
  \[
  TC(Q) = IC + OC
  \]
  \[
  = \frac{hc \cdot Q}{2} + \frac{D \cdot oc}{Q}
  \]
  \[
  Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}}
  \]

- **Total Cost** (optimized)
  \[
  TC(Q^*) = \frac{hc}{2} \sqrt{\frac{2 \cdot D \cdot oc}{hc}} + \frac{D \cdot oc}{\sqrt{\frac{2 \cdot D \cdot oc}{hc}}}
  \]
  \[
  = \sqrt{2 \cdot D \cdot oc \cdot hc}
  \]

**Inventory Policies Comparison Example: Data**

- Data table with various inventory parameters such as purchase price, lead time, holding cost rate, shortage cost, ordering cost, demand rate, and mean demand.
Graph of Cost Tradeoffs for Known Demand

Inventory Policies Comparison: Known Demand (Instantaneous)

\[ Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}} \]
\[ Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100 \]
\[ TC(Q) = IC + OC \]
\[ TC = \frac{100 \cdot 0.2 \cdot 10}{2} + \frac{200}{100} = 200 \]
\[ R = d \cdot LT \]
\[ R = 200 \cdot 0 = 0 \]

Inventory Policies Comparison Example: Results

Optimal EOQ Derivation

\[ \frac{d(TC)}{dQ} = \frac{hc}{2} - \frac{D \cdot oc}{Q} \]
\[ \frac{d^2(TC)}{dQ^2} = \frac{2 \cdot D \cdot oc}{Q^3} > 0 \]
\[ \frac{d(TC)}{dQ} \bigg|_{Q \to 0} = -\frac{D \cdot oc}{Q^2} \]
\[ Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}} \]
Inventory Pattern for Batch Manufacturing

EOQ Formulas
\[ M = Q - d \cdot R = Q - d \left( \frac{Q}{p} \right) = Q \left( 1 - \frac{d}{p} \right) \]
\[ TC = IC + MC \]
\[ = \frac{HC \cdot M \cdot T}{2} + FC + VC \cdot Q \]
\[ = \frac{HC \cdot Q \cdot (1 - d \cdot p) \cdot Q}{2 \cdot d} + FC + VC \cdot Q \]
\[ = FC + VC \cdot Q + \frac{HC \cdot (p - d) \cdot Q^2}{2 \cdot p \cdot d} \]

EOQ Formulas continued
\[ tc = \frac{TC}{Q} = \frac{FC}{Q} + VC + \frac{HC \cdot (p - d)}{2 \cdot pd} \cdot Q \]
\[ d(tc) = \frac{FC}{dQ} = -\frac{HC \cdot (p - d)}{Q^2} + 2 \cdot pd \cdot Q = 0 \]
\[ Q^* = \sqrt{\frac{2 \cdot FC \cdot d}{HC \cdot (1 - \frac{d}{p})}} \]
\[ d^2(tc) = \frac{2 \cdot FC}{Q^3} + \frac{HC \cdot (p - d)}{2 \cdot pd} > 0 \]

Batch Manufacturing Costs and Quantities

- **Total Cost**
  \[ TC = IC + MC = FC + VC \cdot Q + \frac{HC \cdot (p - d)}{2 \cdot p \cdot d} \cdot Q^2 \]

- **Optimal Batch Size**
  \[ Q^* = \sqrt{\frac{2 \cdot FC \cdot d}{HC \cdot (1 - \frac{d}{p})}} \]

- **Optimal Total Cost**
  \[ TC^* = 2 \cdot FC + VC \cdot \sqrt{\frac{2 \cdot pd \cdot FC}{HC \cdot (p - d)}} \]
**Lead Time for Resupply Reorder Point**

- **Deterministic Demand (D)** and Demand rate (d)
- **Deterministic Lead Time (LT)**
- Reorder Point = Quantity On Hand when Order is Placed
  
  \[
  R = d \cdot LT
  \]
  
  \[
  dLT = d \cdot LT
  \]

**If Reorder Point Exceeds Maximum Inventory**

- When lead time exceeds replenishment cycle length
  \[
  LT > \frac{Q}{d}, \quad R > Q
  \]
- Two Equivalent Interpretations
  - Order when on-hand plus on-order inventory falls below reorder point
  - Order when on-hand inventory falls below reorder point modulo order quantity (or order quantity for exact multiple)

**Inventory Policies Comparison: Known Demand**

\[
Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}}
\]

\[
Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100
\]

\[
TC(Q) = IC + OC
\]

\[
TC = \frac{100 \cdot 0.2 \cdot 10}{2} + \frac{200 \cdot 50}{100} = 200
\]

\[
R = d \cdot LT
\]

\[
R = 200 \cdot 0.5 = 100
\]

**Inventory Policies Comparison Example: Results**
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Independent Demand Systems

- Deterministic or Known Demand
- Stochastic or Uncertain Demand

Basic Pull Inventory Policies

- Single Order Quantity
  - Equal Marginal Profit and Loss
  - Cyclical Order Quantities
  - EOQ (Infinite and finite production rate)

Single Order Quantity

- Single Ordering Opportunity
- Tradeoff between Shortage and Excess Costs
- "Newsboy" / Newsvendor Problem
Single Order Quantity Notation

- \( Q \) = purchased inventory
- \( D \) = actual demand
- \( f(x) \) = demand distribution
- \( F(x) \) = cumulative demand distribution
- \( \tilde{d} \) = expected demand
- \( p \) = sales price
- \( c \) = purchase price
- \( s \) = salvage value

Single Order Quantity Expected Cost

\[
G(Q) = c_s \int_{0}^{Q} (Q - x) f(x) dx + c_c \int_{Q}^{\infty} (x - Q) f(x) dx
\]

\[
\frac{dG(Q)}{dQ} = c_s \int_{0}^{Q} f(x) dx + c_c \int_{Q}^{\infty} -f(x) dx
\]

\[
c_s = p - c \quad \text{if } Q \leq D \quad \text{(marginal profit)}
\]

\[
c_c = c - s \quad \text{if } Q > D \quad \text{(marginal loss)}
\]

Single Order Quantity Optimal Inventory

\[
\frac{d^2G(Q)}{dQ^2} = (c_s + c_c) f(Q) \geq 0 \quad \forall Q
\]

\[
\frac{dG(Q)}{dQ} \bigg|_{Q=0} = c_c F(0) - c_s (1 - F(0)) = -c_s < 0
\]

\[
\frac{dG(Q^*)}{dQ} = (c_c + c_s) F(Q^*) - c_s = 0
\]

\[
F(Q^*) = \frac{c_s}{c_c + c_s}
\]

Single Order Quantity by Equilibrium Theory

\* At the optimal inventory level the expected profit of selling one more items equals the expected loss of one item of excess inventory

\[
c_s (1 - F(Q^*)) = c_c F(Q^*)
\]

\[
F(Q^*) = \frac{c_s}{c_c + c_s}
\]
**Optimal Inventory Level From Cumulative Frequency Lookup**

- Compute demand frequencies and sort by increasing demand
- Compute cumulative demand frequencies

\[ F(Q^*) = \frac{c_s}{c_s + c_c} \]

\[ Q^* = \min \left\{ x \mid F(x) \geq \frac{c_s}{c_c + c_s} \right\} \]

---

**Normally Distributed Demand**

- Normally Distributed Demand

- Probability

<table>
<thead>
<tr>
<th>Demand</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>μ</td>
<td>CD = 95 %</td>
</tr>
</tbody>
</table>

---

**Single Order Quantity Optimal Inventory From Distribution**

\[ z^* = N^{-1} \left( \frac{c_s}{c_s + c_c} \right) \]

\[ Q^* = \bar{d} + z^* \sigma_D \]

---

**Testing If Demand Distribution Is Normal**

- \( \chi^2 \) Goodness of Fit Test
- Coefficient of Variance < 0.5
- Probability of Negative Demand Less Than 2 %
- Probability of Demand More Than Mean Plus Two Standard Deviations Less Than 2 %
Testing If Demand Distribution Is Normal (Formulas)

\[
CV = \frac{\sigma}{\mu} \leq 0.5
\]

\[
P[x < 0] = F(0) \leq 0.02
\]

\[
P[x > \mu + 2\sigma] = 1 - F(\mu + 2\sigma) \leq 0.02
\]

Single Order Quantity Example: Data

Data = Actual Sales Plus Unmet Demand

Single Order Quantity Example: Frequency Distributions

\[
c_s = 75 - 25 = 50
\]

\[
c_e = 25 - 10 = 15
\]

\[
F(Q^*) = \frac{50}{50 + 15} = 0.77
\]

\[
Q^* = \min\{x \mid F(x) \geq 0.77\}
\]

\[
= 15
\]

Single Order Quantity Example: Histogram
Single Order Quantity Example: Optimal Inventory

\[ F = \frac{50}{65} = 0.77 \]

\[ 11.73 + 0.74 \times 4.74 = 15.2 \]

Basic Pull Inventory Policies

- **Single Order Quantity**
  - Equal Marginal Profit and Loss
- **Cyclical Order Quantities**
  - EOQ (Infinite and finite production rate)

Safety Stock Inventory Factor

- **Customer Service Measured by:**
  - Probability of Stockout (Type 1, \( \alpha \))
  - Fraction Delivered from Inventory (Type 2, \( \beta \))
- **Common Rule of Thumb:**
  - Safety Inventory Equals Time Length Multiplied by Demand (Linear Policy)
- **Lead Time:** Time From Placement to Arrival of Order

Demand During Lead Time and Stockout Probability (Type 1)
Reorder Point for Uncertain Demand (Sequential Decision)

- Sequentially Determine $Q$ then $R$
- Approximation
- $z$ Derived from Probability In Stock during Lead Time (Type 1)

$$Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}}$$

$$s_{dit} = s_d \sqrt{lt}$$

$$\sigma_{dit}^2 = \sum_{1}^{lt} \sigma_d^2$$

$$R^* = d \cdot lt + z \cdot s_{dit}$$

Average Inventory and Total Cost (No Stockout Cost)

$$AI = CI + SI = \frac{Q}{2} + z \cdot s_{dit}$$

$$TC = D \cdot oc + hc \cdot \frac{Q}{2} + hc \cdot z \cdot s_{dit}$$

Inventory Policies Comparison: Type 1 Service Level, $F(R) < \alpha$

$$Q^* = \sqrt{\frac{2 \cdot D \cdot oc}{hc}} \quad Q = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100$$

$$s_{dit} = s_d \sqrt{LT} \quad s_{dit} = \sqrt{0.5 \cdot 35.4} = 25$$

$$R^* = d \cdot LT + z \cdot s_{dit} \quad R = 200 \cdot 0.5 + 2.05 \cdot 25 = 151$$

$$TC = D \cdot oc + hc \cdot \frac{Q}{2} + hc \cdot z \cdot s_{dit}$$

$$TC = \left( \frac{100}{2} + 2.05 \cdot 25 \right) \cdot 0.2 \cdot 10 + \frac{200}{100} = 50 = 303$$

Inventory Policies Comparison Example: Results
**Expected Units Out of Stock:**

*Unit Normal Loss Function*

\[ n(R) = \int_{R}^{\infty} (x - R)f(x)dx \]

\[ L(z) = \int_{z}^{\infty} (t - z)\phi(t)dt \]

\[ n(R) = s_{dlt}L\left(\frac{R - d \cdot LT}{s_{dlt}}\right) = s_{dlt} \cdot L(z) \]

---

**Unit Loss Function**

---

**Total Cost for Uncertain Demand (Known Shortage Cost)**

\[ TC(Q) = D \frac{oc}{Q} + hc \frac{Q}{2} + hc \cdot z \cdot s_{dlt} + \]

\[ D \frac{sc \cdot s_{dlt} \cdot L(z)}{Q} \]

\[ TC(Q,R) = D \frac{oc + sc \cdot s_{dlt} \cdot L(z)}{Q} + \]

\[ hc \left(\frac{Q}{2} + R - d \cdot LT\right) \]

---

**Inventory Policy Comparison:**

*Known Stockout Costs*

\[ TC(Q,R) = D \frac{oc + sc \cdot s_{dlt} \cdot L(z)}{Q} + \]

\[ hc \left(\frac{Q}{2} + R - d \cdot LT\right) \]

\[ TC = \frac{200}{100} \left(50 + 25 \cdot 25 \cdot 0.0073\right) + \]

\[ = 10 \cdot 0.2 \left(\frac{100}{2} + 151 - 200 \cdot 0.5\right) = 312 \]
**Inventory Policies Comparison Example: Results**

**Optimal Q and R (Iterative Method)**

- Start with Q Determined by EOQ
- Determine Q and R Iteratively until No Further Change

---

**Iterative Formulas for Determination of Q and R**

\[
Q = \sqrt{\frac{2D \left( oc + sc \cdot n(R) \right)}{hc}} = \sqrt{\frac{2D \left( oc + sc \cdot s_{\text{dist}} \cdot L(z) \right)}{hc}}
\]

\[
P_{\text{onstock}} = F(R) = \int_{0}^{R} f(x)dx = 1 - \frac{Q \cdot hc}{sc \cdot D}
\]

---

**Type 1 Service Level Iterations**
Inventory Policies Comparison
Example: Results

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Type</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Deterministic Demand</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>3 Poisson Demand</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>4 Shortage Cost (Seq)</td>
<td>51</td>
<td>101</td>
<td>101</td>
<td>302</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>5 Shortage Cost ( Seq)</td>
<td>111</td>
<td>43</td>
<td>99</td>
<td>309</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>6 Type 2 Service (Seq)</td>
<td>114</td>
<td>24</td>
<td>8</td>
<td>249</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>7 Type 2 Service (Seq)</td>
<td>119</td>
<td>8</td>
<td>99</td>
<td>241</td>
<td>50</td>
<td>200</td>
</tr>
</tbody>
</table>

Service Level Type 2 or Fill Rate

- Ratio of Long Range Expected Total Units Delivered out of Inventory Over Total Demand
- Complement of Expected Total Units Out of Stock Over Total Demand D
- Expected Units Out of Stock per Cycle \( n(R) \)

\[
SL = 1 - \frac{(D/Q)n(R)}{D} = 1 - \frac{n(R)}{Q}
\]

Inventory Comparison Cost: Type 2 Service (Sequential)

\[
AI = CI + SI = \frac{Q}{2} + z \cdot s_{dlt}
\]

\[
TC = D \cdot \frac{oc + hc}{Q} + D \cdot \frac{hc \cdot z \cdot s_{dlt}}{Q}
\]

\[
TC = \left(100 + 1.02 \cdot 25\right) \cdot 0.2 \cdot 10 + \frac{200}{100} \cdot 50 = 251
\]
Inventory Policies Comparison
Example: Results

Optimal Q and R (Iterative Method) Type 2 Service

- Start with Q Determined by EOQ
- Determine Q and R Iteratively until No Further Change

Iterative Formulas for Q and R Type 2 Service

\[
Q_i = \sqrt{\frac{2 \cdot oc \cdot D}{hc}}
\]

\[
Q = \frac{n(R)}{1 - F(R)} + \sqrt{\frac{2 \cdot oc \cdot D}{hc} + \left(\frac{n(R)}{1 - F(R)}\right)^2}
\]

\[
L(z) = \frac{Q(1-SL)}{s_{dlt}}
\]

\[
R = d \cdot LT + z \cdot s_{dlt}
\]
Inventory Comparison Cost: Type 2 Service (Iterative)

\[ AI = CI + SI = \frac{Q}{2} + z \cdot s_{dlt} \]

\[ TC = \frac{D}{Q} \cdot oc + h_c \cdot \frac{Q}{2} + h_c \cdot z \cdot s_{dlt} \]

\[ TC = \left( \frac{114}{2} + 0.95 \cdot 25 \right) \cdot 0.2 \cdot 10 + \frac{200}{114} \cdot 50 = 249 \]

Demand and Lead Time Uncertainty

\[ s_{dlt} = \sqrt{lt \cdot Var_{dlt} + d^2 \cdot Var_{lt}} \]

\[ CV_d = \frac{\sqrt{Var_{dlt}}}{d} \]

\[ Var_{dlt} = (CV_d \cdot d)^2 \]

\[ SI = z \cdot \sqrt{lt \cdot CV_d^2 + Var_{lt} \cdot d} \]

Standard Deviation of the Demand During the Lead Time

\[ s_{dlt} = \sqrt{0.5 \cdot 35.4^2 + 200^2 \cdot 0.125^2} = 35.4 \]
### Inventory Policies Comparison Example: Results

TC = \( \frac{D}{Q} \cdot \alpha c + \frac{Q}{2} + \alpha c \cdot z \cdot s_{\text{dr}} \)

\[
TC = \left( \frac{119}{2} + 1.11 \cdot 35.4 \right) \cdot 0.2 \cdot 10 + \frac{200}{119} = 282
\]

---

### Inventory Policies Comparison Exercise: Data

---

### Inventory Systems Overview

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Periodic Review Inventory Systems

- Review at Fixed Intervals & Schedule
- Joint Ordering, Transportation, and Production Economies
- Simple Manual Implementation
- Reduced Administration Cost
- Slightly Higher Inventory Cost Compared to Continuous Review

Periodic Review (s,S) Policies

- Definitions
  - i = on hand inventory
  - s = ordering trigger level
  - S = order up to level
- Policy
  \[
  \begin{align*}
  &\text{if } i \leq s \quad \text{order } S - i \\
  &i > s \quad \text{do not order}
  \end{align*}
  \]

Periodic Review Inventory Policies

- Optimal Inventory Policies have Very Complex Derivations
- Simple Approximations have Acceptable Accuracy
  \[
  s \approx R \\
  S \approx R + Q
  \]

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**Distribution Requirements Planning (DRP)**

- Integrated Supply Scheduling Throughout Supply Chain
- MRP for Supply Chain
- Forecasted Demand for Products Delivered to Customers
- Bill of Materials (BOM) \([a_{qp}]\)

**Material Requirements Planning (MRP)**

- Precise Timing of Material Flows to meet Production Planning
- Avoid Carrying Items as Work-In-Process (WIP) Inventory
- Scheduling of High-Value, Custom-Made Items
- When Demand is Reasonably Well Known

**DRP Advantages**

- Requires uniform and consistent data collection
- Advance planning for future demands
- New and irregular demands
- Uniform product shortage allocation
- Incorporate other planned demands
- Balance facility utilization
- Control and balance obsolescence

**Clock MRP Example: K36 Demand**

\[ Q = 350 \]
\[ LT = 2 \]
Clock MRP Example:

K36 Schedule

Q=350
LT=2

Clock MRP Example:

M21 Demand

Q=600
LT=1

Clock MRP Example:

M21 Schedule

Q=600
LT=1

Clock MRP Example:

R1063 Demand

Q=1000, LT=2, SS=250
Clock MRP Example: R1063 Schedule

Q = 1000
LT = 2
SS = 250

Bill of Material Matrix

* # Immediate Components q Required to Produce 1 Unit of Product p = A_{qp}
  * Rows = Components q
  * Columns = Products p

\[
A_{qp} = \begin{bmatrix}
\end{bmatrix}
\]

DRP (MRP) Inventory Equation

* Notation
  * OHI = On-Hand Inventory
  * d = gross requirements (outflow)
  * s = scheduled receipts (inflow)
  * r = planned production receipts (inflow)

\[
OHI_{p,t} = OHI_{p,t-1} - d_{p,t} + s_{p,t} + r_{p,t}
\]

\[
d_{q,t-t_p} = a_{qp}f_{p,t}
\]

DRP Inventory Transition Diagram
**DRP (MRP) Characteristics**

- Strongly Dependent on Quality and Stability of Final Product Forecast
  - Forecast changes/errors create WIP
- Uncapacitated (No Manufacturing or Transportation Resources)
  - Basic equations are deterministic
- Can Accommodate Irregular Demand

**DRP (MRP) Implementation Details**

- Production Quantity
  - First cut heuristic approximation by EOQ
  - More Sophisticated Algorithms
    - Silver-Meal heuristic
    - Wagner-Whitin (optimal dynamic programming)
- Safety Inventory
  - Newsboy Methodology

**Supply Chain and Warehousing Trends**

Inventory Facilities Handling

Transactions Transportation Information

**Inventory References**